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MHD nonlinear flow and heat transfer over a stretching porous surface of constant heat flux

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MHD nonlinear steady flow and heat transfer over a porous surface stretching with a power-law velocity and of constant heat flux is investigated. The governing nonlinear partial differential equations are reduced to nonlinear ordinary differential equations by using similarity transformation. As the presented solution method requires the magnetic field to vary in space in a specific manner, a special form for the variable magnetic field is chosen. Resulting equations are numerically solved by Runge–Kutta shooting method. Values of skin-friction and rate of heat transfer are obtained. The effect of magnetic field, stretching parameter, magnetic interaction parameter, suction parameter and Prandtl number over a flow field and other physical quantities have been discussed in detail.

1. Introduction

In recent years, the study of flow problem with heat transfer over stretching surfaces has generated considerable interest because of its numerous industrial applications such as in the manufacture of sheeting material through an extrusion process. A number of technical processes concerning polymers involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid. Information regarding the momentum exchange and heat transfer along such a stretching surface may be useful in determining the heat transfer rates in manufacturing processes.

Application of suction or blowing finds vital role in industrial applications. As it was pointed out recently [4], heat transfer coefficient in film boiling could be substantially enlarged by continuously removing fluid through a porous heated surface. In view of all these practical applications, this work is devoted to a problem of such kind.

Sakiadis [12] was one of the first to discuss the laminar boundary layer flow of a viscous and incompressible fluid caused by a moving rigid surface. The flow over a linearly stretching sheet for the steady two-dimensional problem was analysed by Crane [9]. These types of flows usually occur in the drawing of plastic films and artificial fibres. Subsequently, several investigators studied various aspects of this problem such as the effect of the mass transfer, wall temperature and magnetic field.

Carragher [5] analysed the same problem as Crane to study heat transfer and obtained the Nusselt number for the entire range of Prandtl number (Pr). Dutta *et al.* [10] considered the temperature field in flow over a stretching sheet with uniform heat flux. Chen with Char [8] delineated the effects of variable surface temperature and variable surface heat flux over the heat transfer characteristics of a linearly stretching sheet.

The flow and heat transfer characteristics of an electrically conducting fluid with the effect of magnetic field over a stretching sheet are of vital importance in the industrial applications. Due to such applications, investigations on the convective heat transfer in an electrically conducting and heat generating fluid flowing over a linearly stretching surface with a uniform free stream were carried out.

Chakrabarti and Gupta [7] outlined the hydromagnetic flow and heat transfer in a fluid initially at rest over a stretching sheet at a different uniform temperature. The flow and heat transfer over a stretching sheet in an electrically conducting fluid for both prescribed wall temperature and prescribed heat flux conditions was analyzed by Kumari *et al.* [11]. The governing equations were solved numerically using shooting method. However, the analysis was restricted to the case of linear stretching sheet.

Hydromagnetic boundary layer flow of a viscous incompressible fluid which is caused by a sheet stretching according to a power-law velocity distribution in the presence of a magnetic field was investigated by Chaim [6].

Recently, Anjali Devi and Thiagarajan [2] have analyzed the problem of steady nonlinear hydromagnetic flow and heat transfer over a stretching surface of variable temperature. But so far, no attempt has been made to analyze the problem of nonlinear MHD flow and heat transfer over a porous surface stretching with a power-law velocity and of constant heat flux in the presence of variable transverse magnetic field $B(x)$ and hence this work analyses such problem. Utilizing similarity transformation, the governing nonlinear partial differential equations are transformed into nonlinear ordinary differential equations and they are solved numerically by using fourth-order Runge–Kutta shooting method. Numerical results are obtained for various values of physical parameters. The numerical solutions are illustrated graphically.

