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A survey of methods for discrete optimum structural design

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The available methods and solutions of problems in discrete optimum structural design are reviewed. They are classified into the following categories: branch and bound methods, dual approach, enumeration methods, penalty function approach, simulated annealing and other methods. For the majority of problems, none of the methods is guaranteed to give the exact solution from the mathematical point of view. However, "good practical" solutions can be obtained at an acceptable cost.

1. INTRODUCTION

In many cases, modern technology is based on prefabricated products composed in different ways, which form different engineering systems. Rolled beams, metal sheets, prefabricated concrete elements, bolts, motors, gearboxes, electronic units are good examples of such products. Therefore, modern designing consists to a great extent in composing a system from a set of prefabricated elements specified in specialized catalogues. In most of the available methods of computer-aided optimum structural design, a very necessary practical requirement that the optimum design should employ only the standard discrete component sizes is omitted; it is assumed that components are available in a continuous range of sizes. The omission of this very practical requirement has been one of several contributory factors to the lack of use of computer-aided optimum structural design methods in engineering practice. Due to this fact, discrete optimum structural design has received considerable attention from researchers for the last few years. The first international symposium in this particular field, sponsored by the International Union of Theoretical and Applied Mechanics, was organized in Zakopane, Poland, in 1993 [Gutkowski and Bauer, 1994a].

Discrete structural optimization problems may be expressed mathematically in the following form:

– Minimize or maximize

$$f(\mathbf{x}), \quad \mathbf{x} \equiv x_i, \; i = 1, \ldots, N, \tag{1}$$

subject to

$$g_j(\mathbf{x}) \leq 0; \quad j = 1, \ldots, J, \tag{2}$$

$$h_k(\mathbf{x}) = 0; \quad k = 1, \ldots, K, \tag{3}$$

$$x_i \in S_i = (s_m : m = 1, \ldots, M_i). \tag{4}$$

Equation (1) is the objective function and is a function of design variables x_i. Equations (2) and (3) represent constraints which the design must satisfy. Equation (4) states that each design variable x_i must be selected from the finite set S_i which contains M_i discrete sizes. In some structural designs it may be necessary for only a certain number of the design variables to be chosen from a discrete set while the other variables may take any continuous values. This type of mixed discrete-continuous problem is not the main purpose of this paper although some of the methods presented here also apply to this class of problems. A survey of methods for mixed discrete-continuous problems has recently been presented in the introductory chapter of the paper [Zhang and Wang, 1993a].

The purpose of this paper is to summarize methods for discrete optimum structural design. We confine ourselves to *discrete variables*. The variables that must have *integer values* (in some cases 0 or 1) can be considered as a special case of discrete variables. Due to the fact that the discrete space is disjoint and nonconvex, traditional optimization approaches which are gradient-based cannot be applied directly. Our discrete optimization problem (1)–(4) contains a finite number of feasible solutions. It could then be suggested that such a problem should be solved by means of the *explicit enumeration* method. But simple calculation shows that even for a simple case of 10 variables with the list of available values (the catalogue) containing 10 parameters the number of possible combinations is 10^{10}. If checking one combination took one second only (solution of equilibrium equations and verification of constraints), the enumeration of all possible combinations would last over three hundred years! A natural development of the explicit enumeration method are methods of implicit enumeration or the branch and bound method which are described in the next sections.

A simple solution suggested quite often for structural design is first to solve an equivalent problem without constraints on discreteness of variables, and then round the design variables to the nearest allowable discrete or integer values. But the rounded solution is not always the optimum and may in fact be infeasible [Maciulevicius, 1968a], [Bauer *et al.*, 1984a].

