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Discrete multicriteria reliability-based optimization of spatial trusses

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The paper presents problem of discrete multicriteria optimization of two-layer regular orthogonal spatial trusses. Three criteria of evaluation are taken into account, namely: minimum of weight, maximum of reliability and maximum of stiffness of the structure. To simplify the problem, decomposition techniques are applied. The decision variables are cross-sections of the truss members. The best possible cross section is selected for each bar from a discrete catalogue. Other decision variables (coordinated variables) describe also the geometry of the structure. The multicriteria reliability-based algorithm allows for evaluating the objective functions values and then finding sets of nondominated evaluations and solutions. Reliability of the structure is expressed by the Hasofer–Lind reliability index β.

1. Introduction

Structural reliability of spatial trusses is an important aspect of their design process. During last years many methods have been created for the purpose of determining the probability of structural failure [1, 10, 12, 15, 17]. The most important are the First Order Reliability Method (FORM) and the Second Order Reliability Method (SORM). Both methods are based on approximating the limit state functions by first and second order Taylor series expansions, respectively. The probability of failure can be expressed then by the reliability index. Although the methods of reliability analysis are constantly improved, the optimization problems are generally considered as deterministic ones. Especially discrete multicriteria optimization problems are difficult to solve when random character of selected parameters is taken into account. Solving multicriteria optimization problems with discrete design variables leads to a set of nondominated solutions which is possible to use in engineering practice.

2. Multicriteria optimization problem

The problem of multicriteria optimization, also called vector optimization, has been first formulated in 1896 by Italian economist — Vilfred Pareto. The main difference, as compared with scalar optimization problem, refers to the number of evaluation criteria. In multicriteria problems, several contradictory criteria are assumed to evaluate an object of the analysis [6, 13]. One or a few of the criteria may concern the reliability level [5, 7, 8, 11, 14, 16]. Taken into account more than one criterion requires application of a relatively complex and time-consuming method of solution, especially when random character of design parameters is analyzed. But formulation of a multicriteria problem with discrete constraints leads to solution that is more applicable in engineering practice.

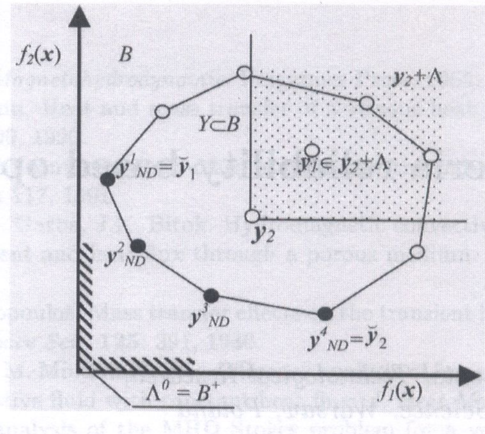

Fig. 1. Cone ordering relation in 2D space

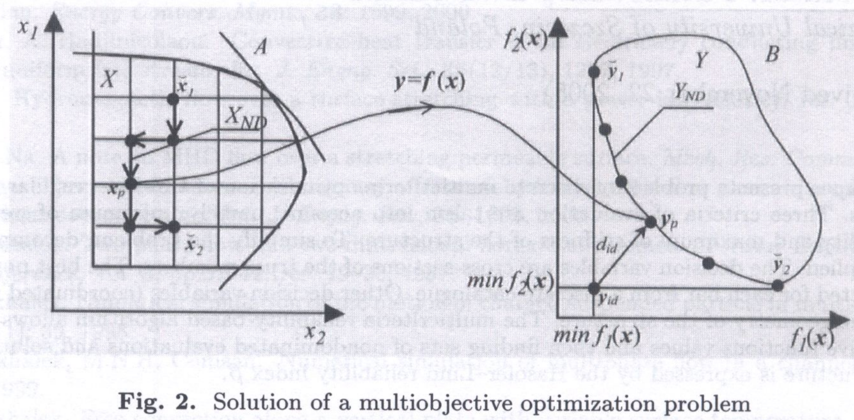

Fig. 2. Solution of a multiobjective optimization problem

Criteria of a vector optimization problem are defined by appropriate objective functions and ordered in a vector of objective functions $\boldsymbol{f}(\boldsymbol{x})$,

$$\boldsymbol{f}(\boldsymbol{x}) = \{f_j(\boldsymbol{x})\}, \qquad j = 1, J. \tag{1}$$

In discrete problems, each solution is evaluated, and the vector of evaluations is an element of discrete set of evaluations Y, included in a J-dimensional vector space, called the space of evaluations B. In the case of engineering objects, a linear ordering relation is usually established to order the set Y. For minimization problems the relation is defined by a cone Λ^0, specified by the positive orthant of the vector space B (Fig. 1). Another significant difference between scalar and vector optimization problems is the form of solution. In the case of multicriteria problems, the result is a K-element set of nondominated solutions X_{ND}. Through a transformation $\boldsymbol{f}(\boldsymbol{x})$ it gives the K-element set of nondominated evaluations Y_{ND} (Fig. 2).

The nondominated evaluations are the ones that cannot be uniquely improved with respect to the assumed ordering relation.

$$Y_{ND} = \{\boldsymbol{y}^k_{ND} \in Y : \neg \boldsymbol{y}_i \neq \boldsymbol{y}^k_{ND} \wedge \boldsymbol{y}^k_{ND} \in \boldsymbol{y}_i + \Lambda\} \tag{2}$$

The set of nondominated solutions X_{ND} is obtained as the result of an inverse transformation of Y_{ND}, defined also as

$$\boldsymbol{x}^k_{ND} \in X_{ND} \iff \neg\exists(\boldsymbol{x}_i \in Y)\ \forall(j \in J)\ f_j(\boldsymbol{x}_i) \leq f_j(\boldsymbol{x}^k_{ND}) \ \wedge \ \exists(j \in J)\ f_j(\boldsymbol{x}_i) < f_j(\boldsymbol{x}^k_{ND}). \tag{3}$$

Although the objective result of vector optimization problems is the set of nondominated solutions, engineering practice requires to select a single solution to be applied. The solution is selected from the set X_{ND}, and called preferred solution $\boldsymbol{x}_p$. The preferred solution is a compromise between the contradictory criteria of evaluation considered.