2. Formulation of the Problem

Nonlinear, steady two-dimensional, laminar flow and heat transfer in the case of an incompressible viscous and electrically conducting fluid over a stretching porous surface of constant heat flux is considered. The surface stretches with a power-law velocity. The x-axis is assumed to be taken along the surface and y-axis is normal to the surface. The boundary layer equations for the velocity field in the presence of a variable magnetic field $\vec{B} = B(x)\hat{j}$ are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)u}{\rho}, \tag{2}$$

where u and v are the velocity components in the x and y directions respectively, ν is fluid kinematic viscosity, σ is the electrical conductivity, ρ is the density of the fluid, $B(x)$ is the applied magnetic field and $\hat{j}$ is the unit vector along the y direction. The boundary conditions are given by

$$\begin{aligned} &\text{At } y=0 \qquad u = U(x) = ax^m, \quad v = -v_w, \\ &\text{As } y\to\infty, \qquad u = 0. \end{aligned} \tag{3}$$

where a is a dimensional constant, m is the index of power-law velocity which is constant and v_w is the suction velocity. Here it is assumed that the induced magnetic field is negligible, the external electrical field is zero and the electric field due to polarization of charges is also negligible.

The following change of variables are introduced as in Banks [3] and Afzal [1] so that

$$\eta(x,y) = \left[\frac{(1+m)U(x)}{2\nu x}\right]^{1/2} y, \tag{4}$$

$$\psi(x,y) = \left[\frac{2\nu x U(x)}{(1+m)}\right]^{1/2} f(\eta), \tag{5}$$

where η is the similarity space variable and $f(\eta)$ is the dimensionless stream function. Now introducing

$$u = \frac{\partial \psi}{\partial y} = U(x)\, f'(\eta), \tag{6}$$

$$v = -\frac{\partial \psi}{\partial x}, \tag{7}$$

equation of continuity is automatically satisfied.

Choosing the variable magnetic field $B(x)$ in the form as $B(x) = B_0 x^{(m-1)/2}$ and applying the transformations (4)–(7) in Eq. (2) and in the boundary conditions (3), we get

$$f''' + ff'' - \beta f'^2 - M^2 f' = 0, \tag{8}$$

$$f(0) = S, \; f'(0) = 1, \; f'(\infty) = 0, \tag{9}$$

where $\beta = \frac{2m}{1+m}$ is the stretching parameter, $M^2 = \frac{2\sigma B_0^2}{\rho a(1+m)}$ is the magnetic parameter, and $S = \sqrt{\frac{2}{(1+m)}} K_s$ is the suction parameter, and where K_s is a non-dimensional constant and $S > 0$. In order to have $f(0)$ as a constant, suction velocity v_w must vary along the surface. Hence v_w is chosen as $v_w = bx^{(m-1)/2}$ where b is a dimensional constant.

The boundary layer energy equation neglecting viscous dissipation, internal heat generation or absorption and Joule's dissipation is given by

$$\rho C_p \left[u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} \right] = K\frac{\partial^2 T}{\partial y^2} \tag{10}$$

where K is the thermal conductivity, C_p is the specific heat at constant pressure.

The boundary conditions pertaining to temperature are

$$\begin{aligned} &\text{At } y = 0 \qquad -K\frac{\partial T}{\partial y} = q_w\,, \\ &\text{As } y \to \infty, \qquad T = T_\infty\,, \end{aligned} \tag{11}$$

where T is the temperature of the fluid, T_∞ is the temperature of the fluid at infinity and q_w is the wall heat flux.

The dimensionless variable θ is defined by introducing the following

$$T = T_\infty + \frac{q_w}{K}\theta(\eta)\sqrt{\frac{2\nu x}{(1+m)U(x)}}. \tag{12}$$

Substituting Eq. (12) in Eq. (10) and in the boundary conditions (11), the non-dimensional boundary layer equation for temperature is obtained as

$$\theta'' + Pr\left[\frac{(m-1)}{(m+1)}f'\theta + f\theta'\right] = 0. \tag{13}$$

Here $Pr = \frac{\mu C_p}{K}$ is the Prandtl number, where μ is the fluid dynamic viscosity. The boundary conditions are

$$\begin{aligned} &\text{At } y = 0 \qquad \theta'(0) = -1, \\ &\text{As } y \to \infty, \qquad \theta(\infty) = 0. \end{aligned} \tag{14}$$

Hence the governing equations of the problem are the following nonlinear differential equations

$$f''' + ff'' - \beta f'^2 - M^2 f' = 0, \tag{15}$$

$$\theta'' + Pr\left[\frac{(m-1)}{(m+1)}f'\theta + f\theta'\right] = 0, \tag{16}$$

with the boundary conditions

$$\begin{aligned} &f(0) = S, \qquad f'(0) = 1, \qquad \theta'(0) = -1, \\ &\qquad\qquad\quad f'(\infty) = 0, \qquad \theta(\infty) = 0. \end{aligned} \tag{17}$$

