

Evolutionary stress minimisation on a turbine blade shank

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The paper describes shape optimisation of a turbine blade shank. The turbine blade shank zone with a compound fillet is a critical location where a high risk of failure exists. The APDL language operating in Ansys environment is used to write a parametric turbine blade shank FEM models generator, which is a basic part of evolutionary optimisation routine. The goal of the optimisation is the 1st principal stress reduction with maximum allowable mass constraint imposed. Parameterisation routine and optimisation results are presented and discussed.

Keywords: AI, evolutionary optimisation, APDL, turbine blade, shape optimisation, Finite Element Method (FEM)

1. INTRODUCTION

A typical turbine blade is an element requiring a very complex design process. Its shape is governed mainly by aero dynamical functions and it operates very often in conditions of extremely high temperature, inertial and gas pressure loads. The blade shape results usually from a detailed engineering analysis and is a compromise between aerodynamic efficiency, dynamic characteristics and structural stress requirements, as a variety of potential failure mechanisms have to be taken into consideration. Usually the blade shape is based on existing and verified solutions (*engineering best*).

Blade shape optimisation is performed usually in a very limited range, with a series of trial-and-error analyses, as complexity of the problem causes the traditional optimisation algorithms to be difficult to implement or operate. The evolutionary algorithm seems an interesting alternative here, as it searches for the optimal solution basing only on fitness function value and does not require meeting the limitations of traditional optimisation algorithms (continuous objective function, sensitivity analysis, narrow search domain, etc.) [2, 3, 7, 8]. The method is also relatively easy to automate and may be fully implemented into standard FEM solver environment, enabling analysing of relatively complex thermo mechanical problems, as described in [6]. If evolutionary algorithm parameters are set basing on some experience and trial analyses, numerical cost of the optimisation

is relatively low. All this renders this approach very attractive to industry, making evolutionary optimisation an engineering tool of great capabilities.

This paper presents optimisation of a turbine blade shank shape with the evolutionary algorithm. The optimisation routine consists of parametric blade FEM models generator and main optimisation subroutine. It operates entirely in Ansys environment and is written in APDL (Ansys Parameter Design Language) [1].

The optimisation is performed on 3D-geometry with a full structural and thermal loads set representing turbine operating conditions. The optimised zone is a compound fillet at blade airfoil/bottom platform junction.

The bottom compound fillet zone is a critical zone of the turbine blade where a high stress concentration exists and potential failure may be initiated, seriously limiting the blade life. Stress distribution in this area is extremely sensitive on small compound fillet geometry changes. There is no complete engineering method of compound fillet parameters determination, usually they are set with a series of trial-and-error analyses, where the designer's experience and engineering practice play major role.

Preliminary optimisation of the blade shank has been performed previously using the approach described here, in order to test the optimisation routine and correctness of all the assumptions [6]. Basing on the gathered experience, some changes were introduced into reproduction algorithm to improve its convergence and robustness, population was enlarged and new optimisation objectives were formulated.

This paper presents results of stress minimisation of the blade shank, with maximum allowable mass constraint imposed. The model represents actual bladed disc segment of aero derivative gas turbine rotor for industrial applications at early design process stage. The source model will be referred to as a *reference model*. It should be stated that the reference model does not represent the ultimate, actual production configuration.

All the results and geometric parameters were normalized in terms of the reference model.

2. PROBLEM ADAPTATION FOR EVOLUTIONARY ALGORITHM

The evolutionary algorithm mimics a natural evolutionary process, evolving from random solutions towards the ones satisfying a fitness function the best. Usually, on the contrary to the natural process, the evolutionary algorithm environment is fixed during all the evolution – it consists of the fitness function, playing the role of the objective function, and imposed constraints.

To perform the optimisation, the object geometry has to be completely parameterised with a set of design variables (geometric parameters). According to evolutionary nomenclature, each design variable is referred to as “gene” and a set of parameters defining the model is a *chromosome* (design variables vector). A set of chromosomes is referred to as a *population* [2, 4, 7].

During the optimisation, populations containing sets of chromosomes have to be generated and realised as FEM models in fully automated mode, as the expected number of models to be generated is at least several hundreds. This requirement imposes the stability of FEM models generation routine, all the errors and geometry inconsistencies have to be found and corrected during the CAD modelling process, otherwise the fitness function assessment for some chromosomes would be impossible and the evolution process would be seriously biased. The problem described here concerns a generation of complex 3D models and a major part of work done during this research was the modification and improvement of model generator to automatically “*heal*” unfeasible chromosomes and offset them into a geometrically feasible domain with smallest possible modification. Another approach to the problem could be the elimination of these chromosomes and their substitution with new ones, generated with the same input data, but it would extend the optimisation process duration beyond a tolerable range.

The evolutionary algorithm is initiated with a set of randomly generated chromosomes (models), which are the 1-st generation from which the evolution process begins.

Each chromosome is an input for the parametric model generator. It is realised as a FEM model and analysed to evaluate the fitness function value. Then, chromosomes with the lowest fitness function from the first generation are selected as “*genetic material*” for the next generation and used as “*parents*” to produce “*offspring*” – the next generation. The parent chromosomes are crossed and mutated, in a way that allows creating a full set of chromosomes for the next generation. The next generation is sent as input to the parametric model generator and the fitness function evaluation process is performed again. This loop is repeated until the maximum iterations (generations) number is reached. The details of the process are described in Sec. 6.

The optimisation process flow chart is presented in Fig. 1.