3. Reliability-based optimization of spatial trusses

3.1. Problem statement

There are given:

- vector of decision variables

$$\boldsymbol{x} = \{x_n\}, \qquad n = \overline{1, N}, \tag{4}$$

- vector of objective functions

$$\boldsymbol{f}(\boldsymbol{x}) = \{f_j(\boldsymbol{x})\}, \qquad j = \overline{1, J}, \tag{5}$$

- sets of inequality and equality constraints

$$\boldsymbol{g}(\boldsymbol{x}) = \{g_k(\boldsymbol{x})\} \leq \mathbf{0}, \qquad k = \overline{1, K}, \tag{6}$$

$$\boldsymbol{h}(\boldsymbol{x}) = \{h_m(\boldsymbol{x})\} = \mathbf{0}, \qquad m = \overline{1, M}, \tag{7}$$

- vector of random variables

$$\boldsymbol{X} = \{X_i\}, \qquad i = \overline{1, I}, \tag{8}$$

- vector of limit state functions

$$\boldsymbol{G}(\boldsymbol{X}) = \{G_t(\boldsymbol{X})\}, \qquad t = \overline{1, T}, \tag{9}$$

and a set of parameters that are constant during optimization process. Find the set of nondominated solutions and evaluations, according to Eqs. (2) and (3).

In reliability-based optimization problems, as distinct from deterministic ones, random character of selected design parameters is taken into account (Eq. (8)). The limit state functions that define the failure of structure are described in Eq. (9). Reliability of each discrete solution is evaluated as a component of objective functions vector. Probability of failure is usually contradictory with economic criteria, so the preferred solution is the compromise between them. The reliability level may be also the component of the inequality constraints vector (6).

3.2. Algorithm of solution

The proposed method of solution is based on the *OPTYTRUSS* system, that has been extended with the algorithm to evaluate reliability of a truss and make it a component of the objective functions vector. The system conducts static analysis, design, and discrete multicriteria optimization of spatial trusses. The cross sections of bars and geometry design parameters are varied in optimization process. The cross sections are selected from a discrete catalogue that contains the available steel products. The process of selection is performed in deterministic way. Others design variables are constrained by equalities (7), making the problem a discrete one. Solution of such problems is quite sophisticated and time-consuming. Gradient methods cannot be applied, especially when objective functions are given in implicit forms. More effective are local search methods, adapted for discrete problems.

To make the solution more efficient, two-level algorithm has been proposed. The algorithm is based on the theory of parametric decomposition and hierarchic optimization [4]. It allows to reduce dimension of the vector of decision variables and considerably simplify the problem. The vector of decisions variables is decomposed into two local vectors

$$\boldsymbol{x} \longrightarrow \boldsymbol{x}_l^{(1)};\ \boldsymbol{x}_l^{(2)}. \tag{10}$$

The components of the vector $\boldsymbol{x}_l^{(1)}$ are cross sections of the bars

$$\boldsymbol{x}_l^{(1)} = \{x_{Ai}\}, \qquad i = \overline{1, k_b}\,, \tag{11}$$

