

# Modelling of concrete cover degradation caused by rebar's corrosion – multicomponent media theory approach

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This paper presents the model based on the theory of multicomponent media that allows modelling of rebar corrosion processes. The presented model extends and consolidates the dissertations that have been described in the papers [8-10]. The aim of the present work is a creation of the model consistent with the thermodynamics of multicomponent media with internal parameters, allowing description and numerical modelling of reinforced concrete structures degradation as the result of corrosion by using FEM.

**Keywords:** theory of plasticity, corrosion of reinforced concrete, mass transport, concrete cover splitting.

## 1. INTRODUCTION

The issue of reinforced concrete structures degradation as the result of corrosion state is a very important industrial problem [17, 18]. This subject matter states both a very complicated and interesting physical problem. In the reinforced concrete, the separation of concrete cover takes place as the result of corrosion products creation. Concrete cover splitting increases rapidly the rate of degradation process. In the previous published papers, e.g. [1, 7, 13, 20] setting up a concrete cover splitting time has been described in an analytical way. The other way of the phenomena description is the analysis of the problem from thermodynamics of multicomponent media point of view by using computer method based on e.g. FEM. This subject has been presented e.g. in papers [8–10]. The first paper presents the use of the multicomponent media theory with mechanical distortion of a problem description. The second one, presents the application of plasticity theory into the description of concrete cover fracturing as the result of distortion caused by corrosion products. The tensor of distortional strains has been defined here as the function of corrosion current intensity. The approaches shown in both papers are inconsistent and require homogenisation, what is the aim of this paper.

## 2. FORMULATION OF THERMODYNAMICAL MODEL

### 2.1. Model assumption

The mixtures theory [2, 3, 19] will be used to describe the problem. The two components model of continuous body (the skeleton  $\alpha = 0$  together with the migrating substance  $\alpha = 1$ ) is taken into consideration. It will be assumed that the components do not react with each other and the aggressive substance only initiates the process of mechanical distortion creation in the media. The process of distortional strains creation depends on the additional internal parameter  $I^e$  (a function

of electric current intensity), which will be defined in the further part of the work. It is assumed in the paper that the skeleton is a dominant component and fulfills the following assumptions [11].

- The skeletons density  $\rho^0$  is notably bigger than the density of the other components of mixture

$$\rho^0 \gg \rho^1, \quad \rho = \sum_{\alpha=0}^1 \rho^\alpha \approx \rho^0. \quad (1)$$

- The mean velocity field can be presented in the form

$$\rho \mathbf{v} = \sum_{\alpha=0}^1 \rho^\alpha \mathbf{v}^\alpha = \rho^0 \mathbf{v}^0 + \rho^1 \mathbf{v}^1 \approx \rho^0 \mathbf{v}^0 \approx \rho \mathbf{v}^0, \quad \mathbf{v}^\alpha = \mathbf{u}^\alpha + \mathbf{v}. \quad (2)$$

In Eqs. (1), (2)  $\rho^\alpha$  is the partial density of the component  $\alpha$ ,  $\mathbf{v}$  is the mean rate of the mixture,  $\mathbf{u}^\alpha$  is the partial diffusion velocity,  $\mathbf{v}^\alpha$  is the velocity of  $\alpha$  component, and  $\rho$  is the total mass density of the body.

## 2.2. Local mass balance equation of the multicomponent media

It will be assumed that there is no chemical reaction between migrating substance  $\alpha = 1$  and a skeleton  $\alpha = 0$ , the density of mass source productivity of both components is equal to zero  $\tilde{c}^\alpha = 0$ . The local mass balance equations of the mixture according to these assumptions [2, 19], respectively for a skeleton and migrating substances, will be defined in the following form

$$\begin{aligned} \rho \frac{d}{dt} (c^\alpha) + \operatorname{div} (\mathbf{j}^\alpha) &= \tilde{c}^\alpha, & c^\alpha &= \frac{\rho^\alpha}{\rho}, \\ \rho \frac{d}{dt} (c^0) &= 0, & \rho \frac{d}{dt} (c^1) + \operatorname{div} (\mathbf{j}^1) &= 0, & \mathbf{j}^0 &= \rho^0 \mathbf{u}^0 \cong 0, & \mathbf{j}^1 &= \rho^1 \mathbf{u}^1, \end{aligned} \quad (3)$$

where  $\mathbf{j}^\alpha$  is the mass flux of the component  $\alpha$ , and  $c^\alpha$  is the concentration of the component  $\alpha$ .