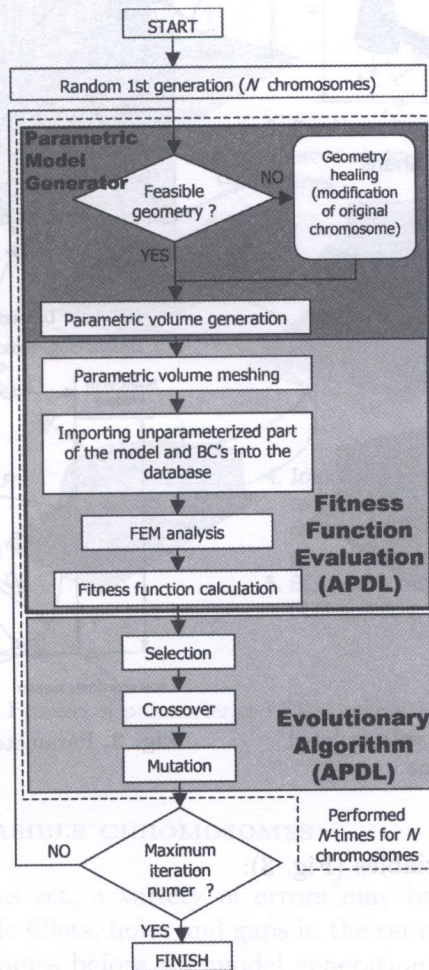


Fig. 1. Optimisation process flow chart

3. GEOMETRY PARAMETERISATION

A typical bladed disc CAD model with isolated parameterisation zone is shown in Fig. 2. The model represents a repetitive segment of a turbine rotor, with angular aperture of $360^\circ/N$ where N stands for the number of the rotor blades.

The optimised zone is a blade shank, a so-called compound fillet, at airfoil/blade bottom platform junction. The compound fillet zone extends up to 10% of blade airfoil height, in this zone compound fillet may be shaped freely with no significant impact on blade aero dynamical efficiency.

The blade airfoil and compound fillet circumference is divided in four parts: Suction Side, Leading Edge, Pressure Side and Trailing Edge. Along each part, a set of parametric locations were chosen

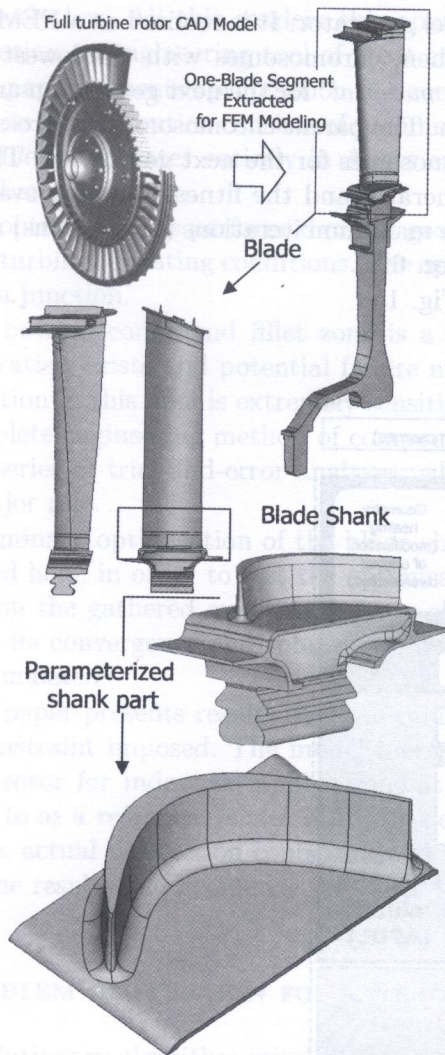


Fig. 2. Typical bladed disc model with isolated parameterisation zone

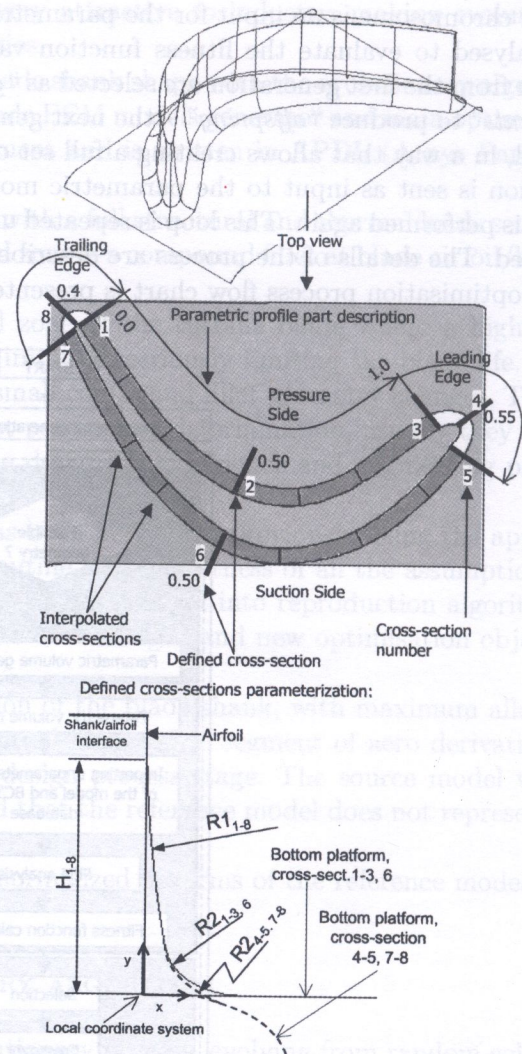


Fig. 3. Parameterisation of compound fillet zone

and described by three design variables (Fig. 3):

$R1$ – upper fillet radius,

$R2$ – lower fillet radius,

H – compound fillet height.