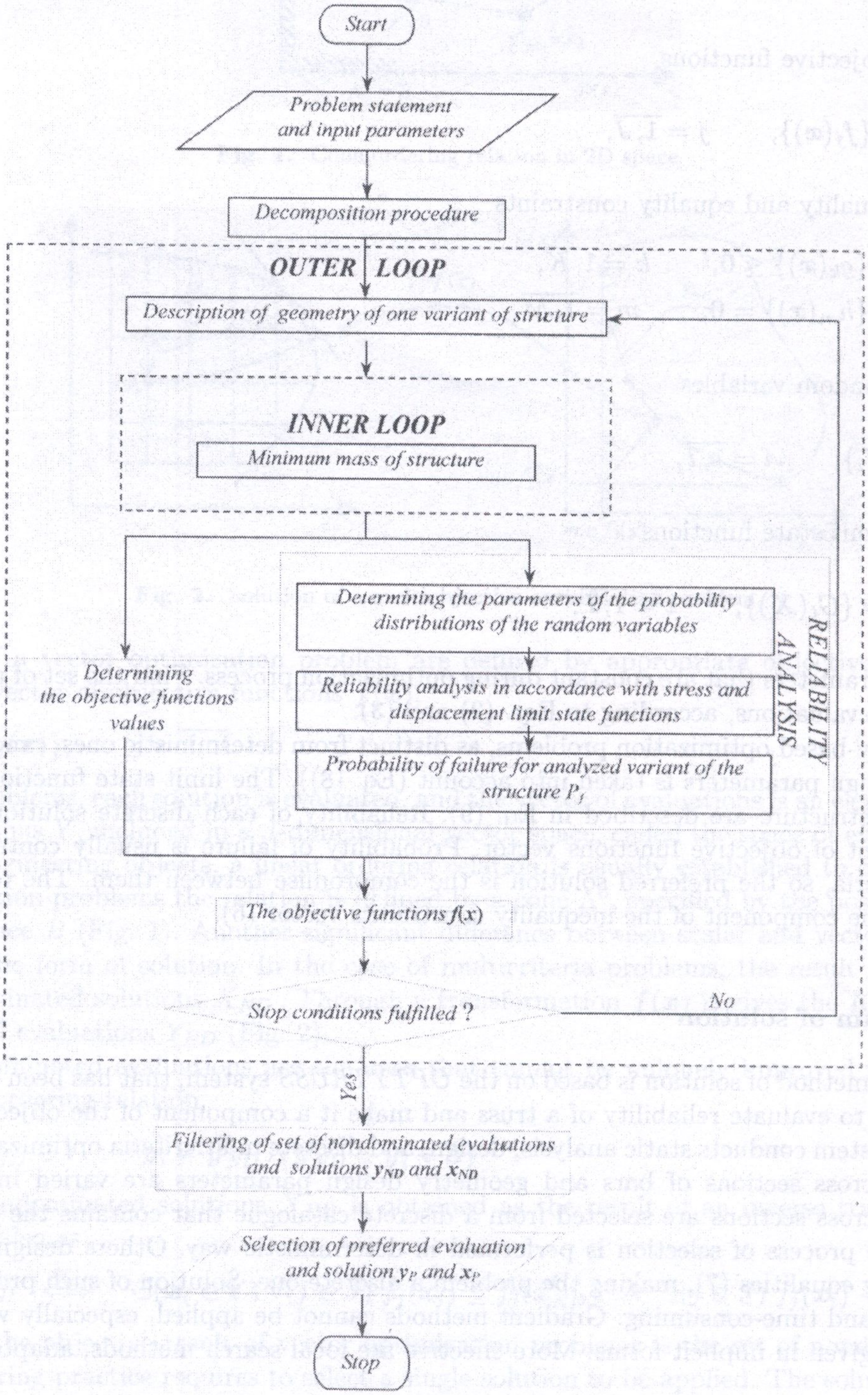

Fig. 3. Algorithm of the *OPTYTRUSS* system

where k_b is the number of the truss bars. Components of the vector $\boldsymbol{x}_l^{(2)}$ are variables that describe geometry of the structure (i.e. shape, height and depth, support system, etc.),

$$\boldsymbol{x}_l^{(2)} = \{x_j\}, \qquad j = \overline{k_b+1, N}, \tag{12}$$

For real structures, dimensionality of the vector $\boldsymbol{x}_l^{(1)}$ is much larger than for $\boldsymbol{x}_l^{(2)}$. The decomposition allows for separating the process of minimization of truss cross sections and for analyzing it as a scalar problem, constituting the inner loop in multicriteria optimization process (Fig. 3). The vector $\boldsymbol{x}_l^{(2)}$ includes the coordinate variables. In the first stage of the analysis, starting values are established to describe geometry of single variant of the structure. Next, the starting values are treated as parameters in the second stage. For each defined variant the optimum bars' cross sections are evaluated in accordance with minimum mass criterion. During the process, dead, live and environmental loads are taken into account in load combinations. After that, for each variant of the truss, values of the objective functions, including reliability, are determined in the outer loop (Fig. 3). The outer loop is controlled by a local search method to minimize the number of analyzed variants and make the process more efficient.

Three alternative methods of structural reliability analysis have been implemented in the *OPTYTRUSS* system. The first and most efficient is *First Order Reliability Method*. The probability of failure is determined by Hasofer–Lind index β, after Rosenblatt transformation. The second method is a crude Monte Carlo method. The last method is integration, referring directly to the definition of probability of failure

$$P_f = \int_{\Omega_f} f_X(\boldsymbol{X})\,\mathrm{d}\boldsymbol{X} \tag{13}$$

where Ω_f is the failure domain, $f_X(\boldsymbol{X})$ — global probability density function, X — vector of random variables. The Monte Carlo and the integral methods have been employed to test correctness of the *FORM* method and Rosenblatt transformation, which are more numerically complex. The

