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Feature extraction using indicators of cyclostationarity for a mechanical diagnosis purpose

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This paper focuses on features extraction based on cyclostationarity for diagnosis purpose. The objective is to derive new indicators for the diagnosis of rotating machinery. These indicators are based on cyclic higher order statistics and generalize some existing ones for the second order statistics. A comprehensive methodology is proposed for obtaining a diagnosis objective; a crucial example is presented, relating to vibration signals of a gearbox. Results demonstrate the effectiveness of these features to detect spalling in gearbox.

1. Introduction

During the last decade, vibration analysis for machine diagnosis has taken advantage from significant advances in different domains. They are related on one hand to the development of modern analysis methods, and on the other hand to a better physical knowledge of the fault symptoms on measured signals. Several methods, complementary to the Fourier technique, were thus proposed such as cepstral analysis [16, 1], parametric modeling techniques [2] as well as time-frequency methods (spectrogram, Wigner–Ville, wavelets,..) [8, 13]. These techniques showed, when used with a good expertise, that they achieve better fault detection. Recently, two important classes of methods, originated from telecommunications, were studied and proposed for vibration signal analysis. The first class supposes that signals are cyclostationary: it analyses more deeply vibration signals exhibiting cyclostationarity by nature [9, 12]. The second class of methods uses higher-order statistics and supposes that analyzed signals are not gaussian any more [3]. This happens when the signals are generated by nonlinear systems. Different features were extracted based respectively on nonlinearity such as the bispectrum [5], the bicoherence [7] and on cyclostationarity such as the spectral correlation [17] and the spectral coherence. Our purpose in this paper is to add to the list of these successful features some new ones based on both cyclostationarity and higher-order statistics. Therefore, concise and global descriptors up to the fourth-order cyclostationarity are proposed. The idea generalizes the previous work of [14] and [18]. Diagnosis being our objective, we shall ask the following questions:

- Which motivation exists behind these descriptors? Which advantages do they have in comparison to other existing features?
- Which diagnosis protocol is involved with these descriptors?

In Sec. 2, the main definitions of cyclostationarity are recalled. In Sec. 3, we try to answer the first question. This leads to discuss some properties of the proposed descriptors. Section 4 gives a few guidelines that answer the second question. Finally, Sec. 5 is dedicated to illustrate a case which is of great practical interest, namely gear signals, for which basic descriptor analyses are provided.

2. Definition of cyclostationarity

In general, a cyclostationary process is a stochastic process that exhibits some hidden periodicities in its structure [10]. Formally, a stochastic process is said to be *strict-sense cyclostationary* with *cycle T* if its joint probability density function $p_x(x_1,\dots,x_n;\ t_1,\dots t_n)$ is periodic in t with period T, i.e. if

$$p_x(x_1,\dots,x_n;\ t_1,\dots t_n) = p_x(x_1,\dots,x_n;\ t_1+T,\dots t_n+T). \tag{1}$$

The most basic cyclostationary signal is that which is cyclostationary at the first order, i.e. whose first-order moment or expected value $m_{1x}(t)$ is periodic with period T:

$$m_{1x}(t) = E\{x(t)\} = m_{1x}(t+T). \tag{2}$$

A more general cyclostationary signal is that which is cyclostationary at the second order, i.e. whose second-order moments are periodic. In particular, the autocorrelation function $R_{2x}(t_1,t_2)$ is a periodic function with period T:

$$R_{2x}(t_1,t_2) = E\{x^*(t_1)x(t_2)\} = R_{2x}(t_1+T,t_2+T). \tag{3}$$

Finally, an n-th order cyclostationary signal is that whose n-th order moments are periodic. In particular, if all moments up to infinity are periodic, then the signal is strict-sense cyclostationary according to Eq.(1).

In rotating machinery, the statistics of their vibration signals are periodic because of different cyclic mechanical phenomena. For example, the cyclic modifications in the geometry of the machine, the cyclic changes in torques, rotations of the anisotropic components, etc. produce periodic modulations in the statistics of vibration signals. Depending on the structure of the physical machines and phenomena which take place there, the signals resulting can exhibit various types of cyclostationarity.