Seventeen parametric positions were chosen to define the compound fillet zone. Eight of them are defined explicitly and the rest is calculated by linear interpolation between defined values. The compound fillet height H is constant along the profile circumference. Thus, there is a set of $8 \times 2 (R1, R2) + 1 (H)$ parameters that is sufficient to define the compound fillet geometry (Fig. 3). The parametric model generator input may be any set of parameters in any parametric positions, the choice is an arbitrary decision depending on the actual model shape and modelling assumptions.

The CAD models are generated directly in Ansys PREP7 pre-processor, input are text file with a set of parameters and an iges format file with a part of model (bottom platform and shank/airfoil division lines). At first, a set of parametric curves is generated, then, B-surface areas are spanned at each 4-curves chain. Finally, the rest of the model is read into the Ansys and a new volume defined with the areas is created.

The main phases of the parameterised FEM model generation are shown in Fig. 4.

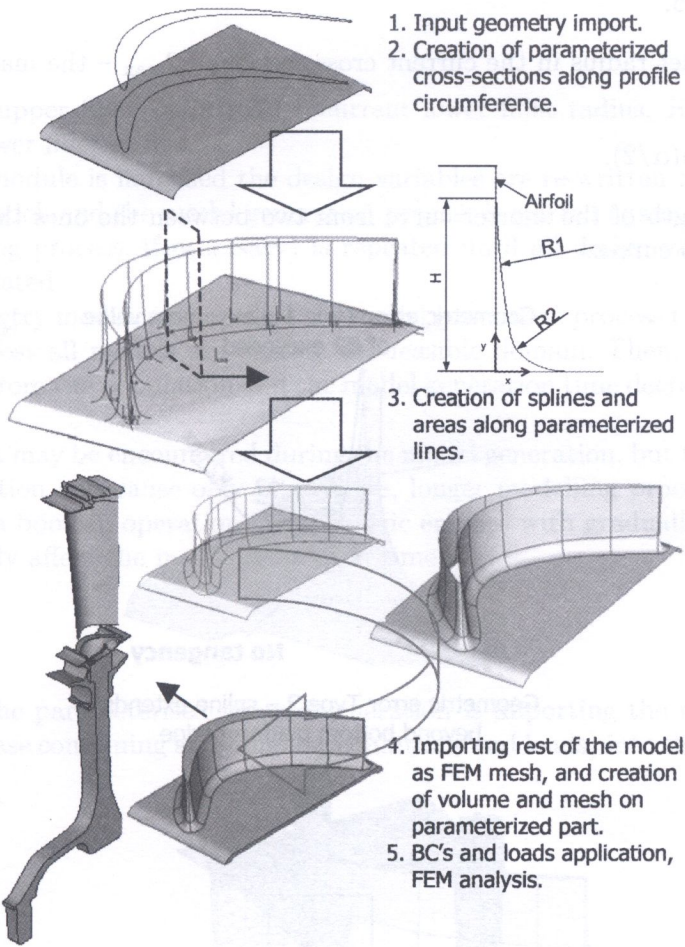


Fig. 4. Process of parameterised FEM model generation

4. MODIFICATION OF UNFEASIBLE CHROMOSOMES

Depending on a design variables set, a variety of errors may be encountered during the model generation, including non-feasible fillets, holes and gaps in the set of areas defining volume, etc. The detection of unfeasible chromosomes before the model generation process launching is impossible. Each cross-section is a function of three independent parameters so a trial model generation must be performed to check if the curves will fit within the bottom platform edges or if curves tangency requirement may be satisfied. The problem is also the shape of the compound fillet areas – some of them may extend beyond the bottom platform edges.

To prevent these problems, a *healing module* was written as a part of the parametric model generator. It modifies unfeasible design variables by changing *R1* and *R2* values in parametric positions closest to geometry failure zone, offsetting a chromosome into a feasible domain.

Two main types of geometric errors are considered.

4.1. Geometric error type 1

The type 1 is an error occurring when the fillet curve cannot be realised with tangency within the bottom platform area because of too large radii values (Fig. 5). A corrective action is taken, by

reducing the $R2$ value according to the formula:

$$R2_N = R2_{\max} \cdot 0.95, \quad (1)$$

where: $R2$ – a new fillet radius in the current cross-section, $R2_{\max}$ – the maximum allowable fillet radius calculated as:

$$R2_{\max} = L_{\min} \cdot \tan(\alpha/2), \quad (2)$$

where: L_{\min} – the length of the shorter curve from two between the ones the fillet is created, α – the angle between two curves.