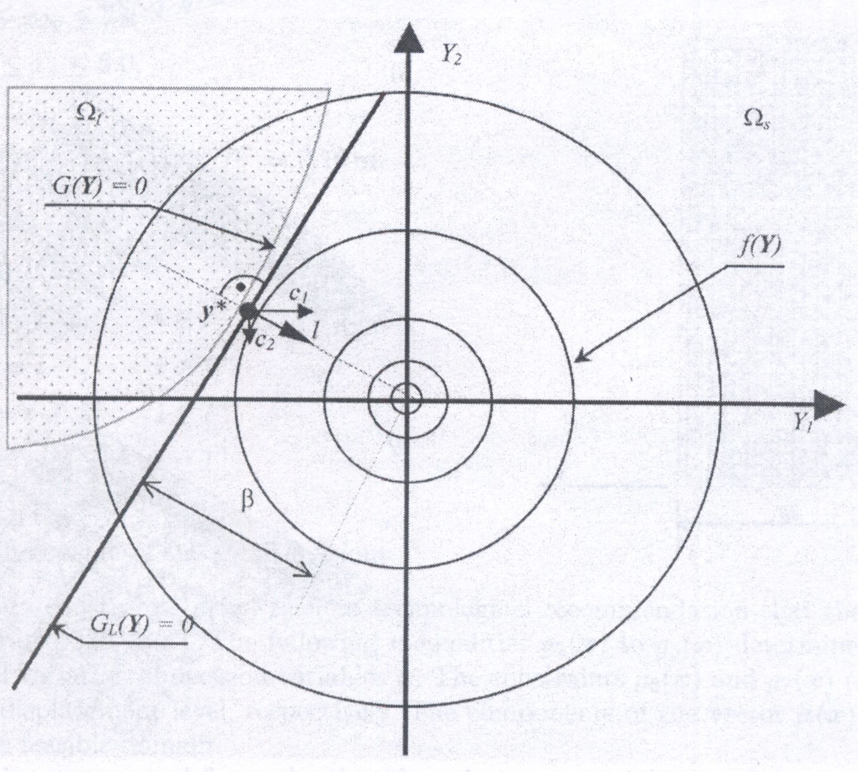

Fig. 4. Linear approximation of a limit state function

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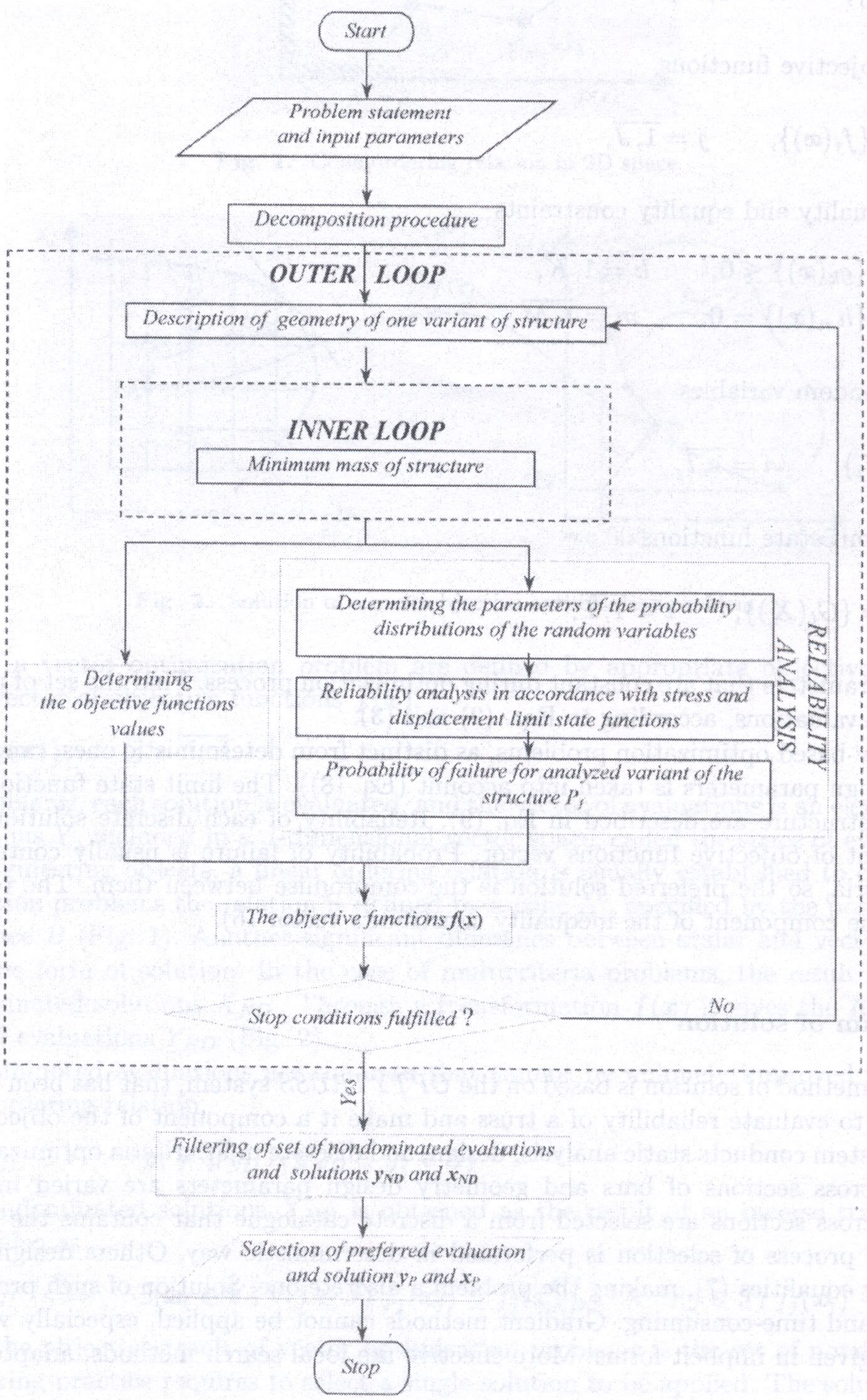

Fig. 3. Algorithm of the *OPTYTRUSS* system

where k_b is the number of the truss bars. Components of the vector $\boldsymbol{x}_l^{(2)}$ are variables that describe geometry of the structure (i.e. shape, height and depth, support system, etc.),

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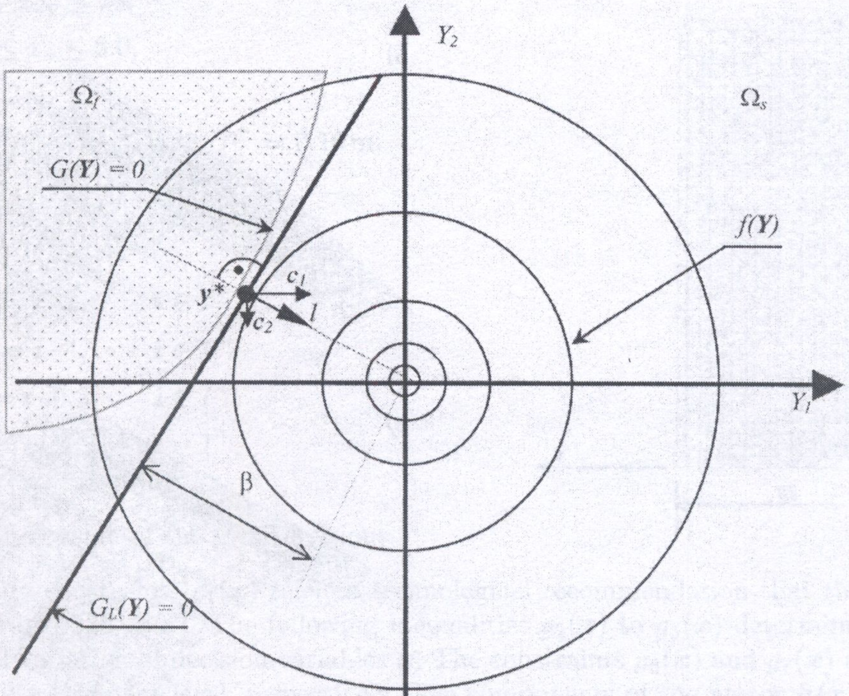