## 2.3. The mean local momentum and angular momentum balance equation of the multicomponent media

The mean local momentum balance equation of the mixture and the angular momentum of the mixture can be presented while skipping long distance influence and using the assumption that the diffusion stress tensor equals zero ( $\mathbf{t}^{D\alpha} = 0$ ), presented [2, 19] in the form

$$\operatorname{div} \boldsymbol{\sigma} + \rho \mathbf{b} = \rho \frac{d\mathbf{v}}{dt}, \quad \boldsymbol{\sigma} = \sum_{\alpha=0}^1 \boldsymbol{\sigma}^\alpha, \quad \mathbf{b} = \sum_{\alpha=0}^1 c^\alpha \mathbf{b}^\alpha, \quad \boldsymbol{\sigma} = \boldsymbol{\sigma}^T, \quad (4)$$

where  $\boldsymbol{\sigma}$  is the mean stress tensor,  $\mathbf{b}$  is the mean mass force,  $\boldsymbol{\sigma}^\alpha$  is the partial stress tensor, and  $\mathbf{b}^\alpha$  is the partial mass force.

## 2.4. Mean local balance equation of energy of the multicomponent media

The mean local balance equation of energy when ignoring long distance influence and small strains assumptions  $\operatorname{grad}^S \mathbf{v} \approx \dot{\boldsymbol{\varepsilon}}$  can be formulated [2, 11, 19] in the form

$$\begin{aligned} \rho \frac{d\eta}{dt} &= \rho r - \operatorname{div} \mathbf{q} + \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} - \sum_{\alpha=0}^1 \operatorname{div} (M^\alpha \mathbf{j}^\alpha), & M^\alpha &= \eta^\alpha + k^\alpha - \frac{p^\alpha}{\rho^\alpha}, & p^\alpha &= \frac{1}{3} \operatorname{tr} \boldsymbol{\sigma}, \\ \eta &= \sum_{\alpha=0}^1 c^\alpha \eta^\alpha, & r &= \sum_{\alpha=0}^1 c^\alpha r^\alpha, & \mathbf{q} &= \sum_{\alpha=0}^1 c^\alpha \mathbf{q}^\alpha. \end{aligned} \quad (5)$$

Here,  $\eta$  is the mean internal energy of the multicomponent media,  $r$  is the mean productivity of heat source,  $\mathbf{q}$  is the mean heat flux,  $\boldsymbol{\varepsilon}$  is the strain tensor,  $\eta^\alpha$  is the partial internal energy,  $k^\alpha$  is the partial kinetic energy density,  $r^\alpha$  is the partial productivity of heat source, and  $\mathbf{q}^\alpha$  is the partial heat flux.

## 2.5. Mean local entropy growth inequality of the multicomponent media

It is assumed [19], that the temperature  $T = T^\alpha$  is common for all the components. The mean local entropy balance equation can be written in the form [2, 19]:

$$\begin{aligned} \rho \frac{ds}{dt} &\geq \rho s^e - \operatorname{div}(\mathbf{h}), & \mathbf{h}^\alpha &= \frac{\mathbf{q}^\alpha}{T}, & s_e^\alpha &= \frac{r^\alpha}{T}, \\ s &= \sum_{\alpha=0}^1 c^\alpha s^\alpha, & \mathbf{h} &= \sum_{\alpha=0}^1 (\mathbf{h}^\alpha - \rho^\alpha s^\alpha u^\alpha) \cong \sum_{\alpha=0}^1 \mathbf{h}^\alpha, & s^e &= \sum_{\alpha=0}^1 c^\alpha \tilde{s}_e^\alpha. \end{aligned} \quad (6)$$

Here,  $s$  is mean entropy of the multicomponent media,  $s^e$  is the mean productivity of entropy sources of the multicomponent media,  $\mathbf{h}$  is the mean entropy flux of the multicomponent media,  $s^\alpha$  is the partial entropy density,  $\mathbf{h}^\alpha$  is the partial entropy flux, and  $\tilde{s}_e^\alpha$  is the partial entropy sources productivity.

## 2.6. The residual inequality of the process and constitutive relationship

Taking into consideration the uniform temperature postulate of all components in the entropy growth inequality (6) and eliminating from the energy balance equation and for the entropy growth inequality the terms describing the mean productivity of heat sources and mean heat flux the residual inequality of the process will be obtained

$$-\rho \left( \dot{\psi} + s\dot{T} \right) + \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} - \sum_{\alpha=0}^1 \mathbf{j}^\alpha \operatorname{grad} M^\alpha - \sum_{\alpha=0}^1 M^\alpha \operatorname{div} \mathbf{j}^\alpha - \frac{\mathbf{q} \cdot \operatorname{grad} T}{T} \geq 0, \quad (7)$$