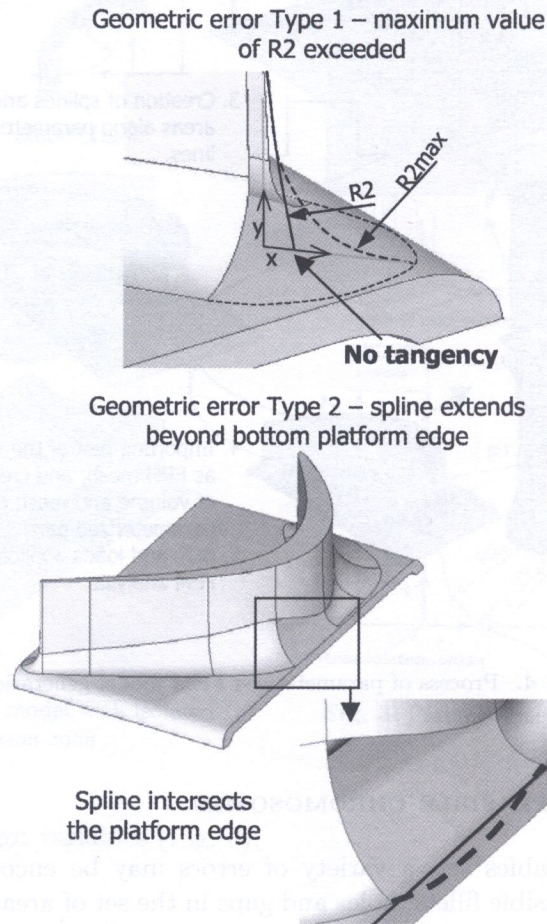


Fig. 5. Most common geometry errors

The equation above is based on an assumption that two curves are straight lines, which is not true in our case, because upper fillet is an arc (of $R1$ radius) and a curve at the platform is a spline following the platform curvature. This causes the $R2_{\max}$ value to be usually too large to be realised. To override this problem, a corrective factor of 0.95 is used. Usually the radius cannot be realised, being too large in spite of this amendment, so $R2_N$ is gradually decreased by 5% until the curves tangency condition is satisfied.

4.2. Geometric error type 2

The type 2 is an error occurring when a B-spline around the bottom airfoil circumference extends beyond the bottom platform area (Fig. 5). A corrective action is taken by changing both $R1$ and

$R2$ values according to equation:

$$\begin{aligned} R1_N &= 1.05 \cdot R1, \\ R2_N &= 0.95 \cdot R2, \end{aligned} \quad (3)$$

where: $R1$ – current upper fillet radius, $R2$ – current lower fillet radius, $R1_N$ – new upper fillet radius, $R2_N$ – new lower fillet radius.

After the healing module is launched the design variables are re-written to the chromosome describing the healed model, and the model generation process is started again. The model generation (and the model healing process, if necessary) is repeated until all design variables are acceptable and a volume is generated.

This mode of geometry modification causes the model generation process to be longer for the first generation where almost all models are outside the feasible domain. Then, unfeasible models are gradually eliminated from the population and the model generation time decreases with a generation number.

Several other errors may be encountered during the model generation, but they do not require the chromosome modification and cause only alternative, longer modelling procedures to be launched (for instance looping a boolean operation on geometric entities with gradually changing tolerance). This may also seriously affect the model generation time.

5. FEM MODEL

The next step after the parameterised volume generation is importing the rest of the bladed disc FEM model (in database containing all boundary conditions and loads) into the Ansys pre-processor.