3. Indicators of cyclostationarity

3.1. State of the art

The problem of defining an appropriate measure of degree of cyclostationarity was first posed in 1975 in relation to the estimation of the least square errors of a cyclostationary signal. This problem was discussed again by Zivanovic and Gardner in 1991 [18]. In this paper, several criteria were discussed. The most frequently used criterion consists in considering the most similar stationary signal having the same spectrum as the cyclostationary signal to be studied. The degree of cyclostationarity (DCS) is defined as follows:

$$\mathrm{DCS} = \frac{\sum\limits_{\alpha\neq 0}\int_{-\infty}^{\infty}|S_{2x}^{\alpha}(f)|^2\mathrm{d}f}{\int_{-\infty}^{\infty}|S_{2x}^{0}(f)|^2\mathrm{d}f} = \frac{\sum\limits_{\alpha\neq 0}\int_{-\infty}^{\infty}|R_{2x}^{\alpha}(\tau)|^2\mathrm{d}\tau}{\int_{-\infty}^{\infty}|R_{2x}^{0}(\tau)|^2\mathrm{d}\tau}, \tag{4}$$

where $S_{2x}^{\alpha}(f)$ is the cyclic spectrum of moments at the cyclic frequency α and $R_{2x}^{\alpha}(\tau)$ its temporal dual. $S_{2x}^{0}(f)$ is the spectral density and $R_{2x}^{0}(\tau)$ the autocorrelation function. The relation between these statistical parameters and their mathematical expressions can be found in [15].

The authors decomposed the DCS for each cyclic frequency α defined by:

$$\mathrm{DCS}^{\alpha} = \frac{\int_{-\infty}^{\infty}|S_{2x}^{\alpha}(f)|^2\mathrm{d}f}{\int_{-\infty}^{\infty}|S_{2x}^{0}(f)|^2\mathrm{d}f} = \frac{\int_{-\infty}^{\infty}|R_{2x}^{\alpha}(\tau)|^2\mathrm{d}\tau}{\int_{-\infty}^{\infty}|R_{2x}^{0}(\tau)|^2\mathrm{d}\tau} \tag{5}$$

with $\mathrm{DSC} = \sum\limits_{\alpha\neq 0}\mathrm{DSC}^{\alpha}$.

Another degree of cyclostationarity $\delta(\alpha)$ was proposed by Hurd in 1991 [11]. It is expressed under the following form:

$$|\delta_x(\alpha)|^2 = \frac{\left|\int_{-\infty}^{+\infty} S_{2x}^{\alpha}(f)\mathrm{d}f\right|^2}{\left|\int_{-\infty}^{+\infty} S_{2x}^{0}(f)\mathrm{d}f\right|^2}. \tag{6}$$

This Eq. (6), although it is very simple, it is very significant to consider. This formalization introduced by Hurd, permits us to generalize the degree of cyclostationarity to the higher orders.

The two indicators described before are only dedicated for second order. These limitations incited Dandawate and Giannakis in 1994 [6], to develop statistical tests for the presence of cyclostationarity for real processes. These tests are the first in their kind to detect the presence of n-th-order cyclostationarity, in the contrary to the work of Hurd and Gardner, which is limited to second order.

In the literature, the only application of degree of cyclostationarity is proposed by Prieur and D'Urso in 1995 [14]. In this study, the degrees of cyclostationarity used are $\delta_x(\alpha)$ and the square root of DCS$^{\alpha}$. Simulations have shown that the two descriptors have similar behavior. Then, they were estimated for a fatigue test for gears. The visual expertise on the gear train teeth was conducted for over 15 days. The high-speed wheel remained intact during the test, whereas, the visual expertise on the low-speed wheel showed an increasing spalling fault during the measurements. The indices relating to the high-speed wheel confirmed this result because they did not change significantly, while those associated to the low-speed wheel increased very clearly. This increase in the indices is related to the spalling increase in the low-speed wheel. In conclusion, these degrees of cyclostationarity provided a relevant description of the degradation state of gear faults.

3.2. Motivation

In the objective of realizing an industrial tool for diagnostic, the descriptors must respect several constraints like simplicity in the implementation and interpretation of the results.

Our indicators benefit from many advantages:

- They are monotonic and increasing functions of the degree of n-th order cyclostationarity.
- They are theoretically zero if the process is stationary.
- They are normalized by the energy of the signal to be without any dimension.
- They generalize the well-known standardized cumulants, i.e. the classical RMS (Root Mean Square), the skewness and the kurtosis, by giving them a 'cyclic' counterpart.

3.3. Definitions

Consider the cyclostationary signal $x(t)$; defined by its first-order moment, its second, third and fourth order cumulants respectively as $m_{1x}(t)$, $c_{2x}(t,\tau)$, $c_{3x}(t,\tau_1,\tau_2)$ and $c_{4x}(t,\tau_1,\tau_2,\tau_3)$. The relation between these statistical parameters and their mathematical expressions can be found in [15].

