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Optimisation of composite shapes with the help of genetic algorithms

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During the manufacturing process of multilayered fibre-reinforced composites with variable fibre orientations, residual stresses build up due to the directional expansion of the single unidirectionally reinforced layers. Dependent on the laminate lay-up, these inhomogeneous residual stresses, which are caused by thermal effects, moisture absorption and chemical shrinkage, can lead to large multistable out-of-plane deformations. Instead of avoiding these laminate's curvatures, they can be advantageously used for technical applications following the near-net-shape technology. However, due to the effect that the laminate curvature depends on huge amount of different parameters such as anisotropic, hygroscopic and thermomechanical material properties, fibre orientations and ply thickness of each single layer as well as technological processing parameters, a search in a multi-dimensional search area is necessary. In order to solve such a task, Genetic Algorithms in combination with a fitness function based on a nonlinear semi-analytical calculation model for the laminate shape prediction have been applied and described in the paper. Using this approach, one can purposefully adapt the laminate lay-up dependent on the loading and process parameters.

1. INTRODUCTION

In multilayered fibre-reinforced composites with variable fibre orientations, the directional expansion of the single unidirectional reinforced (UD) layers due to thermal effects, moisture absorption and chemical shrinkage leads to a discontinous residual stress field over the laminate thickness. In case of unsymmetric laminate plates, these residual stresses can purposefully be adapted dependent on the manufacturing process in order to realize either slightly or highly curved laminates with defined multistable out-of-plane deformations. For the adjustment of laminate curvatures to technical requirements, new optimization methods using Genetic Algorithms have been developed, that can efficiently be applied to find an optimal laminate lay-up dependent on material properties, loading conditions and process parameters. The mathematical description of the large out-of-plane deformation of multilayered plates by a nonlinear theory serves as the basis for the function to be optimized (fitness function). Some papers deal with the calculation of the purely thermally caused fundamental out-of-plane deformations with the help of extended nonlinear displacement-strain relations (e.g. [1, 2, 3]). The important influence of the chemical shrinkage and moisture absorption on the laminate deformation however is not considered, although its contribution to residual stresses can be higher than the thermal contribution and therefore cannot be neglected in advanced design processes [7, 8]. Considering extended stress-displacement relations, the principle of minimizing the elastic potential in combination with the Rayleigh–Ritz method results in a system of nonlinear equations for the calculation of the multistable laminate deformations. Dependent on the size and lay-up of the unsymmetric laminate, the equation system leads to different solutions of stable and unstable deformation states like saddle shapes or cylindrical shapes [3, 4, 5, 6].

Within the considered optimization task of realizing a defined laminate curvature, the fitness function is a multimodal function, which is not given in a closed analytical form due to application of a numerical energy principle. Thus, conventional optimization methods like the Gradient Method or the Simulated Annealing are not suitable to find an optimum, because the function's derivatives are missing or the methods are likely to get stuck in local optima. For the best adjustment of residual stresses, Genetic Algorithms offer special advantages and allow to find the optimal combination of material-, load- and process-specific parameters, wherein the most technically relevant parameters are fibre/matrix combination, number of layers, layer orientation, hygrothermal process and load conditions.

The research presented in the paper consists of the two main parts. In the first part, a non-linear semi-analytical model is described. This model allows calculating the laminate curvature and serves as a base for determining the criteria (fitness) function of Genetic Algorithm. Then, analysis of exemplary unsymmetric composites is depicted together with experimental verification. Finally, adjustments of residual stresses with Genetic Algorithm, which reflects the main of goal of the research, is discussed. The proposed approach has successfully been verified by exemplary experiments and numerical calculations on unsymmetric glass-(GFRP) and carbon-(CFRP)-fibre reinforced plastics and was efficiently applied to the design of multilayered curved hybrid structures.

2. NONLINEAR DEFORMATION THEORY OF UNSYMMETRIC COMPOSITES

For unsymmetric cross-ply laminates, the hygrothermally and chemically caused directional deformations of the single UD layers result in large out-of-plane deformations for instance. The occurring basic shapes of a square $[0_n/90_n]$ laminate are illustrated in Fig. 1.