Most proposed methods appear to work well on selected problems since they generate an exact or sufficiently good approximate solution, however, none of them claims to work well on all problems. The computational complexity [Nemhauser and Wolsey, 1988f], [Walukiewicz, 1991d] of an algorithm for solving a certain problem is defined as the maximum number of computational steps performed to solve any instance of the considered problem measured as the number of elementary bit operations performed on an input string of length s. A *polynomial-time algorithm* ($\mathcal{P}$) is the one whose complexity is limited by a polynomial $p(s)$. A *deterministic algorithm* is the one in which the result of every operation is uniquely defined. A *non-deterministic algorithm* is the one in which the result of each operation is not defined uniquely but belongs to a specified set of possibilities. Discrete optimization problems are $\mathcal{NP}$-class problems ($\mathcal{N}$on-deterministic algorithm that runs in $\mathcal{P}$olynomial time). Computational complexity for minimum weight trusses with discrete member sizes is given in the paper [Yates *et al.*, 1982a]. Truss weight minimization with deflection constraint and with a catalogue of cross-sectional areas of bars is a $\mathcal{NP}$-hard problem.

In the first papers on discrete variable structural optimization, the cutting plane method for linear programming problems (the Gomory algorithm) was used [Maciulevicius, 1968a], [Toakley, 1968b]. Small scale engineering optimization problems for truss and frame structures were solved. Toakley concluded that because of convergence difficulties the Gomory algorithm cannot be used with confidence. In some cases, linear discrete programming problems are transformed to (0-1) linear programming problems and well-established numerical algorithms of logical programming are applied.

A survey of methods for discrete structural optimization has been presented by Bauer *et al.* [1981a], Bremicker *et al.* [1990h] and Vanderplats and Thanedar [1991c]. The methods reviewed in the papers mentioned above are the branch and bound method, the cutting plane method, decomposition methods, dynamic programming methods, enumeration methods, heuristic (approximation) techniques and "ad hoc" methods. In the present paper, the numerical methods for solving discrete optimum structural design problems are classified as follows:

(a) Branch and bound method

(b) Dual approach

(c) Enumeration methods

(d) Penalty function approach

(e) Simulated annealing

(f) Other methods — heuristic methods, genetic algorithms, integer gradient direction methods, etc.

The methods are described in the subsequent sections.

2. Branch and Bound Method

A detailed explanation of the branch and bound approach is available in monographs [Korbut and Finkelsztejn, 1969a], [Garfinkel and Nemhauser, 1972a]. First, a problem without constraints on discreteness of variables is analysed. This gives a starting point, as well as a lower boundary (in the case of a minimization problem) on the discrete solution. If this solution is discrete (all obtained values are in the catalogue), the process is terminated. If one of the desired variables is not discrete, its value lies between two discrete values, e.g. $s_{i,j} \leq x_i \leq s_{i,j+1}$. Now two subproblems are defined, one with constraints $x_i \leq s_{i,j}$ and the other with $x_i \geq s_{i,j+1}$. This process is called branching. It eliminates part of the continuous feasible region which is not feasible for the discrete problem. It does not eliminate any of the discrete feasible solutions. The two subproblems are solved again and the process is repeated. The general framework involves decomposing the original problem into subproblems, modifying constraints to enlarge feasible domains, and finally a process referred to as fathoming. This involves checking a solution for feasibility and establishing optimality.

The method has been applied by Cella and Soosar [1973a] in the optimization of a space frame and a box-shaped beam and by Haftka and Walsh [1992c] in the optimization of the buckling of laminated plates. A modified version of the branch and bound method for a mixed integer and discrete programming problem has been used by Hajela and Shih [1989d] in the optimization of a cantilever composite laminate beam for minimum weight and with constraints on strength, displacements and natural frequencies. The main disadvantage of the branch and bound method is that a multitude of nonlinear optimization tasks must be performed, which is very expensive.

3. Dual Methods

Dual concepts are the most popular ones among all discrete optimization approaches. For continuous sizing problems, an approximation scheme creating a series of strictly convex, separable problems obtained by approximating the original problem is often used. The approximate problems can be solved by means of dual methods. Dual functions are concave and differentiable so that the optimum primary solution can be constructed from the dual solution. For a discrete sizing problem, a similar scheme creating a series of discrete problems approximating the original problems can be used [Schmit and Fleury, 1980a], [Ringertz, 1988a]. The dual functions are still concave but piecewise linear, not everywhere differentiable and standard gradient methods cannot be used in the maximization of the dual function. Further, solving the dual problem is not equivalent to solving the primary one, since the primary problem is nonconvex. This fact can create the so-called duality gap. In maximizing the dual function, various methods are proposed, e.g. the gradient projection method [Schmit and Fleury, 1980a] and the steepest subgradient method [Sepulveda and Cassis, 1986d]. Among the dual approaches one can distinguish a family of methods that might be described as Lagrangean relaxation combined with subgradient optimization. Reviews of this method can be found in [Fisher *et al.*, 1975a], [Shapiro, 1979a], [Nemhauser and Wolsey, 1988f], [Walukiewicz, 1991d]. These methods proceed in the following manner. We construct the