The cyclic moment M_{1x}^{α} is defined as the Fourier coefficients of the first-order moment $m_{1x}(t)$. Similarly, the cyclic cumulants $C_{nx}^{\alpha}(\boldsymbol{\tau})$ are the Fourier coefficients of n-th order cumulants with respect to the variable t where $\boldsymbol{\tau} = (\tau_1, \tau_2, ..., \tau_{n-1})$. They summarize all the spectral information of a cyclostationary signal according to the following equations:

$$\begin{aligned} C_{2x}^{\alpha}(0) &= \int S_{2x}^{\alpha}(f)\mathrm{d}f, \\ C_{3x}^{\alpha}(0,0) &= \int S_{3x}^{\alpha}(f_1,f_2)\mathrm{d}f_1\mathrm{d}f_2, \\ C_{4x}^{\alpha}(0,0,0) &= \int S_{4x}^{\alpha}(f_1,f_2,f_3)\mathrm{d}f_1\mathrm{d}f_2\mathrm{d}f_3, \end{aligned} \tag{7}$$

where $C_{nx}^{\alpha}(\boldsymbol{\tau})$ represents the Fourier coefficients of $C_{nx}(t,\boldsymbol{\tau})$ with respect to t. $S_{nx}^{\alpha}(\mathbf{f})$ with $\mathbf{f} = (f_1, f_2, ..., f_{n-1})$ is the cyclic multispectrum of $x(t)$ at the cyclic frequency α, i.e. the Fourier transform of $C_{nx}^{\alpha}(\boldsymbol{\tau})$ with respect to $\boldsymbol{\tau}$.

The simplified descriptors from the first to the fourth-order are defined, as follows:

$$\begin{aligned} I_{1x} &= \sum_{\alpha\neq 0} |m_{1x}^{\alpha}|^2, \\ I_{2x} &= \sum_{\alpha\neq 0} |C_{2x}^{\alpha}(0)|^2, \\ I_{3x} &= \sum_{\alpha\neq 0} |C_{3x}^{\alpha}(0,0)|^2, \\ I_{4x} &= \sum_{\alpha\neq 0} |C_{4x}^{\alpha}(0,0,0)|. \end{aligned} \tag{8}$$

After an appropriate normalization, the final form of descriptors is given by the following equations:

$$\begin{aligned} I_{1x}^{n} &= \frac{I_{1x}}{|c_{2x}^{0}(0)|}, \\ I_{2x}^{n} &= \frac{I_{2x}}{|c_{2x}^{0}(0)|^2}, \\ I_{3x}^{n} &= \frac{I_{3x}}{|c_{2x}^{0}(0)|^3}, \\ I_{4x}^{n} &= \frac{I_{4x}}{|c_{2x}^{0}(0)|^4}, \end{aligned} \tag{9}$$

I_{3x}^{n} and I_{4x}^{n} are the measure of cyclic asymmetry and cyclic kurtosis.

In summary, (9) is simplified as follows:

$$I_{ix}^{n} = \frac{\sum_{\alpha\neq 0} |P_{ix}^{\alpha}|^2}{c_{2x}^{0}(0)^i} \tag{10}$$

where $P_{1x}^{\alpha} = m_{1x}^{\alpha}$ and $P_{ix}^{\alpha} = C_{ix}^{\alpha}(\mathbf{0})$ for $i = 2,3,4$.

The estimation techniques are not the objective of this paper. However, the estimated indicators $\hat{I}_{1x}^{n}$, $\hat{I}_{2x}^{n}$, $\hat{I}_{3x}^{n}$ and $\hat{I}_{4x}^{n}$ are used in Sec. 5. The statistical properties of these descriptors are not considered in this paper. Their statistical properties (bias and variance) are capable to define useful thresholds for statistical tests. Their computations are very complicated and the mathematical development will be the object of a future journal paper. However, in our application, thresholds are used based on these calculations.

4. Diagnosis protocol

The objective of this paragraph is to propose an application of the mentioned indicators. In this application, the diagnostic of several mechanical systems is established, which are consisted of two or (several) classes ϖ_1 and ϖ_2, characterized by a set of specific cyclic frequencies. The main purpose is to monitor the evolution of these indicators at each class; in order to detect any abnormal change.