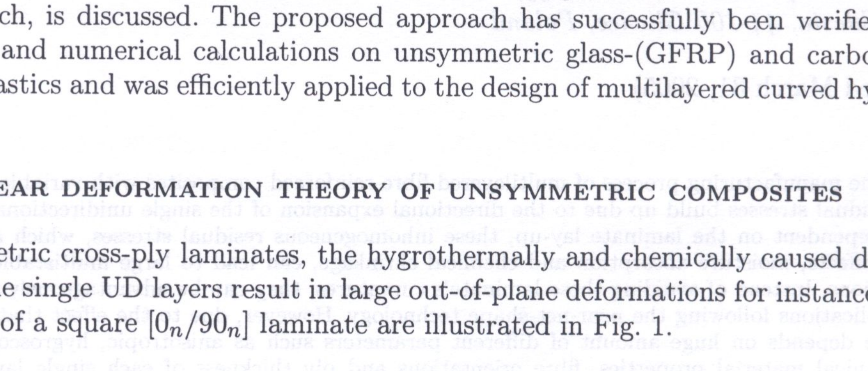

Fig. 1. Basic shapes of square $[0_n/90_n]$ laminates; a) reference state at elevated curing temperature, b) – d) saddle and cylinder shapes at room temperature

Starting from the plane shape, which is considered as the reference state (Fig. 1a), the residual stresses lead – dependent on the laminate dimensions and non-mechanical loads – to a saddle shape (b) or to either of the two stable cylindrical shapes (c, d). For $[0_n/90_m]$ laminates ($n \ll m$ or $n \gg m$) however only one cylindrical shape occurs.

To take the large deformations into account, which are often many times the laminate thickness, the linear strain-displacement relations must be extended by nonlinear terms (see e.g. [12, 13]):

$$\begin{aligned}
\varepsilon_x &= \varepsilon_x^0 - z\frac{\partial^2 w}{\partial x^2} = \frac{\partial u^0}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - z\frac{\partial^2 w}{\partial x^2},\\
\varepsilon_y &= \varepsilon_y^0 - z\frac{\partial^2 w}{\partial x^2} = \frac{\partial v^0}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 - z\frac{\partial^2 w}{\partial y^2},\\
\varepsilon_{xy} &= \varepsilon_{xy}^0 - z\frac{\partial^2 w}{\partial x \partial y} = \frac{1}{2}\left(\frac{\partial u^0}{\partial y} + \frac{\partial v^0}{\partial x} + \frac{\partial w \partial w}{\partial x \partial y}\right)^2 - z\frac{\partial^2 w}{\partial x \partial y},
\end{aligned} \tag{1}$$

where u^0, u^0 denote the displacement of the laminate mid-plane in x, y directions, respectively, w denotes the displacement in the direction z perpendicular to the laminate mid-plane, ε_x, ε_y, ε_{xy} denote the strains and ε_x^0, ε_y^0, ε_{xy}^0 are the mid-plane strains.

The applied theory is based on the principle of minimizing the total potential energy of a composite of volume V, which is given here by [6]

$$\Pi = \int_V \left(\frac{1}{2}\overline{Q}_{ij}\varepsilon_i\varepsilon_j - \eta_{Ti}\varepsilon_i\Delta T - \eta_{Mi}\varepsilon_i\Delta M - \eta_{Si}\varepsilon_i\right) \mathrm{d}V \tag{2}$$

with $(i, j = 1, 2, 6)$, where the $\overline{Q}_{ij}$ are the reduced transformed stiffnesses, ΔT and ΔM are the differences in temperature and relative media concentration and η_{Ti}, η_{Mi}, η_{Si} are related to the elastic constants and to the

- thermal expansion coefficients α_j $(\eta_{Ti} = \overline{Q}_{ij}\alpha_j)$,
- swelling coefficients β_j $(\eta_{Mi} = \overline{Q}_{ij}\beta_j)$ and
- shrinkage s_j $(\eta_{Si} = \overline{Q}_{ij}s_j)$ respectively.