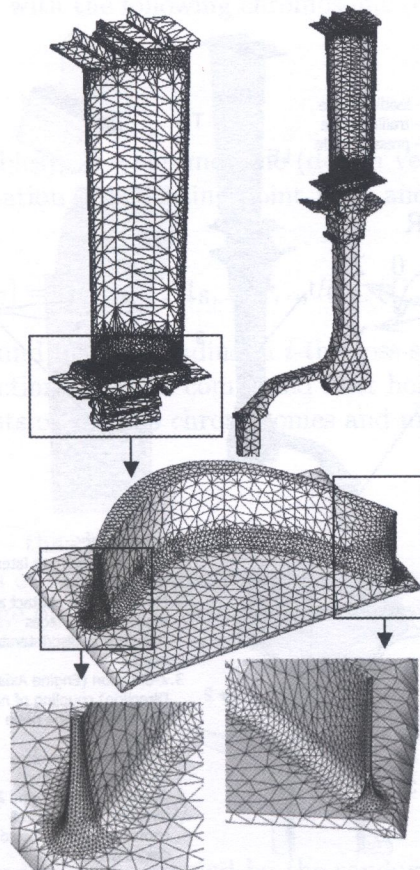


Fig. 6. Bladed disc of FEM model

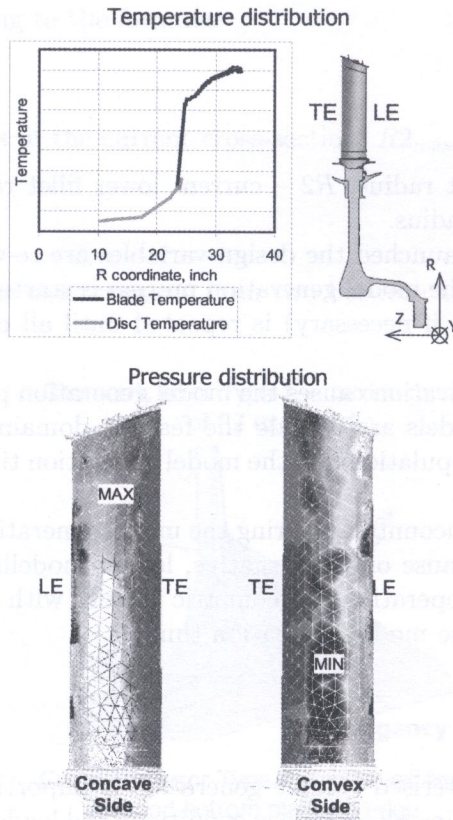


Fig. 7. FEM model loads

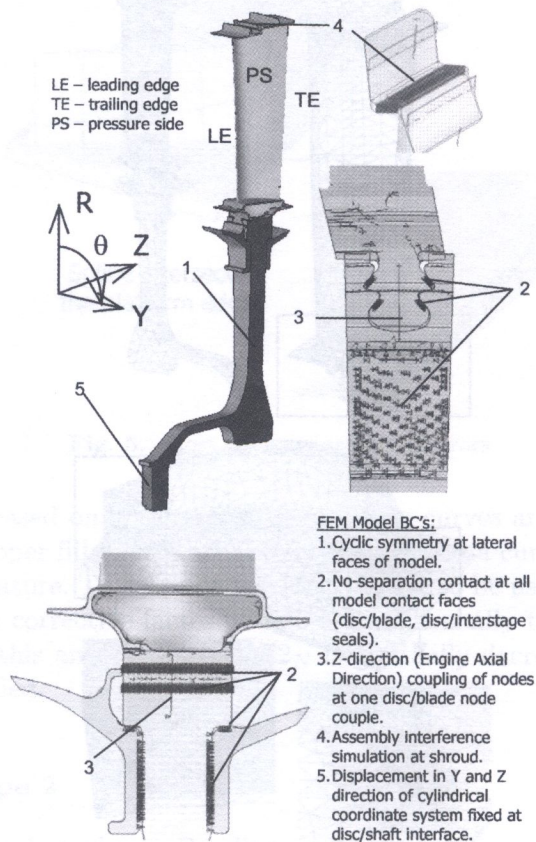


Fig. 8. FEM model boundary conditions

The model representing the rest of the bladed-disc geometry is meshed very roughly to reduce DOF number. It was verified that the reduced model yields < 3% stress error in compound fillet zone comparing to model with overall fine mesh, with mass difference between fine and coarse model below 1%. The element type is 10-node tetrahedron.

The parameterised volume is meshed using areas containing mesh at the interfaces of parameterised zone with the rest of the model. This way, mesh consistency at the interface areas is secured. A very fine mesh is used in the interest area (a compound fillet). The FEM model description is presented Figs. 6–8.

The next step is the application of nodal temperature, pressure and a rotational velocity along Z global cylindrical coordinate system onto a newly generated FEM model (Fig. 7). Displacement boundary conditions are applied as shown in Figure 8. Materials of blade and disc are alloys with strongly temperature-dependent properties.

The model size is usually (depending on the parameterised part shape) around 185000 DOFs.

FEM analysis and stress/mass assessment are final operations performed for each chromosome. A set of chromosomes representing a generation is then processed by the evolutionary algorithm.

6. EVOLUTIONARY ALGORITHM

The criterion of evolutionary optimisation is to minimize the maximum 1-st principal stress. The fitness function is expressed as follows:

$$F = \sigma_{1st_principal}, \quad (4)$$

where: $\sigma_{1st_principal}$ – optimised part maximum 1-st principal stress.

The stress is read from superficial nodes of the optimised part FEM mesh.

The model is fully described with the following chromosome (design variables vector) containing 17 design variables (genes):

$$X = [x_1, x_2, \dots, x_{17}], \quad (5)$$

where: x_i – genes (design variables), X – chromosome (design vector).

The chromosomal representation is a floating-point type and each gene represents the actual design variable:

$$X = [x_1, \dots, x_8, x_9, \dots, x_{16}, x_{17}] = [R1_1, \dots, R1_8, R2_1, \dots, R2_8, H], \quad (6)$$

where: $R1_i$ – the upper compound fillet arc radius in i -th cross-section, $R2_i$ – the lower compound fillet arc radius in i -th cross-section, H – the compound fillet height.

The entire generation consists of $K = 30$ chromosomes and may be described as:

$$X = [X_1, X_2, \dots, X_{30}], \quad (7)$$

where: X – the generation, X_i – the chromosomes.

The first generation consists of randomly generated chromosomes. For each design variable type ($R1$, $R2$, H) a permissible range of values is defined. The random gene generation process may be represented as:

$$x_i = \begin{cases} \xi_U(R1_{\min}, R1_{\max}) & \text{if } i \in \langle 1, 8 \rangle \\ \xi_U(H_{\min}, H_{\max}) & \text{if } i = 17 \\ \xi_U(R2_{\min}, H^*) & \text{if } i \in \langle 9, 16 \rangle \end{cases} \quad (8)$$

where: $\xi_U(r_1, r_2)$ – represents a variable obtained by the random generation with the uniform distribution, where r_1 is the lower limit and r_2 the upper limit, $R1_{\min}$, $R1_{\max}$ – the allowable range

of $R1$ parameter, $R2_{\min}$ – the minimum value of $R2$ parameter, H^* – the H value for the current chromosome (in case $R2 = H$ the cross-section is reduced to simple fillet shape, H_{\min} , H_{\max} – the allowable range of H parameter.

The allowable parameters range is specified in Table 1.

Table 1. Normalized allowable gene range

	Min, –	Max, –
$R1$	0.02	6.75
$R2$	0.05	H of current model*
H	0.40	1.20

*in case $R2 = H$ a cross-section is reduced to simple fillet shape.

The random generation is performed until all K chromosomes are created. Each chromosome is sent as input for parametric model generator. After the fitness function evaluation, the reproduction process begins. The reproduction consists of three phases.