where  $\psi = \psi(c^\alpha, \boldsymbol{\varepsilon}^e, \boldsymbol{\alpha}, I^e, T)$  is the Helmholtz free energy,  $\boldsymbol{\varepsilon}^e = \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p$  is the elastic strain tensor,  $\boldsymbol{\varepsilon}^p$  is the plastic strain tensor, and  $\boldsymbol{\alpha}$  is the internal plastic variable. Taking into consideration in Eq. (7) the local mass balance equation of the multicomponent continua (3), the Clausius–Duhem inequality will be obtained

$$\left( -\rho \frac{\partial \psi}{\partial \boldsymbol{\varepsilon}^e} + \boldsymbol{\sigma} \right) : \dot{\boldsymbol{\varepsilon}}^e + \left( -\rho \frac{\partial \psi}{\partial T} - \rho s \right) \dot{T} + \left( -\rho \frac{\partial \psi}{\partial c^1} + \rho M^1 \right) \dot{c}^1 + D \geq 0, \quad (8)$$

$$D = \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}^p - \mathbf{X}^\alpha \bullet \dot{\boldsymbol{\alpha}} - X^I \dot{I}^e - \mathbf{j}^1 \cdot \operatorname{grad}(M^1) - \frac{\mathbf{q} \cdot \operatorname{grad}(T)}{T} \geq 0, \quad (9)$$

where the abstract operator  $\bullet$  denote the scalar product. Hence, this inequality must hold  $\{\boldsymbol{\varepsilon}^e, T, c^\alpha\}$  independently the way of field choice the constitutive relationships will have the form

$$\boldsymbol{\sigma} = \rho \frac{\partial \psi}{\partial \boldsymbol{\varepsilon}^e}, \quad \rho s = -\rho \frac{\partial \psi}{\partial T}, \quad \rho M^1 = \rho \frac{\partial \psi}{\partial c^1}. \quad (10)$$

The quantities  $\mathbf{X}^\alpha$  and  $X^I$  occurring in the relationship (9) are coupled thermodynamical forces defined by using the formulae

$$\mathbf{X}^\alpha = \rho \frac{\partial \psi}{\partial \boldsymbol{\alpha}}, \quad X^I = \rho \frac{\partial \psi}{\partial I^e}. \quad (11)$$

## 2.7. The evolution equations of internal parameter

Fulfilment of the Clausius–Duhem inequality requires introduction of dissipative potential function [12], scalar and convex function of dual variables  $\Theta = \Theta(\boldsymbol{\sigma}, \mathbf{X}^\alpha, X^I, \text{grad } T, \text{grad } M^1)$ . It will be assumed that potential function can be treated as superposition of independent functions that describe the mechanical effect, the effect connected with the internal parameter  $I^e$  and the part describing thermo-diffusion effect

$$\Theta = \Theta_m(\boldsymbol{\sigma}, \mathbf{X}^\alpha) + \Theta_c(X^I) + \Theta_{td}(\text{grad } T, \text{grad } M^1). \quad (12)$$

The evolution equation of internal parameters will be assumed in the form of following equations [12]

$$\dot{\boldsymbol{\varepsilon}}^p = \gamma \frac{\partial \Theta_m(\boldsymbol{\sigma}, \mathbf{X}^\alpha)}{\partial \boldsymbol{\sigma}}, \quad (13)$$

$$-\dot{\boldsymbol{\alpha}} = \gamma \frac{\partial \Theta_m(\boldsymbol{\sigma}, \mathbf{X}^\alpha)}{\partial \mathbf{X}^\alpha}, \quad (14)$$

$$-\dot{I}^e = \frac{\partial \Theta_c(X^I)}{\partial X^I}, \quad (15)$$

$$-\mathbf{j}_1 = \frac{\partial \Theta_{td}(\text{grad } T, \text{grad } M^1)}{\partial (\text{grad } M^1)}, \quad (16)$$

$$-\frac{\mathbf{q}}{T} = \frac{\partial \Theta_{td}(\text{grad } T, \text{grad } M^1)}{\partial (\text{grad } T)}, \quad (17)$$

where in Eqs. (13) and (14)  $\gamma$  is a scalar function.

## 3. THE FORMULATION OF PLASTICITY EQUATION FOR THE ISOTHERMAL MEDIA

### 3.1. Constitutive relationship

It will be assumed that in the isothermal condition the free energy function will have form of the following function  $\psi = \psi(c^1, \boldsymbol{\varepsilon}^e, \boldsymbol{\alpha}, I^e)$ , cf. [12, 16]

$$\rho\psi(c^1, \boldsymbol{\varepsilon}^e, \boldsymbol{\alpha}, I^e) = \frac{1}{2}\boldsymbol{\varepsilon}^e : \mathbf{C} : \boldsymbol{\varepsilon}^e - \boldsymbol{\varepsilon}^e : \mathbf{C} : \boldsymbol{\chi}(I^e - I_0^e) + \frac{1}{2}\boldsymbol{\alpha} \bullet \mathbf{D} \bullet \boldsymbol{\alpha} + \psi_d(c^1), \quad (18)$$