In the following, the Rayleigh–Ritz method is applied to obtain approximate solutions for the displacement fields. Therefore, general approaches in the form of polynomials are used. Geometrical assumptions for the out-of-plane displacements of cross-ply laminates

$$w(x,y) = w(-x,y), \qquad w(x,y) = w(x,-y), \qquad w(0,0) = 0 \tag{3}$$

lead to the general second order approach

$$w(x,y) = \frac{1}{2}(a_0x^2 + b_0y^2), \tag{4}$$

where the coefficients a_0 and b_0 define the laminate curvatures along the x- and y-axes [11]. For the description of the in-plane deformations several approximations can be found in the literature [3, 4, 9, 10, 14]. The following geometrical assumptions

$$\begin{aligned}
u^0(x,y) &= u^0(x,-y), & u^0(x,y) &= -u^0(-x,y), & u^0(0,y) &= 0,\\
v^0(x,y) &= v^0(-x,y), & v^0(x,y) &= -v^0(x,-y), & v^0(x,0) &= 0
\end{aligned} \tag{5}$$

are fulfilled by the general approaches

$$\begin{aligned}
u^0(x,y) &= x(a_1 + a_2x^2 + a_3y^2 + a_4x^2y^2 + \ldots),\\
v^0(x,y) &= y(b_1 + b_2y^2 + b_3x^2 + b_4x^2y^2 + \ldots).
\end{aligned} \tag{6}$$

The assumptions that ε_x^0 is independent on x, ε_y^0 is independent on y and the consideration especially of the nonlinear terms in Eq. (1) reduce the amount of coefficients and finally lead to the approaches

$$\begin{aligned} u^0(x,y) &= a_1 x - \frac{1}{6}a_0^2 x^3 + a_3 x y^2, \\ v^0(x,y) &= b_1 y - \frac{1}{6}b_0^2 y^3 + b_3 y x^2. \end{aligned} \tag{7}$$

Using the displacement approximations Eq. (4) and (7) in the strain displacement relations (1) and substituting the resulting expressions into Eq. (2), the total potential energy of the laminate becomes a function dependent on the coefficients a_k, b_k $(k = 0, 1, 3)$.

The principle of the minimum total potential energy requires the first variation to be zero, which means:

$$\partial \Pi = \frac{\partial \Pi}{\partial a_k}\delta a_k + \frac{\partial \Pi}{\partial b_k}\delta b_k \equiv 0. \tag{8}$$

To satisfy this condition, every term in Eq. (8) must be zero, which results in a coupled non-linear algebraic equation system in a_k, b_k. Dependent on the laminate lay-up and the laminate size, more than one solution can be obtained. These solutions have to be checked for their stability by means of $\partial^2 \Pi$ according to the Hessian matrix, which has to be positive definite.

3. ANALYSIS OF EXEMPLARY UNSYMMETRIC COMPOSITES

Applying the above mentioned theory, the solutions expressed in terms of the curvatures of an exemplary $[0_n, 90_n]$ CFRP laminate $(n = 4)$ are shown in Fig. 2 dependent on the laminate edge length. Branch AB shows the curvature of the stable saddle form $(a_0 = -b_0)$, which occurs in the case of small laminates. The critical length of the saddle shape is, where the saddle form becomes instable, which defines the bifurcation point (B). After this point the saddle shape only occurs theoretically but not in reality (BC). Two further equivalent stable solutions are calculated (BD and BE). Branch BD represents the curvature a_0 of the cylindrical shape (Fig. 1d) and the curvature $-b_0$ of the cylindrical shape (Fig. 1c). Branch BE represents the curvature $-b_0$ of the cylindrical shape (1d) and the curvature a_0 of the cylindrical shape (1c), which asymptotically approach zero with an increasing edge length L.

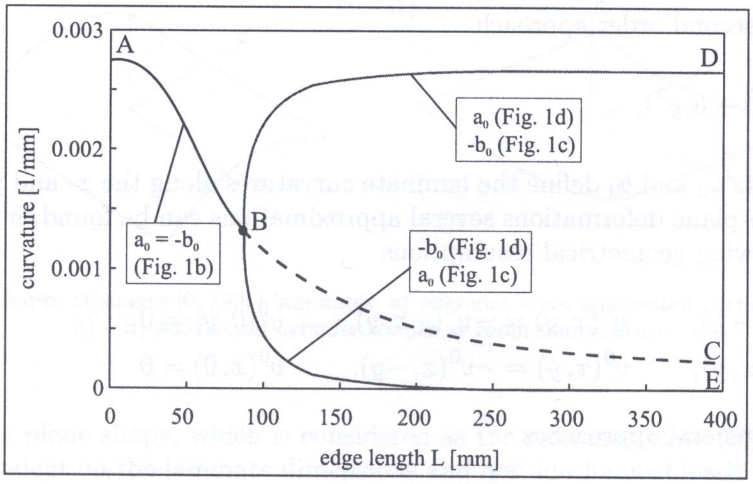

Fig. 2. Exemplary curvatures of a $[0_4, 90_4]$ CFRP laminate dependent on laminate edge length