In the general case of rotating machinery, consider for example $\varpi_1 = \{k/T_{1c}\}$ and $\varpi_2 = \{k/T_{2c}\}$ two sets of harmonically related cyclic frequencies of finite cardinal $2K+1$ (it is the harmonic number in the frequency band of interest). T_{ic} is the fundamental period of the class i. Suppose also a non-empty intersection between the cyclic frequencies of the two classes defined by $\varpi_1 \cap \varpi_2 = \{N_1 k/T_{1c}\}$.

The diagnosis protocol is resumed in Table 1 where CSi characterizes the cyclostationarity of order $i = 1 \ldots 4$ and $\varpi_1 - \varpi_1 \cap \varpi_2$ indicates the set of cyclic frequencies which belong to ϖ_1, while removing the common cyclic frequencies to ϖ_1 and ϖ_2:

Table 1. Diagnosis Protocol

	Set 1	Set 2
CS1	$I_{1x}^n(\varpi_1 - \varpi_1 \cap \varpi_2)$	$I_{1x}^n(\varpi_2 - \varpi_1 \cap \varpi_2)$
CS2	$I_{2x}^n(\varpi_1 - \varpi_1 \cap \varpi_2)$	$I_{2x}^n(\varpi_2 - \varpi_1 \cap \varpi_2)$
CS3	$I_{3x}^n(\varpi_1 - \varpi_1 \cap \varpi_2)$	$I_{3x}^n(\varpi_2 - \varpi_1 \cap \varpi_2)$
CS4	$I_{4x}^n(\varpi_1 - \varpi_1 \cap \varpi_2)$	$I_{4x}^n(\varpi_2 - \varpi_1 \cap \varpi_2)$

The diagnosis protocol is described as the following: for set 1, the four descriptors are calculated for the specific cyclic frequencies which are exclusive for this set. The computation occurs on real-time, giving an evolution curve for each descriptor. Idem for set 2.

An interesting application is related to characterizing the cyclostationarity for gears with two wheels (their rotation frequency are respectively f_{r_1} and f_{r_2}). It is important to recall that gear vibration signals are demonstrated to be cyclostationary at higher orders [15]. More precisely, it has been shown that:

- the geometric errors induce a periodic part in gear signals and consist the base of first order cyclostationarity,
- the contact phenomena induce transitory phenomena at high frequencies and higher order cyclostationarity,
- the speed variations contribute to the higher order cyclostationarity if and only if these variations are cyclostationary processes at these orders.

Based on these conclusions, a monitor protocol for wheel 1 and 2 can be used, based on cyclostationary indicators. In this case, the set of cyclic frequencies is different for the two wheels. For the wheel 1 and 2, the two classes of the cyclic frequencies and the relations between the two classes can be resumed by:,

$$\begin{aligned} \varpi_1 &= \left\{0, \frac{1}{f_{r_1}}, \frac{2}{f_{r_1}}, \ldots, \frac{n}{f_{r_1}}\right\}, \\ \varpi_2 &= \left\{0, \frac{1}{f_{r_2}}, \frac{2}{f_{r_2}}, \ldots, \frac{n}{f_{r_2}}\right\}, \\ \varpi_1 \cap \varpi_2 &= \left\{0, \frac{N_1}{f_{r_1}}, \ldots, n\frac{N_1}{f_{r_1}}\right\}, \end{aligned} \tag{11}$$

where N_1 is the tooth number of the wheel 1.

It results from the Eq. (11), that the set of cyclic frequencies $\varpi_1 - \varpi_1 \cap \varpi_2$ characterizing exclusively the wheel 1, does not contain neither the gear mesh frequency $f_e = N_1/f_{r1}$ nor any one of its harmonics $nf_e = nN_1/f_{r1}$. Idem for the wheel 2.

5. Application to gear signals

The system under examination is a power circulating gear-testing machine. It is composed of two single-stage gear units mounted back to back. Both units contain a pair of spur gears. The first pair under study has an equal number of 20 teeth, whereas the second has a ratio of 40/40. The speed rotation is of 1000 rev/min, so, a rotation frequency of 16,67 Hz. The mesh frequency f_e is equal to 330 Hz. Data were collected with a piezoelectric accelerometer with a resonance band localized at 20 kHz. A digital card PCI6052 digitizes vibration signals issued with a sampling frequency 80 kHz, the sample size is about 160000 (which corresponds to 2s).