3. Solution of the problem

Equations (15) and (16) with the boundary conditions (17) are nonlinear ordinary differential equations and they are solved numerically using the fourth-order Runge–Kutta shooting method. Initial guessing for the values of $f''(0)$ and $\theta(0)$ is an important thing to initiate the shooting process. The success of the procedure depends very much on how good this guess is. Flow field and temperature are analyzed for different values of the physical parameters S, m, M^2, β and Pr.

4. Results and discussion

The numerical solution of MHD nonlinear steady flow and heat transfer over a porous surface stretching with a power-law velocity and of constant heat flux is sought for various values of the physical parameters. The numerical values obtained are shown graphically.

The effect of suction on velocity is demonstrated with the help of Fig. 1. It is observed from this figure that the effect of suction is to decelerate the velocity. It is interesting to note the effect of suction on the boundary layer thickness in this figure. The boundary layer thickness is reduced as suction increases. Away from the wall the velocity profiles asymptotically tends to zero.

The effect of suction over the temperature is still dominant over the temperature and is noted in Fig. 2. As suction increases, temperature decreases elucidating the fact that the effect of suction is to reduce the temperature. Thermal boundary layer thickness is steeply reduced due to the effect of suction.

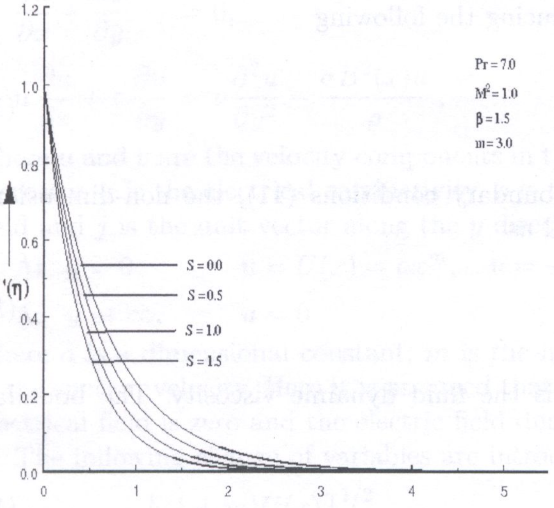


Fig. 1. Velocity profiles for various S

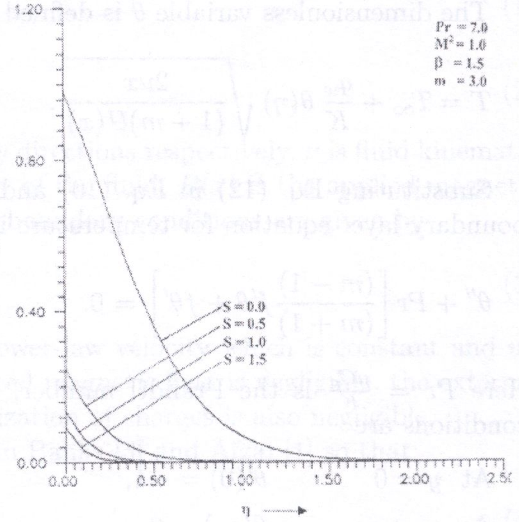


Fig. 2. Temperature profiles for different values of S

Figure 3 demonstrates the effect of magnetic field over dimensionless velocity profiles. The effect of magnetic field is to decelerate the velocity. The boundary layer thickness is reduced sharply due to the effect of magnetic field. In comparison to the effect of magnetic field over velocity, the effect of magnetic field over temperature is different. It has no effect on the temperature and all these facts are displayed in Fig. 4.