In Fig. 3 the curvatures of GFRP and CFRP $[0_2/90_2]$ laminates cured at 125°C are compared dependent on the laminate's edge length. It can clearly be seen, that for this special stacking sequence the GFRP composite has a higher curvature than the CFRP laminate. However, several theoretical and experimental investigations on different unsymmetric composites have shown, that a general statement about the comparison of curvature magnitudes of laminates made of different materials cannot be given because of the significant influence of many different material parameters like $E_{||}$, $E_{\perp}$, $\nu_{||}$, $G_{||\perp}$, $\alpha_{||}$, $\alpha_{\perp}$ and lay-up parameters like number of layers, layer thicknesses and layer arrangement. Instead, it has to be distinguished in every single case dependent on the lay-up, whether for example the GFRP or the CFRP laminate shows a higher curvature. Moreover, the comparison of Fig. 2 and Fig. 3 illustrates that in general thinner laminates show a higher curvature than thicker laminates of the same material.

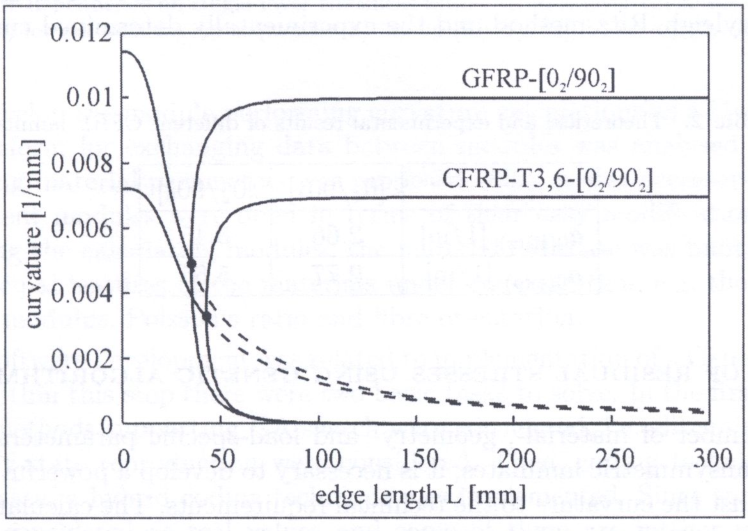

Fig. 3. Curvatures of different cross-ply laminates dependent on the laminate edge length L

4. EXPERIMENTAL VERIFICATION

Within the current research, several experimental investigations have been carried out on carbon-fibre reinforced and glass-fibre reinforced laminates. Exemplarily the results on the basis of the CFRP prepreg system KUBD 1408 of the August Krempel Soehne GmbH+Co. with the material properties given in Table 1 are discussed here.

Table 1. Material properties of CFRP

$E_{\|\|}$ [GPa]	135	$\nu_{\|\|\perp}$ [–]	0.286
$E_{\perp}$ [GPa]	7	$\alpha_{\|\|}$ [1/K]	-0.35×10^{-6}
$G_{\|\|\perp}$ [GPa]	5.4	$\alpha_{\perp}$ [1/K]	36×10^{-6}

In Fig. 4 two different kinds of CFRP laminates are shown with the stacking sequences $[0_4/90_4]$ and $[0_2/90_6]$ (single layer thickness: 0.125 mm) and a laminate size of 400 mm × 400 mm, which have been processed at a curing temperature CT = 125°C and slowly cooled down to room temperature RT = 20°C. After the manufacture and during the experimental investigations the laminates have been stored in dry conditions, so that moisture effects could be neglected for the present.

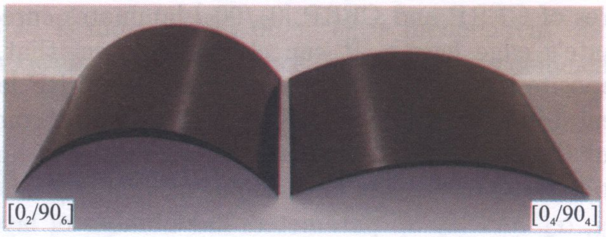

Fig. 4. Shapes of laminates with different lay-ups

Exemplarily, the calculated and measured curvatures for the $[0_4/90_4]$ as well as for the $[0_2/90_6]$ laminate are shown in Table 2. It can be seen that the calculated curvatures of the laminates according to the Rayleigh–Ritz method and the experimentally determined curvatures are in good agreement.