6.1. Selection

The probability of chromosome selection for the reproduction is calculated according to the following formula:

$$P_k = \begin{cases} 0 & \text{if } f_k \geq \frac{f_{\max} - f_{\min}}{2}, \\ 1 & \text{if } f_k < \frac{f_{\max} - f_{\min}}{2}, \end{cases} \quad (9)$$

where: P_k – the probability of the selection of the chromosome k , f_k – the fitness function value for the chromosome k , f_{\min} – the minimum fitness function value for the current generation, f_{\max} – the maximum fitness function value for the current generation.

All chromosomes of the generation are selected with probability P_k for the crossover and the mutation. Such a selection pattern results in rejecting a half of original generation with the highest fitness function value. For the next reproduction phase, $K/2 = 15$ chromosomes are chosen. This set of chromosomes is a temporary generation and will be denoted by Y . The selection process may be represented as:

$$X = [X_1, X_2, \dots, X_{30}] \xrightarrow{\text{Selection}} Y = [Y_1, Y_2, \dots, Y_{15}]. \quad (10)$$

6.2. Crossover

The crossover is an operation that re-creates a new, full generation starting from a selected half of original generation (the temporary generation) chosen in the selection process. The pattern used here requires three “parent” chromosomes to produce one “child” chromosome. The process may be described with the following formula:

$$\left. \begin{matrix} Y_{k1} \\ Y_{k2} \\ Y_{k3} \end{matrix} \right\} \rightarrow Z = [y_1^{k1}, \dots, y_{c1}^{k1}, y_{c1+1}^{k2}, \dots, y_{c2}^{k2}, y_{c2+1}^{k3}, \dots, y_{17}^{k3}], \quad (11)$$

where: Y_{k1} , Y_{k2} , Y_{k3} – randomly selected chromosomes of Y , Z_k – the k -th chromosome of the temporary generation Z , $c1$, $c2$ – randomly selected cutting points of Z chromosome (normal distribution),

$c1 = \xi_N(1,8)$, $c1 = \xi_N(9,16)$, y_i^{kj} – the i -th gene of the chromosome Y_{kj} , or:

$$Y = [Y_1, Y_2, \dots, Y_{15}] \xrightarrow{\text{Crossover}} Z = [Z_1, Z_2, \dots, Z_{30}] \quad (12)$$

where: Z – the temporary generation after the selection and the crossover.

The best chromosome of the generation Y is passed with no change to the generation Z . The genes swapping for the temporary generation Y is performed $K - 1 = 29$ times to obtain a new temporary generation denoted as Z . Chromosomes Y_{k1} , Y_{k2} , may not be the same, and the combination Y_{k1} , Y_{k2} , Y_{k3} , may not be repeated within the generation.

6.3. Mutation

The last reproduction phase is a mutation, an operation changing the gene value randomly within a given range. It may be described with the equation:

$$x' = z + \xi_G(0, 0.1) \cdot z, \quad (13)$$

where: x' – a gene value after the mutation, z – a mutated gene of the temporary generation Z , $\xi_G(r_1, r_2)$ – represents a variable obtained by the random generation with the gaussian distribution, where r_1 is the expected value and r_2 a standard deviation.

The mutation is performed on two randomly selected genes of each chromosome. It may be represented as:

$$Z = [Z_1, Z_2, \dots, Z_{30}] \xrightarrow{\text{Mutation}} X' = [X'_1, X'_2, \dots, X'_{30}], \quad (14)$$

where: X' – the generation after the reproduction operation, X'_i – the chromosomes after the reproduction operation.

After the mutation, a gene values check is performed to correct values lying outside a feasible domain. All the values lying outside are truncated to the closest minimum/maximum value (see Table 1).

Entire reproduction process results in a new generation that is passed as input for the parametric model generator. After the fitness function evaluation the reproduction process is repeated.

The Table 1 shows the allowable gene values range. The values are normalized in terms of the reference model, i.e. the values of the minimum and maximum $R1$ are divided by the average $R1$ of reference model, the values of the minimum and maximum $R2$ are divided by average $R2$ of reference model and the values of minimum and maximum H are divided by the reference model H .

7. RESULTS

The optimisation goal was the 1-st principal stress minimisation with the maximum allowable mass constraint. The maximum allowable mass value was equal to 1.00 of the reference model mass. All models were checked by a penalty function and their reported stress was doubled in the case of mass limit violation.

In total, the optimisation process was performed in 60 generations and 1500 models were analysed. It seems that the fitness function is levelled and evolution process is in stagnation point.

All presented results are normalized in terms of reference model results. The reference model is one of preliminary design geometric blade configurations, where compound fillet geometry and blade mass are close to the ultimate production configuration, but no stress compensation and minimisation was performed yet.

The optimisation results are shown in Figs. 9–14 and presented in Table 2.

The fitness function (which is equal to 1-st principal stress) is shown in two diagrams, alone (Fig. 9) and together with average fitness function of each generation (Fig. 10).