Fig. 4. Linear approximation of a limit state function

reliability analysis proceeds in parallel to the other deterministic objective functions (Fig. 3). Apart from the method of the analysis, the input data have the same form. It is required to formulate the vector of the random variables X. For each variable, appropriate probability distribution and its parameters are established. To simplify the problem, it is assumed that the variables are statistically independent.

For real engineering problems the *FORM* method is the most efficient and precise enough. In case of spatial trusses, stress and displacement limit state functions are regular and may be approximated by a first order Taylor expansion (Fig. 4).

4. Numerical examples

The numerical example for the proposed algorithm considers the analysis of a steel spatial truss which is a cover of a sports hall. The dimensions of the cover are 40×80 m (Fig. 5). The upper and lower layers are realized as orthogonal and parallel grids. The structure is supported along the longer walls of the hall. Bars of the truss are made of hot-rolled tubes, connected at nodes as joints [2, 3, 9].

In the presented example, random character of design parameters and loads is considered. During the analysis, the polyoptimum shape of the cover (Fig. 5), the modular distance between nodes, and

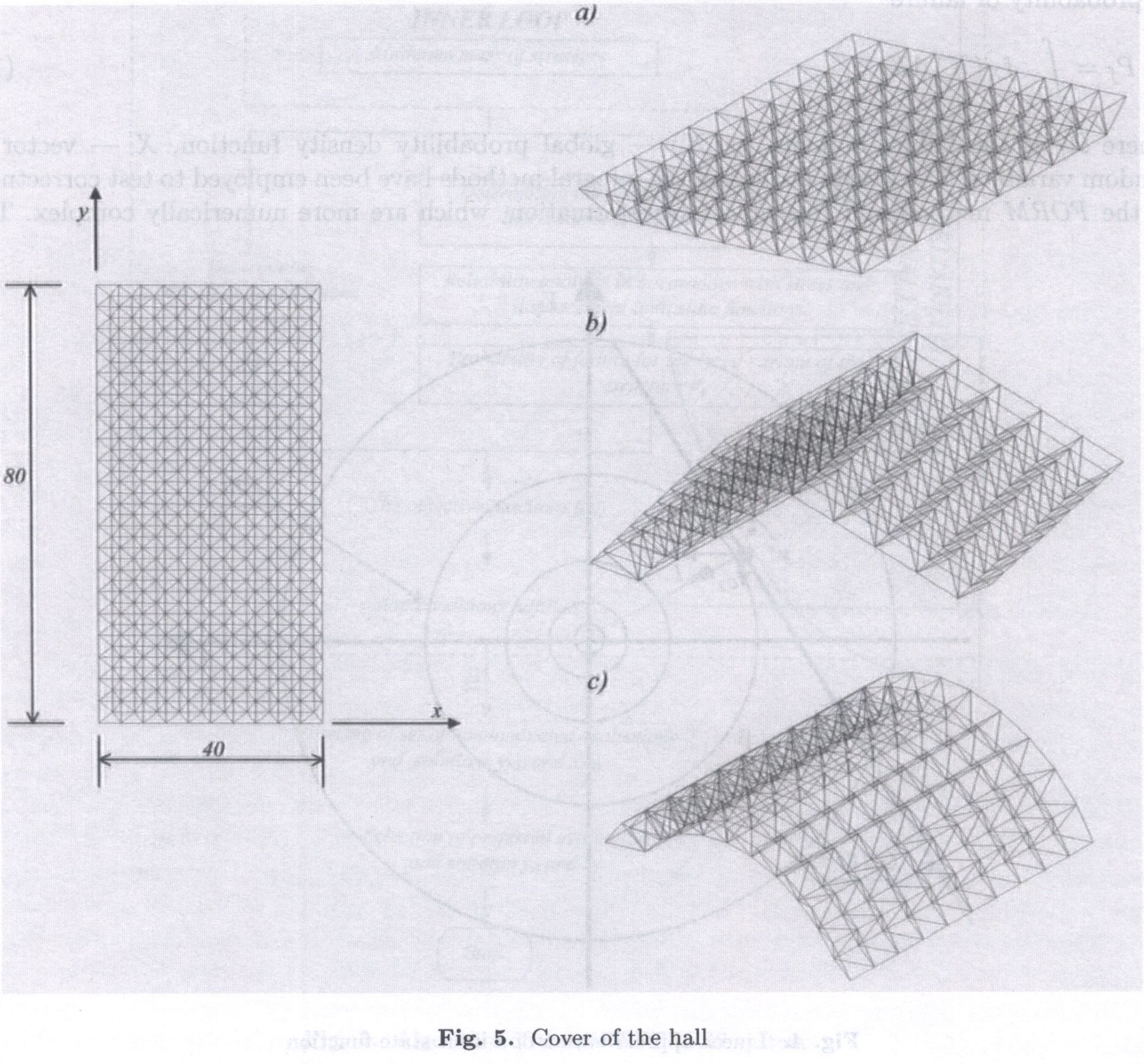

Fig. 5. Cover of the hall

the depth and rise of the cover are determined. Thus, the four-element vector of decisions variables x is taken into account

$$\boldsymbol{x} = \{x_n\}, \qquad n = \overline{1,4}, \tag{14}$$

where

x_1 – number of the shape of the cover,
  $x_1 = 1$ – one-sloped plane cover,
  $x_1 = 2$ – two-sloped cover,
  $x_1 = 3$ – cylindrical cover with circular arc,
x_2 – number of modular divisions, described by a pair of numbers x_{2x} and x_{2y},
x_3 – depth of the cover,
x_4 – rise of the cover.