Table 2. Theoretical and experimental results of different CFRP laminates

	$[0_4/90_4]$	$[0_2/90_6]$
$a_{0(\mathrm{Ritz})}$ [1/m]	2.66	5.45
$a_{0(\mathrm{exp})}$ [1/m]	2.77	5.65

5. ADJUSTMENT OF RESIDUAL STRESSES USING GENETIC ALGORITHM

Due to the high number of material-, geometry- and load-specific parameters, that influence the residual stresses of unsymmetric laminates, it is necessary to develop a powerful optimization procedure in order to adjust the curvature to the technical requirements. The calculation of the curvature requires an iterative procedure and the solution cannot be given in a closed analytical form due to application of a numerical energy principle. Thus, conventional optimization methods, like the hillclimbing methods are not suitable to find an optimum, because the function's derivatives are missing. Next, the investigations have shown that the relationship between the laminate's lay-up and the laminate's curvature is often described by a multimodal function with several local and global maxima, so that conventional optimization procedures like hillclimbing methods or Simulated Annealing can fail in finding an optimal solution since they are likely to stack in a local optimum. This can be depicted by a criteria surface of a basic example shown in Fig. 5. These criteria surfaces are built by the single curvature values of square CFRP laminates cured at 125°C (edge length $L =$ 400 mm, global height $h = 2.5$ mm) with three single layers of varying thickness ratios and lay-up sequences.

It can be clearly noticed that for this basic example – considered as simple – the optimization procedure has already to search a multimodal, non-continous surface in order to find the optimum laminate lay-up.

For the adjustment of residual stresses to technical requirements, Genetic Algorithms [15] in combination with the above mentioned nonlinear calculation method offer special advantages and allow to find the optimal combination of material, load and process parameters. These optimization methods are robust and can be used to efficiently search the multidimensional search space and they do not need any derivatives of the function to be optimized [16]. The algorithms are applied to determine the optimum laminate lay-up either in order to maximize the laminate curvature or to adjust the curvature to defined limits by changing the number of layers, layer thickness, layer orientation, layer material, fibre volume content, the laminates size and the process parameters like temperature or moisture.

Regarding the developed software, it is emphasised that several points have been taken into consideration at designing stage. The software has been developed in the form of calculating modules.

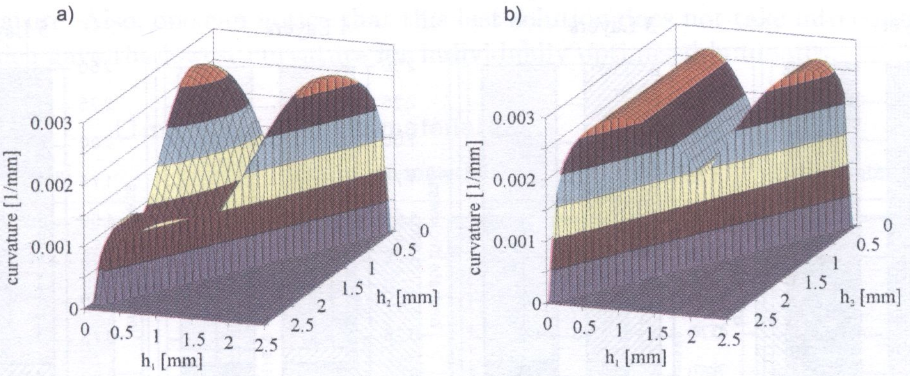

Fig. 5. Curvature dependence on single layer thickness ratio (h_1: first layer thickness, h_2: second layer thickness) for different lay-up sequences: a) lay-up $[0_i/90_j/0_k]$, b) lay-up $[0_i/90_j/90_k]$

It was decided to develop the module performing curvature calculations as a Dynamic Link Library (DLL). Next, a platform for exchanging data between modules was analysed. Here, a quick and efficient way of varing material parameters was proposed. Also, it was necessary to assure that the developed independent modules were open in terms of their easy modification and exchange. In parallel to developing the calculating modules, the material database was built, as well. The database contained principal features of the materials under consideration, e.g. the thermal expansion coefficient, Young's modulus, Poisson's ratio and fibre orientation.