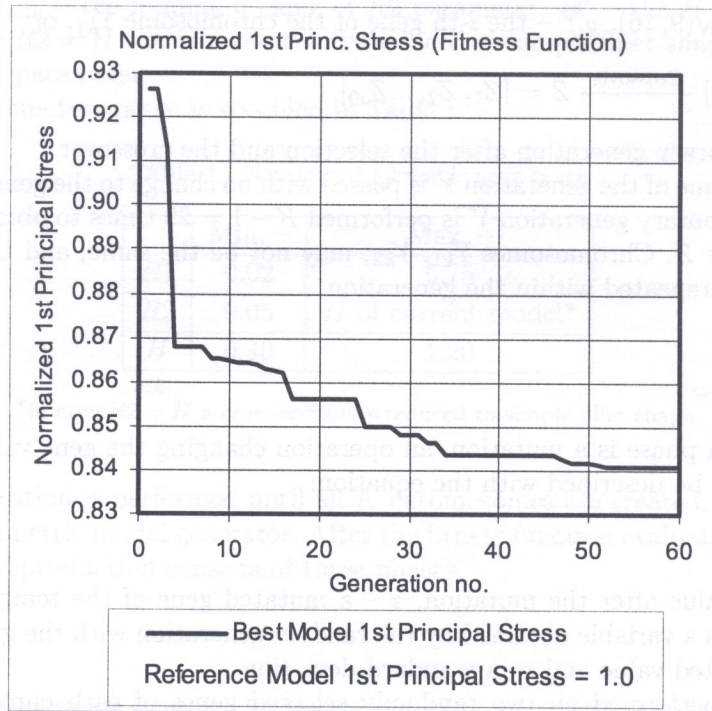


Fig. 9. Normalized fitness function diagram – best solution

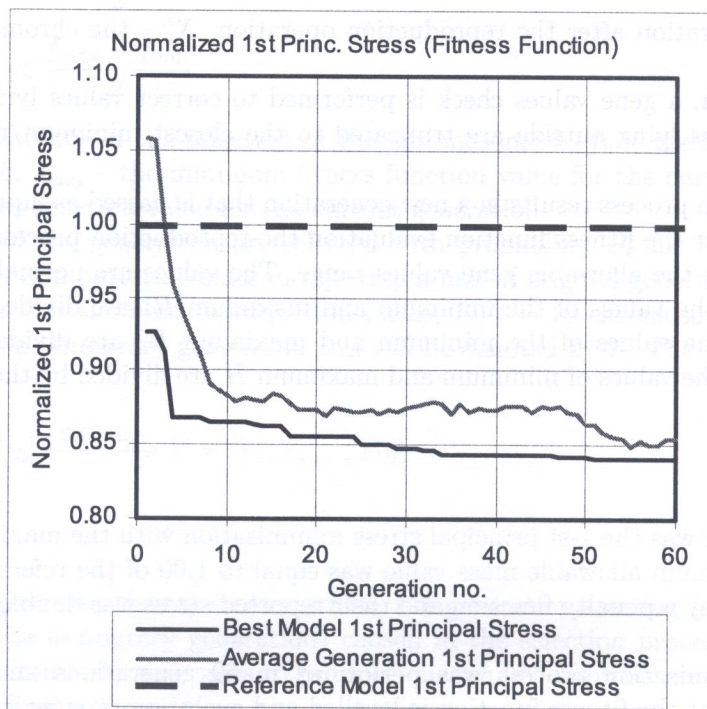


Fig. 10. Normalized fitness function diagram – average vs. best solution

Figure 11 shows the best model mass with respect of the generation number.

The optimisation process statistics (average fitness function evaluation time, the number of chromosomes with mass penalty) is shown in Fig. 12. Optimisation was performed on PC with P4 3.2GHz CPU and 1.5GB RAM.