The constraints define the feasible domain of the problem

$$X = \{\boldsymbol{x} \in A : \boldsymbol{g}(\boldsymbol{x}) \leq \boldsymbol{0},\ \boldsymbol{h}(\boldsymbol{x}) = \boldsymbol{0}\}. \tag{15}$$

The decision variables must satisfy the inequalities $\boldsymbol{g}(\boldsymbol{x})$ and equalities $\boldsymbol{h}(\boldsymbol{x})$

$$\boldsymbol{g}(\boldsymbol{x}) = \{g_k(\boldsymbol{x})\} \leq \boldsymbol{0}, \qquad k = \overline{1,7}, \tag{16}$$

$$\boldsymbol{h}(\boldsymbol{x}) = \{h_m(\boldsymbol{x})\} = \boldsymbol{0}, \qquad m = \overline{1,4}, \tag{17}$$

where

$$g_1(\boldsymbol{x}): \ \frac{\sqrt{6}}{6} < \frac{x_3}{a} < \frac{\sqrt{6}}{2}, \qquad a = \frac{40}{x_{2x}} = \frac{80}{x_{2y}}, \tag{18}$$

$$g_2(\boldsymbol{x}): \ 1 \leq x_1 \leq 3, \tag{19}$$

$$g_3(\boldsymbol{x}): \ 10 \leq x_{2x} \leq 14, \tag{20}$$

$$g_4(\boldsymbol{x}): \ 20 \leq x_{2y} \leq 28, \tag{21}$$

$$g_5(\boldsymbol{x}): \ 2.0 \leq x_4 \leq 6.0, \tag{22}$$

$$g_6(\boldsymbol{x}): \ \beta \geq 2.0, \tag{23}$$

$$g_7(\boldsymbol{x}): \ f \leq f^{gr}, \qquad f^{gr} = 0.16\,\text{m}, \tag{24}$$

and

$$h_1(\boldsymbol{x}): \ x_1 \in I^+, \tag{25}$$

$$h_2(\boldsymbol{x}): \ x_{2x} = i \cdot 2, \qquad i \in I^+, \tag{26}$$

$$h_3(\boldsymbol{x}): \ x_{2y} = i \cdot 4, \qquad i \in I^+, \tag{27}$$

$$h_4(\boldsymbol{x}): \ x_3 = i \cdot 0.6, \qquad i \in I^+, \tag{28}$$

$$h_5(\boldsymbol{x}): \ x_4 = i \cdot 2, \qquad i \in I^+, \tag{29}$$

I^+ – the set of integers,
a – modular dimension of the layer divisions.

The inequality constraints $g_1(\boldsymbol{x})$ refer to technological recommendation that the angle between bars is within range $[30°, 60°]$. The following inequalities $g_2(\boldsymbol{x})$ to $g_5(\boldsymbol{x})$ determine the upper and lower bounds of variation of decision variables $\boldsymbol{x}$. The constraints $g_6(\boldsymbol{x})$ and $g_7(\boldsymbol{x})$ restrict the limit reliability and displacement level, respectively. The components of the vector $\boldsymbol{h}(\boldsymbol{x})$ assure discrete character of the feasible domain.

Three criteria are assumed for evaluating the solutions — minimum of the mass of the cover, minimum of the greatest nodal displacement, and minimum of probability of failure or maximum of reliability level. The first and the second criteria are treated as deterministic functions. All criteria are expressed formally in the vector of objective functions

$$f(x) = \{f_j(x)\}, \qquad j = \overline{1,3}, \tag{30}$$

where

$$f_1(x) = \sum A_i l_i \rho, \tag{31}$$

$$f_2(x) = \sum \frac{N_i \overline{N_i} l_i}{E A_i}, \tag{32}$$

$$f_3(x) = P_f(-\beta), \tag{33}$$

where

$\beta = \min[(y^T y)^{\frac{1}{2}}]$,
i – the number of a truss bar,
A_i – cross sectional area of the i-th bar,
l_i – length of the i-th bar,
ρ – density of steel, $\rho = 7850$ kg/m^3,
k_b – the number of bars in the truss,
$\overline{N_i}$ – force in the i-th bar caused by virtual load,
N_i – force in the i-th bar caused by real load,
E – Young modulus,
y – vector of coordinates of the point on the failure surface.

The cross sections of bars are selected from a five-element discrete catalogue [18] (Table 1).

The vector of random variables contains fifteen components, presented in Table 2. The unbounded distributions has been truncated ($\pm 5\sigma$ in case of double-sided unbounded distributions) and normalized with the use of Rosenblatt transformation.

Two limit state functions are assumed — the stress limit function (34) and the displacement limit function (36),

$$G_l^i = 1 - \frac{|\sigma_i|}{\sigma_i^{lt}}, \qquad i = \overline{1, k_b}, \tag{34}$$

where σ_i is the stress in i-th element and σ_i^{lt} — the limit stress, defined as

$$\sigma_i^{lt} = \{\sigma_i^0 \Leftrightarrow \sigma i \geq 0\} \vee \{\sigma_i^{cr} \Leftrightarrow \sigma_i < 0\}, \tag{35}$$

$$G_2^i = 1 - \frac{|\, q_i \,|}{q^{lt}}, \qquad i = \overline{1, k_n}, \tag{36}$$

where q_i is the i-th node displacement and q^{lt} — the limit displacement. In Eq. (35), σ_i^0 is the yield stress of steel and σ_i^{cr} is the Euler buckling stress.