The last step of software development was related to implementation of a Genetic Algorithm (GA) based procedure. Within this step there were two basic tasks to solve. In the first case, analysis and implementation of methods supporting GA search were considered, i.e. elitism strategy, multipoint crossover and steady-state reproduction were considered. Then, coding techniques were analysed and tested. In this case, a hybrid coding technique was implemented. Since some of the optimised parameters can be considered as real values and some of them are integer values, a specialised procedure imposing, so-called, mutation mask was applied. The procedure varies the mutation range for each gene depending whether the genes represent integer or real value. Finally, since performance of the Genetic Algorithm may depend on the specific task, a hybrid Genetic Algorithm was also taken into consideration. In this case, the selection and implementation of methods that potentially could be applied in the hybrid procedure were done. Consequently, enumerative search procedure was developed as a means for tuning the results obtained with the Genetic Algorithm.

In order to exemplary present capabilities of the developed software, the main steps of analysis of a certain set of materials are considered. First, it is assumed that different materials, e.g. materials based on glass or carbon fibre with altered matrix contents, are considered. Obviously, such materials can substantially differ in terms of their mechanical and thermal properties. Therefore, many laminate lay-ups can be developed with regard to an assumed or maximal curvature. However, in order to determine an influence of each material it is convenient to start with analysis of each material, separately. In such a case, the task to be solved consists in determining the number of layers and orientation and thickness (height) of each layer. From the point of view of the Genetic Algorithm procedure, it means that the algorithm deals with chromosomes representing the thickness of the layers and orientation of each layer. The laminate global height is considered constant, e.g. $h = 2.5$ mm. Next, it is assumed that the Genetic Algorithm procedure is supposed to find the laminate lay-up giving the maximum curvature. An example of tests performed for a certain material is depicted in Fig. 6. In this figure, four lay-ups of the laminate are presented in terms of layer height and orientation. The thickness of each layer is marked with different texture, so one can clearly see the laminate lay-up for an assumed number of layers. The layer orientation is marked, however, with two textures only, which means that orientation of 0 degree or 90 degree is considered here. For example, four layer laminate (Fig. 6) consists of layers of the following orientations: layer 1: 0°, layer 2: 90°; layer 3: 0° and layer 4: 0°.