Lagrangean function by including in the objective function complicating constraints priced by Lagrangean multipliers. The Lagrangean function in terms of Lagrange multipliers is a continuous, not everywhere differentiable, concave function. In solving the dual problem, the ascent method is constructed with the use of the subgradient. The application of the above mentioned approach in the optimization of truss structures has been given by Jonsson and Larsson [1990a] and in the optimization of plastic circular plates with step-wise varying thickness by Bauer [1992d]. The dual approach is efficient in mixed discrete-continuous programming problems [Schmit and Fleury, 1980a], [Ringertz, 1988a].

4. Enumeration methods

The enumeration method for solving discrete problems often has advantages over other methods. Obviously, since the variables take on discrete values only, they can be listed easily, although they may be numerous. For the procedure to be manageable, the enumeration should be ordered in such a way that solutions are obtained with the minimum amount of calculation. Thus, a certain strategy for eliminating some of the solutions is necessary. A possible method, suggested in [Greenberg, 1971b], may consist in examining the solutions in such a way that monotonicity of the objective function is ensured. The first encountered solution satisfying the constraints is the solution of the problem. But the algorithm presented by Greenberg is practically of no use because of its memory requirements and time consumption. A significant improvement of the algorithm, which significantly reduces memory requirements and allows us to start with any value of the objective function, has been presented by Iwanow [1981e and 1990e]. The algorithm in [Iwanow, 1990e] has been described with details for both 0-1 variables and real variables from a finite catalogue. Some applications of the algorithm in discrete optimum design of truss structures have been presented in [Gutkowski *et al.*, 1986b and 1994a (17)] and [Gutkowski, 1992h].

A systematic search procedure in the application of the backtrack method in discrete optimum structural design has been presented in [Farkas and Szabo, 1980b]. This combinatorial method can be successfully applied to problems with nonlinear objective functions and constraints. In the algorithm, a partial search is carried out for each variable and if the possibilities are exhausted, a backtrack and a new partial search are performed. This scheme has been used for several optimum structural problems such as hybrid I-beam or portal frames [Janczura and Sieczkowski, 1986c], [Janczura, 1994a (23)]. The accuracy and efficiency of this method has been demonstrated with standard numerical examples [Yuan *et al.*, 1990d]. An efficient implicit enumeration method with systematic elimination of nonoptimum solutions has been applied by [Hua, 1983b]. The optimum design of four- and ten-bar trusses and an eighteen-member wing box has been demonstrated.

5. Penalty function approach

By applying the penalty function method, we create a pseudo-objective function by combining the original objective function and the constraint equations. The constraints are added to the objective function in such a way so as to penalize it if the constraint relations are not satisfied. The main idea has been presented by S.S. Rao in his monograph [Rao, 1978d]. Its application in discrete optimum structural design has been presented by Shin *et al.* [1988e and 1990b]. A simple penalty approach combined with the extended interior penalty function technique has been given. The procedure has been demonstrated on well-known benchmark problems: three-bar truss, ten-bar truss and twenty-five-bar truss. The improved penalty function method has been presented by Cai and Thierauf [1993b]. Efficient solutions have been obtained for ten- and twenty-five-bar trusses under static loading and a 44-element plane frame under dynamic loading. In both cases, the authors conclude that further numerical tests with more complex problems are required to use the penalty method techniques with confidence. The penalty function has been applied in the transformation of a nonlinear integer programming problem into a global optimization problem [Ge and Huang, 1989g]. The authors investigated unconstrained nonlinear integer programming and mixed nonlinear integer programming.