Figure 5 displays the nondimensional velocity profiles for various β, the stretching parameter. The effect of stretching parameter over velocity is to accelerate it for its increasing values. The

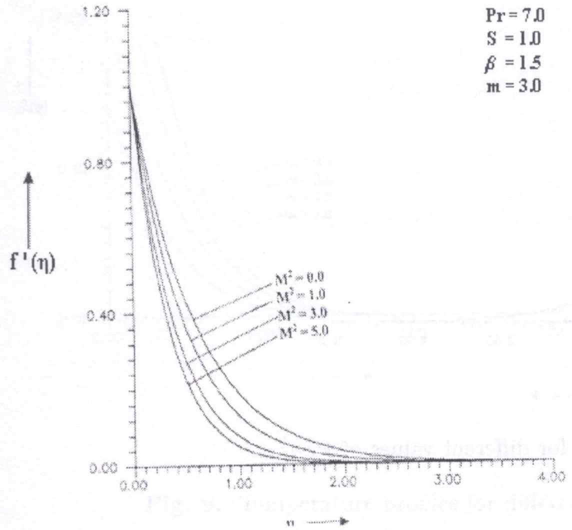


Fig. 3. Velocity profiles for various M^2

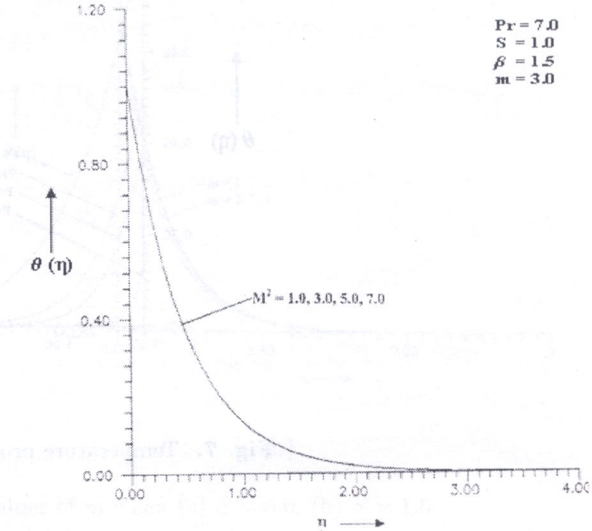


Fig. 4. Temperature profiles for different values of M^2

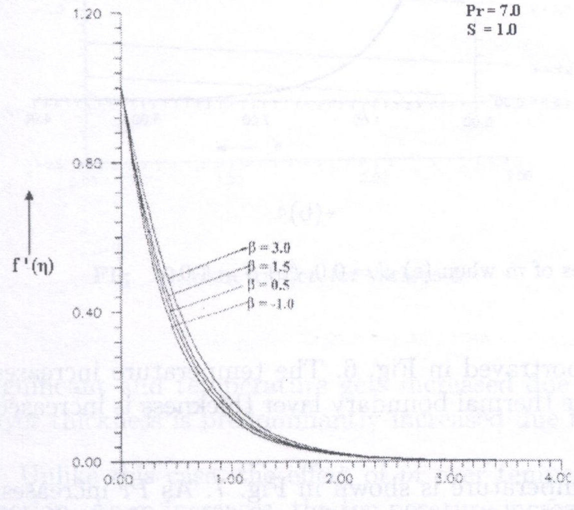


Fig. 5. Velocity profiles for different values of β

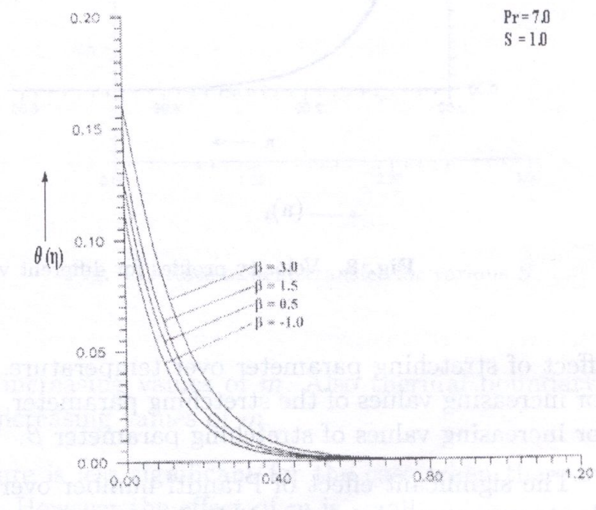


Fig. 6. Temperature profiles for different values of β

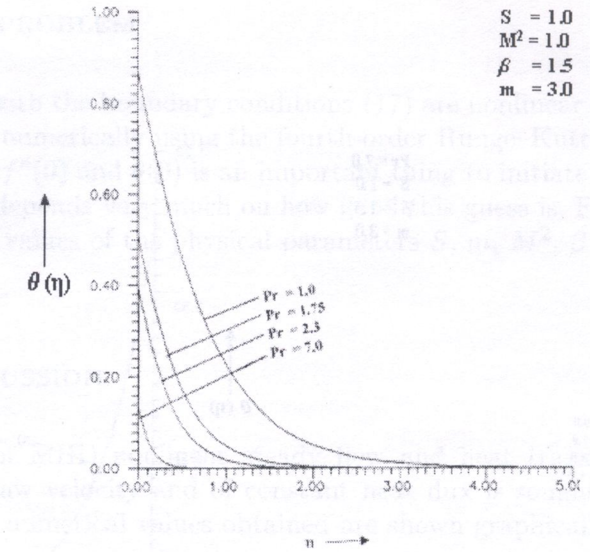


Fig. 7. Temperature profiles for different values of *Pr*

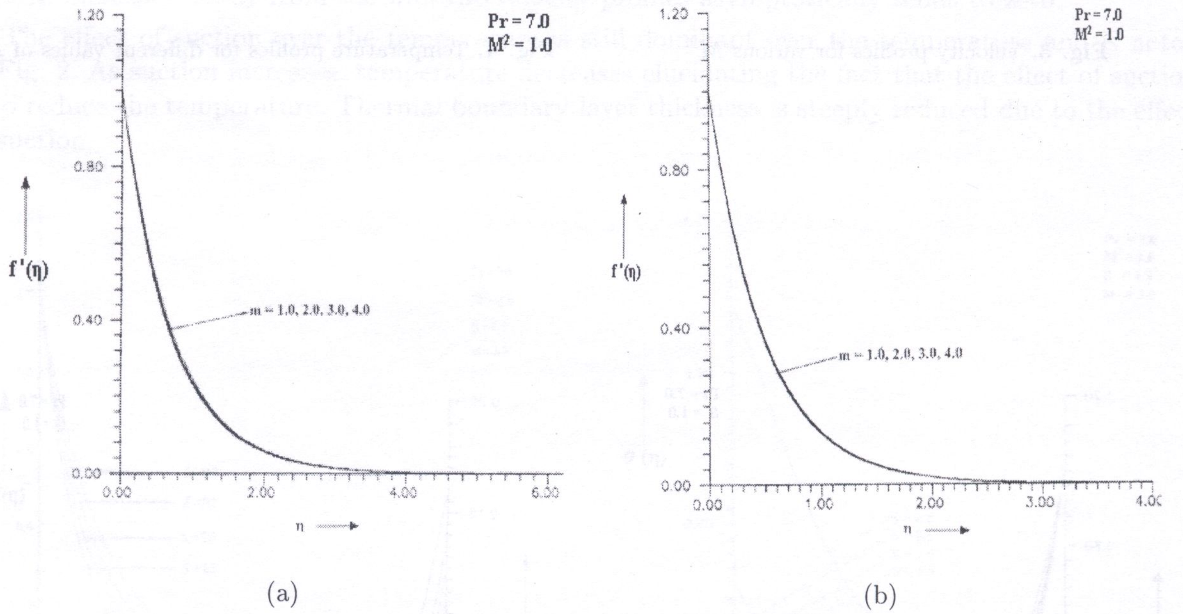


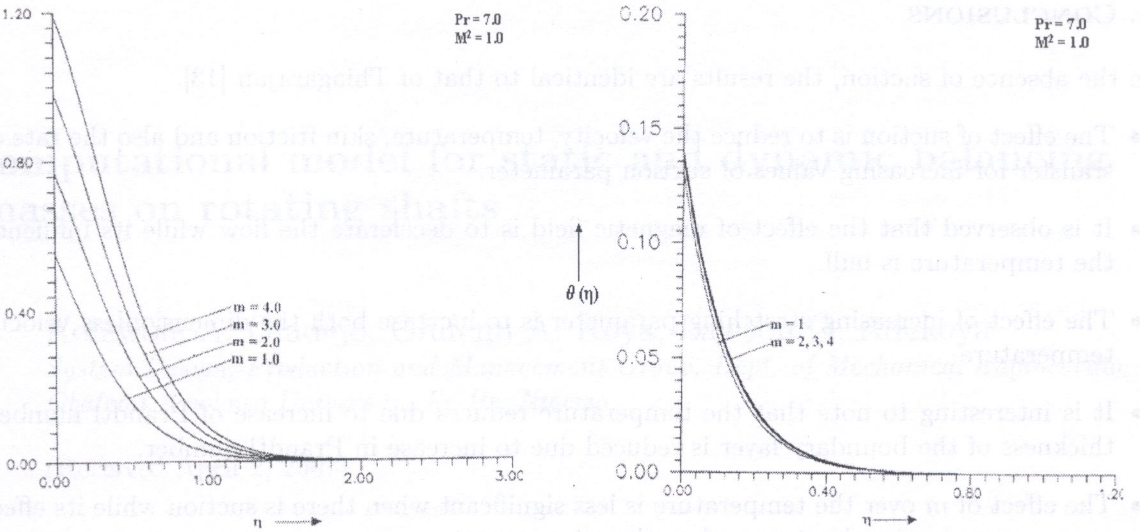
Fig. 8. Velocity profiles for different values of m when (a) $S = 0.0$, (b) $S = 1.0$

effect of stretching parameter over temperature is portrayed in Fig. 6. The temperature increases for increasing values of the stretching parameter. The thermal boundary layer thickness is increased for increasing values of stretching parameter β.

The significant effect of Prandtl number over temperature is shown in Fig. 7. As Pr increases, the temperature decreases. It is fascinating to note that the thermal boundary layer thickness is dominantly reduced due to increase of Prandtl number.