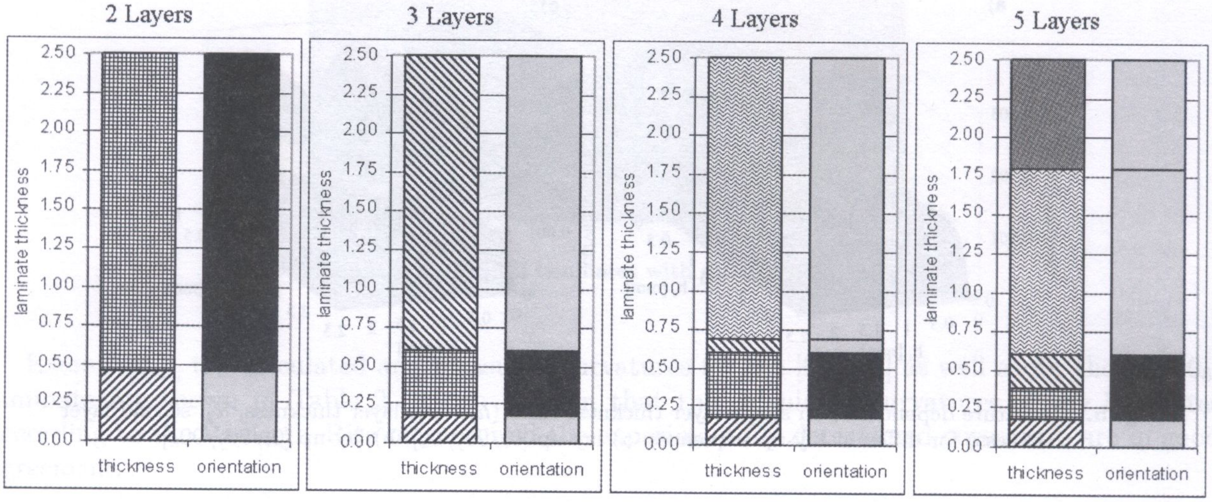

Fig. 6. Examples of optimised laminate lay-ups giving the maximum curvature

Analysing lay-ups shown in Fig. 6, one can easily notice that the number of layers affects the the lay-ups up to three layer laminate only. Increasing number of layers above three does not increase the curvature, at the same time. Also, it can be emphasised that the four and five layer laminates should be considered as a three layer laminate with thickness of each layer driven according to the layer orientation. However, this interesting finding was verified for other materials and it was revealed that the maximal number of layers giving the maximal curvature depends on material properties, e.g. already two layers can maximise the curvature. Also, some other specific capabilities of the GA optimisation procedure can be underlined based on the results shown in Fig. 6. The tests with four or five layer laminates can be considered as tests aiming not only in deciding the layer height and orientation but as tests allowing to select the optimal number of layers, as well. Assuming a relatively high initial number of layers, the optimisation procedure is capable of selecting such a solution for which the final number of layers can be substantially lower.

The exemplary results presented above were obtained for a certain assumed global laminate height and dimensions. The developed optimisation software allows conducting the tests in which these parameters can vary. Such tests can reveal that that the maximum curvature can be obtained by changing not only relative height of each layer but by changing the height of the laminate, as well. In such a way, the global height of the laminate can also become an optimised parameter.

The search for maximum curvature is just a one way of optimising laminate lay-up. Potentially, it can be also interesting to find a laminate lay-up giving a certain pre-defined curvature. This task can be considered as a search for points or curves representing intersection of criteria surface (shown for example in Fig. 5) with a plane reflecting a pre-defined curvature. From GA point of view it means that this algorithm can give different solutions each time the search for optimal lay-up is conducted. However, the solutions should differ only in terms of accuracy expressed as a difference between the pre-defined curvature and the curvature calculated with the GA.

Summarising the description of the developed software, an example of simultaneous analysis of three different materials is depicted. Such a task is already relatively difficult since the GA based procedure must consider several parameters for the optimisation (namely: layer height, layer orientation, number of layers and different material properties for each layer). An example of the results obtained for this task is shown in Fig. 7. First, the optimum structure of the laminate built with material 1 was considered, only. Then, the material 2 was introduced as an optimised parameter. Introduction of this material allowed obtaining the laminate with much higher curvature than in the first case. In the last step, the optimisation procedure dealt with possibility of assigning one of the three materials to each layer. Such a possibility is reflected in the form of the highest obtained curvature. Also, one can notice that this last solution does not take into consideration the material 2 which gave the lowest curvature for individually optimised laminates.