The reliability index β is defined as

$$\beta = \min(\min \beta_i\,,\, \beta^f) \tag{37}$$

where β_i concerns the stress in the i-th bar and β^f — the displacement of the truss.

During the analysis, six types of loads are taken into account:

- dead load — mass of the structure and the roofing,
- snow load,
- wind load in the direction $(+X)$ (Fig. 5),

Table 1. Catalogue of the feasible cross sections of the bars

D_0 [mm]	t_0 [mm]
30.0	2.9
70.0	3.6
139.7	6.3
273.0	12.5
355.6	16.0

Table 2. Random variables assumed in the example

Variable		Probability distribution		
Name	Symbol	Type	Expected value	Coefficient of variation
Diameter of bars	$X_1 - X_5$	log-normal	nominal value (Table 1)	0.01
Thickness of the cross section wall	$X_6 - X_{10}$	log-normal	nominal value (Table 1)	0.01
Yield stress of steel	X_{11}	log-normal	360 MPa	0.10
Young modulus	X_{12}	log-normal	225 GPa	0.05
Dead load multiplier	X_{13}	normal	1.0	0.10
Snow load multiplier	X_{14}	Frechet	1.0	0.20
Wind load multiplier	X_{15}	Gumbel (max)	1.0	0.20

- wind load in the direction $(-X)$ (Fig. 5),
- wind load in the direction $(+Y)$ (Fig. 5),
- wind load in the direction $(-Y)$ (Fig. 5).

The directions and the areas of actions of loads are assumed in accordance with the design codes. Ten combinations of loads are analyzed:

- dead load,
- dead load + snow load,
- dead load + wind load in the direction $(+X)$,
- dead load + wind load in the direction $(-X)$,
- dead load + wind load in the direction $(+Y)$,
- dead load + wind load in the direction $(-Y)$,
- dead load + snow load + wind load in the direction $(+X)$,
- dead load + snow load + wind load in the direction $(-X)$,
- dead load + snow load + wind load in the direction $(+Y)$,
- dead load + snow load + wind load in the direction $(-Y)$.

Table 3. Set of nondominated solutions and nondominated evaluations of the analyzed cover

Number of realization	Decisions variables				Objective functions		
	Shape	Number of divisions	Depth	Rise	Mass [kg/m²]	Displacement [cm]	Reliability index β
	x_1	x_2	x_3	x_4	$f_1(x)$	$f_2(x)$	$f_3(x)$
1	one-sloped cover	10 x 20	3.0	2.0	21.627	10.85	2.389
2		10 x 20	3.0	4.0	21.595	10.91	2.384
5		10 x 20	3.6	4.0	22.836	8.28	2.632
6		10 x 20	3.6	6.0	22.993	8.39	3.151
7		10 x 20	4.2	2.0	22.944	5.92	2.216
1	two-sloped cover	10 x 20	3.0	2.0	21.462	10.73	2.119
4		10 x 20	3.6	2.0	22.687	8.25	2.637
5		10 x 20	3.6	4.0	22.553	8.45	3.112
13		12 x 24	3.6	2.0	25.615	7.95	2.057
2	cylindrical with circular arc cover	10 x 20	3.0	4.0	21.313	11.09	2.399
3		10 x 20	3.0	6.0	23.472	11.13	2.514
5		10 x 20	3.6	4.0	22.051	8.75	2.091
6		10 x 20	3.6	6.0	21.517	9.33	2.068
7		10 x 20	4.2	2.0	25.513	6.13	2.510
8		**10 x 20**	**4.2**	**4.0**	**25.316**	**6.35**	**3.975**
10		12 x 24	3.0	2.0	23.464	9.86	2.530
11		12 x 24	3.0	4.0	22.734	10.56	2.386
20		14 x 28	3.0	4.0	25.034	9.50	2.180

During the process of selection of the cross section areas, the most disadvantageous load combinations is taken into account for each bar.

The problem is solved according to the presented algorithm as a hierarchical optimization problem. Parametric decomposition of decision variables is applied. The variable x_1 that describes shape of the structure is treated as the coordinated one. Thus three local problems are obtained.

The results of the analysis, namely the set of nondominated evaluations and nondominated solutions, are presented in Table 3.

In order to solve the problem, the exhaustive search method is employed. The dimension of the solutions space and the small number of discrete alternatives of the decision variables make this method efficient enough. The local search methods for discrete problems can be also employed for the example, but the numerical effectiveness is similar. The important advantage of the exhaustive search method is its ability to find the global set of nondominated solutions.

Values of objective functions are normalized and the preferred evaluation is selected by a distance function method. The preferred evaluation $\boldsymbol{y}_p = \boldsymbol{y}_8$ and corresponding preferred solution $\boldsymbol{x}_p = \boldsymbol{x}_8$ is marked in Table 3,

$$\boldsymbol{y}_p = \{25.3;\ 6.35;\ 3.975\}, \tag{38}$$

$$\boldsymbol{x}_p = \{3;\ (10 \times 20);\ 4.2;\ 4.0)\}. \tag{39}$$

The preferred solution is the cylindrical cover, with modular divisions of 10 × 20, depth of 4.2 m and rise of 4.0 m. Mass of the preferred structure is 25.3 kg/m² of the projection of the cover, maximum displacement of the node is 6.35 cm and reliability index is 3.975. The reliability level of the structure equals 0.99996.