Figures 8a–b demonstrate the nondimensional velocity profiles for different values of m. In both the cases when there is suction or not, it is noted that there is no effect of m on the velocity.

The effect of m over the temperature is shown in Figs. 9a–b for the cases in the presence and absence of suction. For the case, when there is no suction the effect of m over temperature is



(a) (b)

Fig. 9. Temperature profiles for different values of m when (a) $S = 0.0$, (b) $S = 1.0$

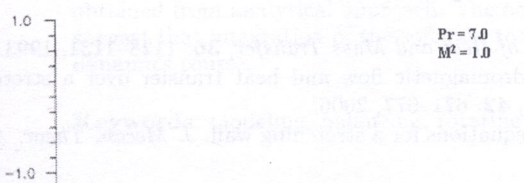


Fig. 10. Skin friction for various S

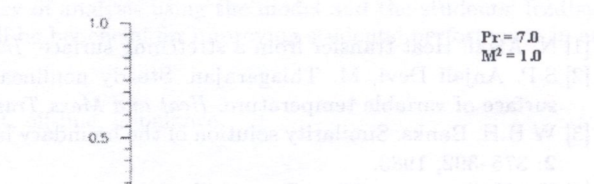


Fig. 11. Rate of heat transfer for various S

significant and temperature gets increased due to increasing values of m. Also thermal boundary layer thickness is predominantly increased due to increasing values of m.

Unlike this case, the effect of m over temperature is less significant for the case when there is suction. As m increases, the temperature increases. However the effect of m is small.

Figure 10 displays the effect of suction on skin friction. As S increases, skin friction decreases disclosing the fact that the effect of suction is to reduce the skin friction.

Rate of heat transfer against β for various values of suction parameter is portrayed in Fig. 11. The effect of suction is to reduce the rate of heat transfer where as the effect of stretching parameter is to increase the rate of heat transfer for each given S.

5. Conclusions

In the absence of suction, the results are identical to that of Thiagarajan [13].

- The effect of suction is to reduce the velocity, temperature, skin friction and also the rate of heat transfer for increasing values of suction parameter.
- It is observed that the effect of magnetic field is to decelerate the flow while its influence over the temperature is null.
- The effect of increasing stretching parameter is to increase both the dimensionless velocity and temperature.
- It is interesting to note that the temperature reduces due to increase of Prandtl number. The thickness of the boundary layer is reduced due to increase in Prandtl number.
- The effect of m over the temperature is less significant when there is suction while its effect over temperature is very dominant when there is no suction.
- The increase of m on non-dimensional velocity has no effect on it whether there is suction or not.

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