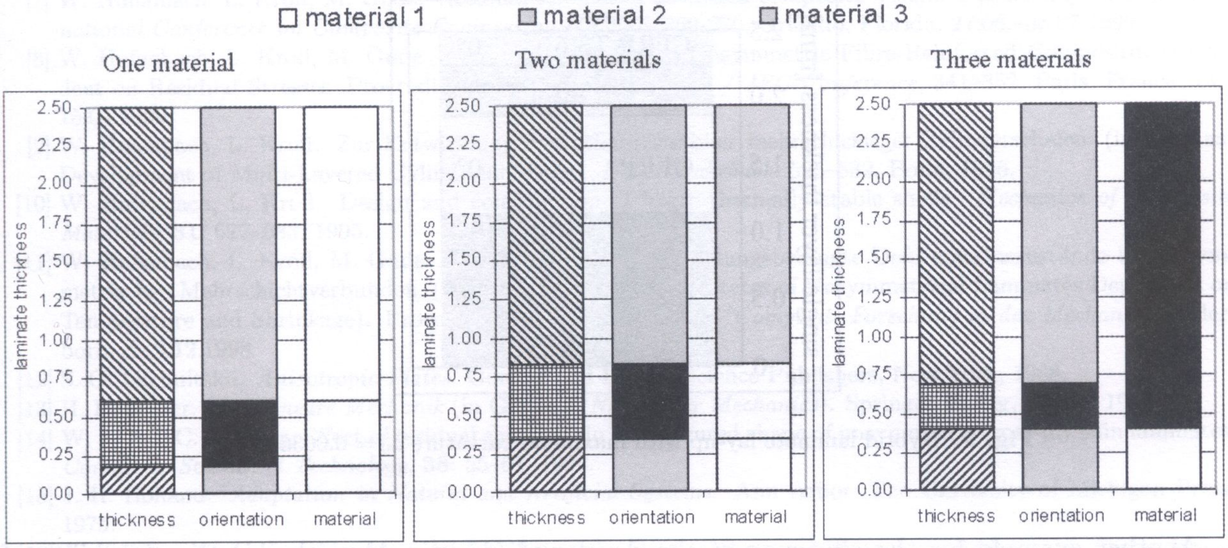

Fig. 7. Optimised laminate structures giving the maximum curvature

In order to emphasise the practical aspects of the tests conducted, in the following part of this section the explicitly defined materials are considered. The Genetic Algorithm was used to optimise the laminate's lay-up in terms of layer arrangement, layer thickness and layer orientation, so that the main cylindrical curvature of different cross-ply laminates (400 mm × 400 mm), consisting either of GFRP or CFRP with different fibres, is maximized. It was found that dependent on the laminate material the maximum curvature has to be realized by different lay-ups. The left bar graph in Figure 8 shows the optimum single layer thicknesses and the layer arrangement of the investigated composites with a global height of 2.5 mm. While the curvature of a GFRP laminate can be achieved by manufacturing a simple $[0_i/90_j]$ lay-up with a ratio shown in Fig. 8, in case of the investigated CFRP laminates the maximum curvatures are realized by different $[0_i/90_j/0_k]$ lay-ups. Furthermore it is evident from the right bar graph in Fig. 8, that in contrast to the curvatures of the $[0_2/90_2]$ laminates shown in Fig. 3 now the CFRP laminates have higher curvature values.

It was furthermore observed, that independent on the global height of the laminates the ratios of the single layer thicknesses which result in the maximum curvature are the same, so that once

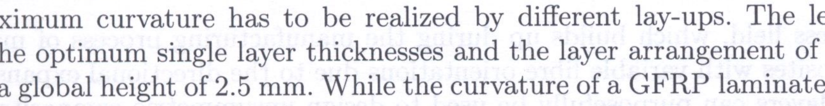

Fig. 8. Lay-up of CFRP and GFRP cross-ply laminates resulting in maximum curvatures

the optimum lay-up ratio is found for a special material, it can be transferred to any laminate of the same material with a varying total thickness.

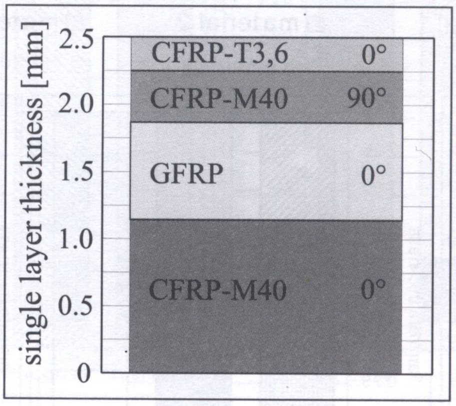

Fig. 9. Hybrid laminate lay-up with maximum curvature $a_0 = 0.0036\,\mathrm{mm}^{-1}$

Another example for the efficiency of the developed Genetic Algorithm is the possibility of generating the design of hybrid stuctures. In order to further increase the curvature of the above mentioned laminates with constant global height, it is possible for the algorithm to choose materials from a given material table and combine them to a hybrid structure with different material layers. Figure 9 shows an example of a hybrid composite structure, which leads to an increased curvature compared to the above mentioned pure CFRP or GFRP laminates.

6. CONCLUSIONS

The residual stress field, which builds up during the manufacturing process of multilayered fibre-reinforced composites with variable fibre orientations due to the directional expansion of the single orthotropic UD layers can purposefully be used to design unsymmetric composites with a defined curvature. Due to the high number of parameters which influence the curvature like the number of layers, layer thickness, layer orientation, layer material, fibre volume content, the laminates size and the process parameters, Genetic Algorithms in combination with a nonlinear calculation method have been developed, which enable an efficient search in the resulting multidimensional search space. It has been shown, that the optimization procedures enable the efficient design of multilayered and hybrid composites and the adjustment of the curvature to the technical demands. This does not only meet the single technical requirements within a near-net-shape technology but also helps to reduce production costs by replacing complexly shaped tools by plane tools.

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