Tests concerned the progressive development of spalling on localized teeth. The campaign was conducted over two weeks. The last day, the spalling was in an advanced stage very close to the rupture of two teeth. Each day, the test bench was stopped to make the visual inspection of teeth in order to correlate with the acquired signals. 256 states were gathered during the measurement campaign. Each state corresponds to a measured signal during the measurement campaign. At the beginning of the campaign, measurements were taken each hour whereas at the end, they were taken each half an hour. During the weekend, the test bench was stopped, and for some days, measurements were conducted only for some period of time. These signals were angular re-sampled by using a top reference and interpolation techniques. The angular re-sampling is identical by respect to the two wheels because their teeth number is identical ($f_{r1} = f_{r2} = 16.67$ Hz). By consequence, no distinction can be possible between the two wheels.

The inspection of the spectrum of a vibration signal showed that it was essentially discrete below 16 kHz. The discrete part corresponds to the rotation frequency, mesh frequency and its harmonics.

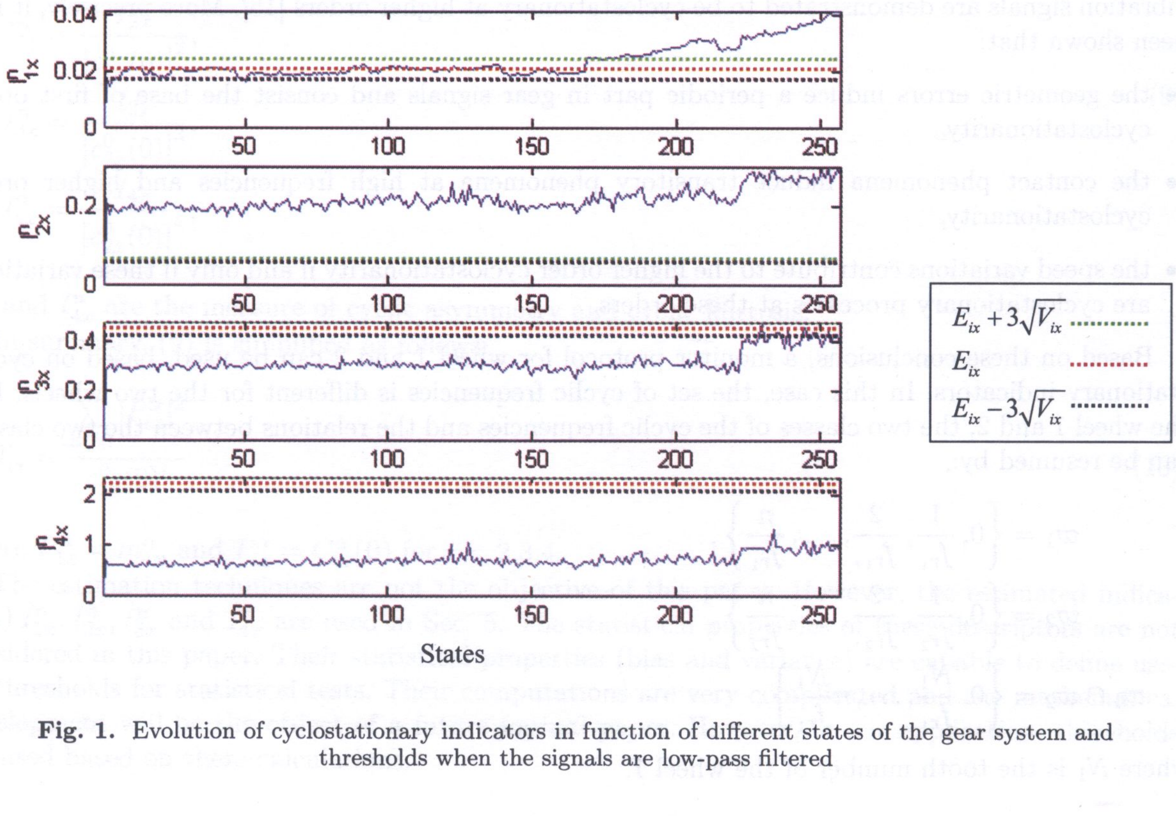

Fig. 1. Evolution of cyclostationary indicators in function of different states of the gear system and thresholds when the signals are low-pass filtered

Figure 1 illustrates the four estimated indicators $\hat{I}_{1x}^{n}, \hat{I}_{2x}^{n}, \hat{I}_{3x}^{n}$ and $\hat{I}_{4x}^{n}$ when all signals are low-pass filtered (the cutting frequency is equal to 16 kHz). The CS1 indicator clearly increases at the end of the campaign, this being a classical result conforming to those obtained in [4]. For the other three indicators, a little increase of cyclostationarity can be observed from the state 232 which corresponds to the last day of acquisition. We can note that the amplitudes of the indicators increase with the orders.