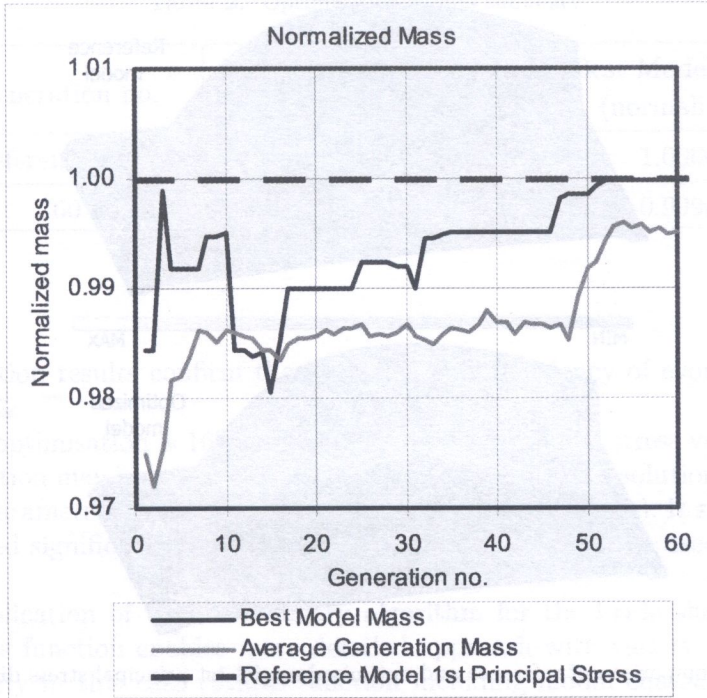


Fig. 11. Normalized mass diagram

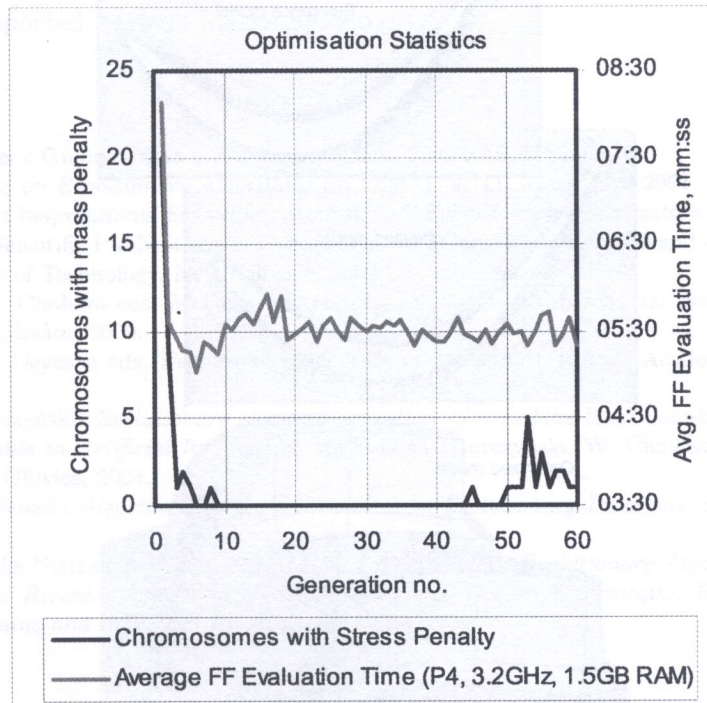


Fig. 12. Optimisation statistics

Comparison of the stress distribution on the blade shank between the reference and optimised models is shown in Fig. 13.

The optimised blade shank shape is compared with reference model in Fig. 14.

Table 2 shows the optimisation results summary.

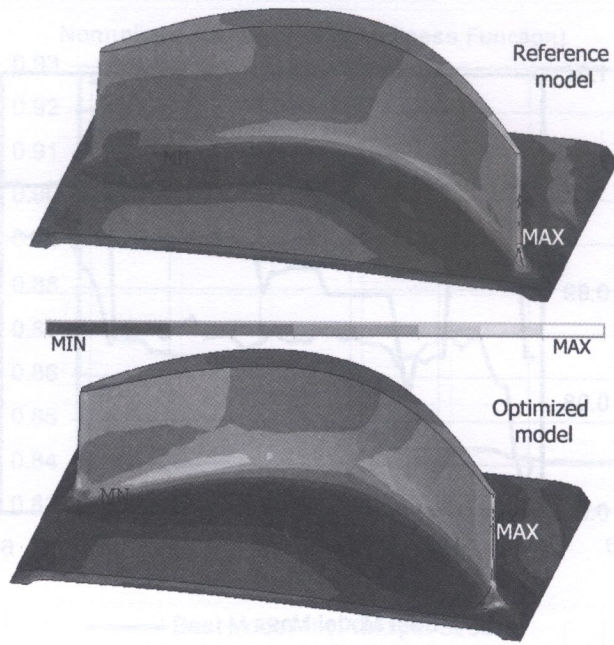


Fig. 13. Comparison of reference and optimised model 1st principal stress distribution

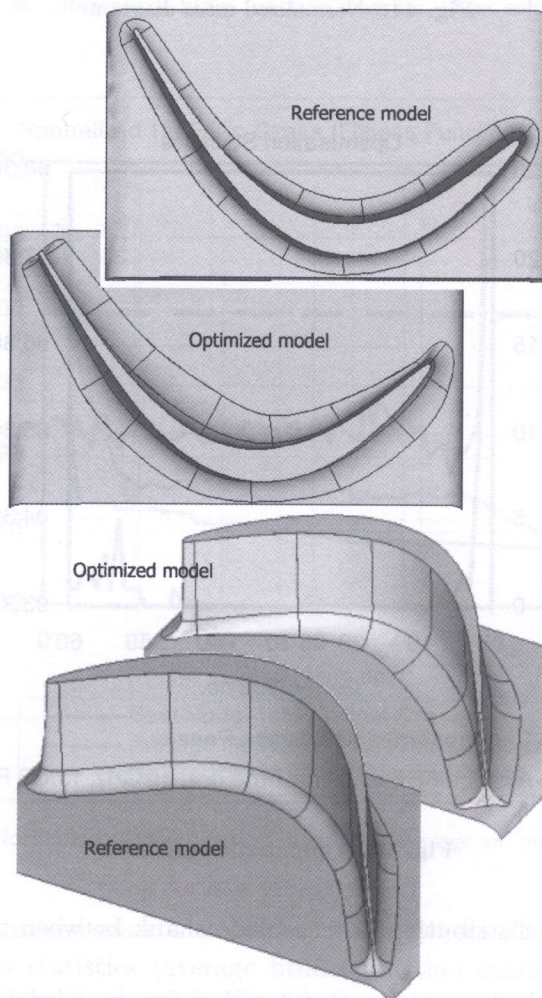


Fig. 14. Comparison of reference and optimised model shape

Table 2. Optimisation results summary

Generation no.	Best Model 1-st Principal Stress (normalized)	Best Model Mass (normalized)
0 (reference model)	1.0000	1.0000
60	0.8407	0.9998

8. CONCLUSIONS

The blade optimisation results confirm the robustness and efficiency of evolutionary algorithm in this application.

A final result of optimisation is 16% decrease of the 1-st principal stress vs. the reference model.

Further optimisation may be performed starting from the obtained solution, with different evolutionary algorithm parameters (the selection and the mutation operators). It seems that the solution may not be improved significantly in further generations with algorithm used in this stage of optimisation.

A successful application of the evolutionary algorithm for the blade shank optimisation with the described fitness function enables more detailed approach with variety of factors, which were omitted for simplicity in this case (fitness function including modal characteristics requirements, considering various turbine operating conditions, creep requirements, etc.).

ACKNOWLEDGMENT

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