6. SIMULATED ANNEALING

The method makes use of the connection between statistical mechanics (the behaviour of systems with many degrees of freedom in thermal equilibrium at a finite temperature) and combinatorial optimization [Kirkpatrick *et al.*, 1983c].

In the paper [Metropolis *et al.*, 1953a], a simple algorithm introduced that can be used to provide an efficient simulation of a collection of atoms in equilibrium at a given temperature is introduced. In each step of this algorithm, an atom is given a small random displacement and the resulting change, ΔE, in the energy of the system is compared.

If $\Delta E \leq 0$, the displacement is accepted and the configuration with the displaced atom is used as the starting point for the next step.

The case $\Delta E > 0$ is treated probabilistically: the probability that the configuration is accepted is

$$P(\Delta E) = \exp\left(\frac{-\Delta E}{k_B T}\right)$$

where k_B — Boltzmann's constant, T — temperature.

Random numbers uniformly distributed in the interval $(0, 1)$ are convenient means of implementing the random part of the algorithm. One such number is selected and compared with $P(\Delta E)$.

If it is less than $P(\Delta E)$, the new configuration is retained; otherwise the original configuration is used to start the next step. By repeating the basic step many times, the thermal motion of atoms in thermal contact with a heat bath at temperature T is simulated.

Using the cost function instead of the energy and defining configurations by a set of parameters, it is easy to generate a population of configurations of a given optimization problem at certain effective temperature with the Metropolis procedure. This temperature is simply a control parameter of the process. The simulated annealing process consists of first "melting" the system being optimized at a high effective temperature, then lowering the temperature in small steps until the system "freezes" and no further changes occur. At each temperature level, the simulation must proceed long enough for the system to reach a steady state. As in the physical process, the performance of the simulated annealing algorithm is sensitive to step sizes of moves and the cooling schedule which includes the determination of the initial temperature, the rate at which the temperature is reduced and the stopping criterion.

The basic elements of a simulated annealing algorithm are the following (see [Zhang and Wang, 1993a]):

1. configuration: a solution to the problem

2. neighbourhood move: a transition from one configuration to another

3. neighbouring configuration: a result of a neighbourhood move

4. objective function: a measure of how good the solution is

5. cooling schedule: how high the starting temperature should be, and the rules to determine (1) when the current temperature should be lowered, (2) how much the temperature should be lowered, and (3) when the annealing process should be terminated.

These elements appear in most simulated annealing implementations, however; they may be in different forms depending on the applications.

A detailed description of the method with applications to problems arising in the optimum design of computers (partitioning, component placement and wiring of electronic systems) and the travelling salesman problem has been presented in the article [Kirkpatrick *et al.*, 1983c]. The efficiency of the simulated annealing algorithm for mixed-discrete nonlinear optimization has been demonstrated in [Zhang and Wang, 1993a]. The discrete optimization of a three-dimensional steel frame has been presented by Balling [1991b]. Certain improvements of the simulated annealing strategy in the discrete optimization of a steel frame with all strength and serviceability constraints have been presented in the paper [May and Balling, 1992e] with non-trivial numbers of variables and of elements in the catalogues. Seven variables (for columns) with 47 elements in the catalogue and four variables (beams) with 46 elements in the catalogue give $47^7 \times 46^4$ (approx. 10^{18}) possibilities.

7. OTHER METHODS AND PARTICULAR APPLICATIONS

A simple genetic algorithm proposed in [Goldberg, 1989h], based on natural genetics, has been modified to an artificial genetic approach and applied with great success in a number of papers on discrete structural optimization. Rajeev and Krishnamoorthy [1992b] have solved two standard problems known from literature and a 160-bar transmission tower. Hajela and Lee [1994a(5)] presented a genetic algorithm in the topological design of grillage structures. Trompette *et al.* [1994a(1)] applied a genetic algorithm in the optimum damping of beams and plates. An application of the neural network has been presented by Kishi *et al.* [1994a(3)].