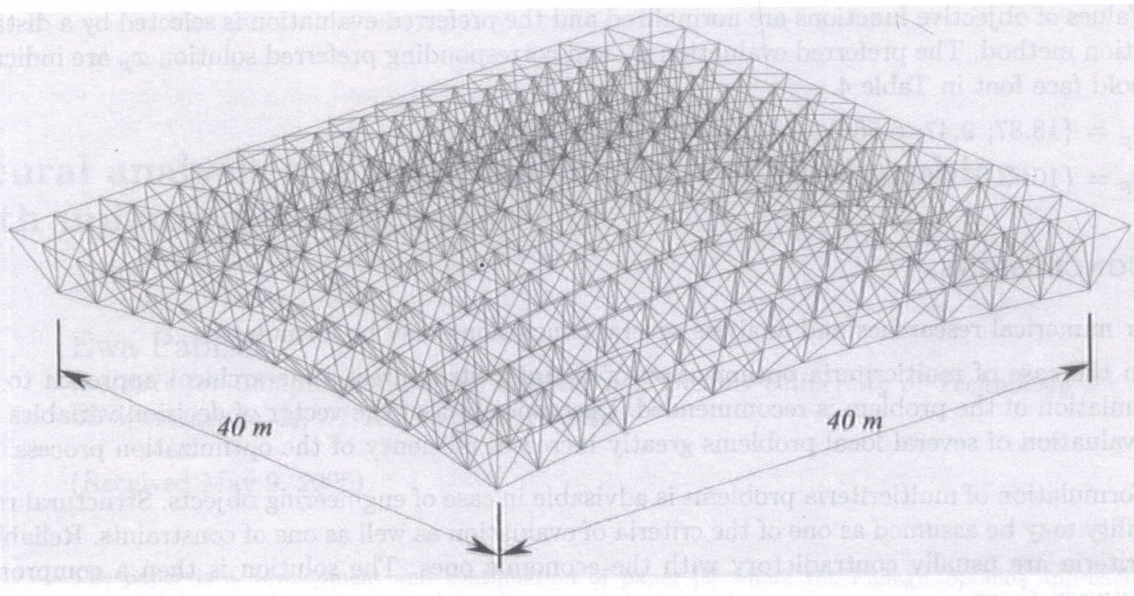

Fig. 6. Analyzed spatial truss

Another numerical example of proposed algorithm is polyoptimum design of the truss of dimension 40×40 m, shaped in Fig. 6. Similar as above, random character of design parameters and loads is considered. During the analysis, the polyoptimum modular distance between nodes, the depth, and rise of the cover are determined. Thus, the three-element vector of decision variables $\boldsymbol{x}$ is taken into account,

$$\boldsymbol{x} = \{x_n\}, \qquad n = \overline{1,3}. \tag{40}$$

The decision variables must satisfy the inequalities $\boldsymbol{g}(\boldsymbol{x})$ and equalities $\boldsymbol{h}(\boldsymbol{x})$,

$$\boldsymbol{g}(\boldsymbol{x}) = \{g_k(\boldsymbol{x})\} \leq \boldsymbol{0}, \qquad k = \overline{1,6}, \tag{41}$$

$$\boldsymbol{h}(\boldsymbol{x}) = \{h_m(\boldsymbol{x})\} = \boldsymbol{0}, \qquad m = \overline{1,3}. \tag{42}$$

As above, the inequality constraints refer to technological recommendations, determine the upper and lower bounds of variation of decision variables $\boldsymbol{x}$ and restrict the limit displacement and reliability level. The components of the vector $\boldsymbol{h}(\boldsymbol{x})$ assure discrete character of the feasible domain.

Another elements of the problem statement, like objective functions, random variables, limit function and load combinations, are established as in the example before. The results of the analysis, namely the set of nondominated evaluations and nondominated solutions, are presented in Table 4.

Table 4. Set of nondominated solutions and nondominated evaluations of the shell

Number of realization	Decision variables			Objective functions		
	Number of divisions	Depth	Rise	Mass [kg/m²]	Displacement [cm]	Reliability index β
	x_1	x_2	x_3	$f_1(x)$	$f_2(x)$	$f_3(x)$
1	**10**	**3.0**	**2.0**	**18.87**	**9.47**	**6.566**
2	10	3.0	4.0	17.86	10.23	2.729
3	10	3.0	6.0	18.18	8.58	4.981
4	10	3.6	6.0	18.26	7.98	2.157
5	12	3.6	4.0	22.09	8.39	2.248
6	12	3.6	6.0	21.80	8.03	2.842
7	14	3.6	2.0	23.06	7.96	2.129
8	14	3.6	4.0	23.33	7.92	2.498

Values of objective functions are normalized and the preferred evaluation is selected by a distance function method. The preferred evaluation $\boldsymbol{y}_p$ and corresponding preferred solution $\boldsymbol{x}_p$ are indicated by bold face font in Table 4.

$$\boldsymbol{y}_p = \{18.87;\ 9.47;\ 6.566\}, \tag{43}$$

$$\boldsymbol{x}_p = \{10;\ 3.0;\ 2.0)\}. \tag{44}$$

5. Conclusions

After numerical researches and analysis several conclusions may be formulated.

- In the case of multicriteria optimization of complex structures, a hierarchical approach to formulation of the problem is recommended. Decomposition of the vector of decision variables and evaluation of several local problems greatly increases efficiency of the optimization process.
- Formulation of multicriteria problems is advisable in case of engineering objects. Structural reliability may be assumed as one of the criteria of evaluation as well as one of constraints. Reliability criteria are usually contradictory with the economic ones. The solution is then a compromise between them.
- The presented algorithm is efficient enough to become a commonly applied engineering tool. All technological and design requirements and real loads combinations may be taken into account in the analysis.

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