In order to evaluate the indicators properties, we calculated their bias E_{ix} and variance V_{ix} for $i = 1..4$. The thresholds corresponding to $E_{ix} + 3\sqrt{V_{ix}}$, E_{ix} and $E_{ix} - 3\sqrt{V_{ix}}$, are exposed in the same figure. Several important conclusions can be deduced. When the signals are low-pass filtered, the first order indicator presents an increasing cyclostationarity along the evolution of the measurements. Idem for order 2. However, the third and fourth order indicator do not exhibit any cyclostationarity, the threshold is always higher than the values of the indicators.

Figure 2 shows the evolution of the four indicators when the signals are high-pass filtered. As it was predicted from the inspection of the spectrum, the synchronous mean amplitude decreases by comparison with Fig. 1. In the contrary, $\hat{I}_{2x}^{n}$, $\hat{I}_{3x}^{n}$ and $\hat{I}_{4x}^{n}$ increase in a spectacular way. This result is very clear on the 174-th state, which corresponds to the 8-th day, i.e. 3 days before the end of measurement. Furthermore, the cyclostationarity is visible at all orders. To be able to confirm this result, we calculate also the thresholds corresponding to $E_{ix} + 3\sqrt{V_{ix}}$, E_{ix} and $E_{ix} - 3\sqrt{V_{ix}}$, $i = 2..4$. Intentionally, we did not put a threshold for the first order indicator because we already know that this indicator will not show any cyclostationarity after the high-pass filtering.

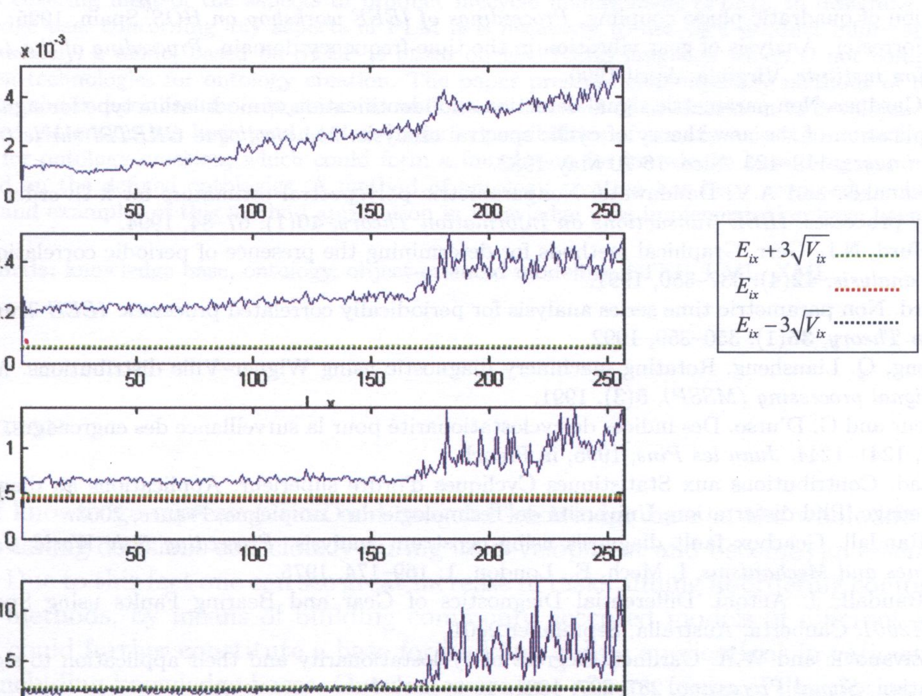

Fig. 2. Evolution of cyclostationary indicators in function of different states of the gear system and thresholds when the signals are high-pass filtered

6. Conclusion

It has been suggested in precursory works that inherent periodicities in the kinematics of rotating machinery are prone to make their vibration signals cyclostationary for different orders. The objective in the present paper was to exploit cyclostationarity in vibration signals to extract simple features, compact and easy in real-time implementation for diagnosis purposes. The proposed indicators are expressed in terms of cumulants and are normalized. They are therefore useful for several applications, and in particular for diagnostics. By consequent, a diagnosis protocol was designed in general to enable their application to any mechanical systems. To illustrate the benefit of such indicators, an application was detailed in this paper, where the cyclostationary indicators were used to monitor a mechanical system. First results are promising and encourage further use of the proposed indicators in future work and in any classification schema like neural networks.

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