Among heuristic methods, one can distinguish the method proposed by Yates *et al.* [1983a] for the optimization of truss structures. Templeman [1988c] proposed an original "segmental member method" in the discrete optimization of truss and plane structures. This method was also applied in the reliability based discrete optimization [da Cruz Simões, 1992g]. Fox and Liebman [1981b] presented a modified nonlinear simplex algorithm for the constrained optimization problem with discrete values. The modifications include the incorporation of a one-dimensional search, a new acceleration and regeneration method and decomposition strategies. Liebman *et al.* [1981c] converted a constrained problem into a sequence of unconstrained ones by means of penalty functions and solved the unconstrained discrete problems by means of the integer gradient direction method.

The hybrid method involving both the integer gradient direction method and the modified Resenbrock ortogonalization procedure has been applied in the papers [Amir and Hasegawa, 1988b and 1989b] and [Amir, 1989c].

The shape optimization of plastic structures by means of 0-1 programming has been presented by Zavelani *et al.* [1975c]. The optimum structural design problem with sizing and shape variables for truss structures has been solved in [Salajegheh and Vanderplaats, 1993c]. Both sizing and shape variables can be continuous, discrete or mixed discrete-continuous. Numerical examples including a 47-bar plane tower have been presented. A semi-analytical approach combined with logical programming has been applied in the discrete optimization of space structures [Bauer *et al.*, 1981d and 1984a], [Gawkowska, 1982b and 1987a].

The linearization of nonlinear problems is a popular approach among many researchers. The sequential linear discrete programming method [Olsen and Vanderplaats, 1989e], [Bremicker *et al.*, 1990h], [Loh and Papalambros, 1991e] begins with the creation of a linear integer approximate problem from the nonlinear discrete problem. The existing integer programming techniques are then used in the approximate problem directly. A series of approximations and optimizations is carried out until convergence occurs. In some cases, this method is combined with the branch and bound method [Bremicker *et al.*, 1990h].

Among interesting engineering applications one can distinguish the optimum allocation of supports or optimum segmentation of structures [Sui *et al.*, 1991f], [Dems and Mróz, 1994a(6)], [Gutkowski *et al.*, 1994a(17)].

An efficient approximation method for complex problems in the optimization of frames (55 variables, 2718 nonlinear constraints) has been applied in [Kalinin, 1989a]. The problem of optimum location of actuators or sensors has been solved by Haftka and Adelman [1985a], Holnicki-Szulc *et al.* [1994a (19)] and Korbicz and Uciński [1994a (18)]. The application of mixed discrete-continuous programming in the optimization of machine tools has been presented in [Weck and Kölsch, 1988d].

Among many versions of enumeration methods applied in discrete design one can distinguish the following:

- [Reinschmidt, 1971a] — plastic design of frames and elastic design of trusses,
- [Leśniak, 1975b], [Leśniak *et al.*, 1978a] — decomposition strategy in the optimization of steel industrial buildings,
- [Garstecki *et al.*, 1978b] — optimization of lightweight steel industrial buildings,
- [Relahan and Gaddy, 1978c]— adaptive random search procedure,
- [Pyrz, 1990f and 1990g] — optimization of geometrically nonlinear truss structures.
- [Mottl, 1992a and 1994a (2)] — "voting method".

Efficient applications of the optimality criteria method have been presented in [Grierson and Lee, 1986a], [Chan, 1992f], [Rozvany and Zhou, 1994a (13)].

A number of papers are devoted to the design of reinforced concrete structures and prestressing problems [Eimer and Mączyński, 1976a], [Choi and Kwak, 1990c], [Marks and Trochymiak, 1991a].

A combination of expert systems with other methods has been applied by Niczyj and Paczkowski [1994a (10)] and Pyrz [1994a (22)].

All above mentioned papers are devoted to optimum problems with scalar objective functions. However, in recent years also multicriteria optimization problems have been solved [Eswaran *et al.*, 1989f], [Jendo and Paczkowski, 1993d and 1994a (21)], [Osyczka and Montusiewicz, 1994a (8)].

8. Final remarks

For a narrow class of problems some methods like the branch and bound method or the dual approach have a solid mathematical basis. For the majority of problems none of the methods guarantees that the global optimum will be achieved. However, these methods quite often provide reasonable solutions at an acceptable cost.

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