where  $\boldsymbol{\chi}$  is a tensor of material volumetric increase as the result of corrosion products creation,  $\mathbf{D}$  is the tensor of generalized plastic module, and  $\psi_d$  is the diffusion part of free energy function. Using Eq. (18) in the relationship (10) the constitutive equation describing the behaviour of two components media with diffusion depending on internal parameter  $I^e$  that characterize the rebar corrosion will be obtained

$$\boldsymbol{\sigma} = \rho \frac{\partial \psi}{\partial \boldsymbol{\varepsilon}^e} = \mathbf{C} : (\boldsymbol{\varepsilon}^e - \boldsymbol{\chi}(I^e - I_0^e)), \quad \boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^e + \boldsymbol{\varepsilon}^p, \quad (19)$$

$$M^1 = \frac{\partial \psi}{\partial c^1} = \frac{\partial \psi_d}{\partial c^1} = M^1(c^1) \quad (20)$$

### 3.2. The formulation of plasticity equation of the media

The elastic-plastic relationship can be obtained according to [16] by using the consistency condition  $\dot{f}(\boldsymbol{\sigma}, \mathbf{X}^\alpha) = 0$

$$\dot{f}(\boldsymbol{\sigma}, \mathbf{X}^\alpha) = \frac{\partial f}{\partial \boldsymbol{\sigma}} : \dot{\boldsymbol{\sigma}} + \frac{\partial f}{\partial \mathbf{X}^\alpha} \bullet \dot{\mathbf{X}}^\alpha = 0 \quad (21)$$

where  $f$  is a yield function. Taking into consideration Eqs. (11), (13), and (14) in Eq. (21) the formulation of the dependence describing the  $\gamma$  function will be obtained in the form

$$\gamma = \frac{1}{H} \left( \frac{\partial f}{\partial \boldsymbol{\sigma}} : \mathbf{C} : \dot{\boldsymbol{\varepsilon}} - \frac{\partial f}{\partial \boldsymbol{\sigma}} : \mathbf{C} : \boldsymbol{\chi} \dot{I}^e \right), \quad H = \frac{\partial f}{\partial \boldsymbol{\sigma}} : \mathbf{C} : \frac{\partial \theta_m}{\partial \boldsymbol{\sigma}} + \frac{\partial f}{\partial \mathbf{X}^\alpha} \bullet \mathbf{D} \bullet \frac{\partial \theta_m}{\partial \mathbf{X}^\alpha}, \quad (22)$$

Using Eq. (22) in (19) the rate form of elastic-plastic constitutive equations will be obtained

$$\dot{\boldsymbol{\sigma}} = \mathbf{C}^{\text{ep}} : (\dot{\boldsymbol{\varepsilon}} - \boldsymbol{\chi} \dot{I}^e), \quad \mathbf{C}^{\text{ep}} = \mathbf{C} - \frac{1}{H} \mathbf{C} : \left( \frac{\partial \theta_m}{\partial \boldsymbol{\sigma}} \otimes \frac{\partial f}{\partial \boldsymbol{\sigma}} \right) : \mathbf{C}, \quad (23)$$

where  $\mathbf{C}^{\text{ep}}$  is elastic plastic tangent tensor. Using the backward Euler scheme in (23), cf. [16], allows the formulation of the incremental form of elastic-plastic constitutive relationships

$$\begin{aligned} \boldsymbol{\sigma}_{n+1} &= \boldsymbol{\sigma}_n + \mathbf{C}^{\text{ep}} : (\Delta \boldsymbol{\varepsilon}_{n+1} - \gamma \Delta I_{n+1}^e), \\ \Delta \boldsymbol{\varepsilon}_{n+1} &= \dot{\boldsymbol{\varepsilon}}(t_{n+1}) \Delta t, \quad \Delta I_{n+1}^e = \dot{I}^e(t_{n+1}) \Delta t_{n+1}, \quad t_{n+1} = t_n + \Delta t_{n+1}. \end{aligned} \quad (24)$$

### 3.3. The description of distortional strain evolution equation

Using the definition of the first invariant of distortional strain tensor and assuming the orthotropy of distortional strain tensor (corrosion products causing distortions only in the plane perpendicular to the rebar axis, therefore  $\varepsilon_{33}^d = 0$  and  $\varepsilon_{11}^d = \varepsilon_{22}^d = \varepsilon$ ), the following relationships can be formulated [9, 10, 14]

$$I_{\varepsilon^d} = \frac{V^{\text{p}}(t)}{V_0} \cong \text{tr}(\boldsymbol{\varepsilon}^d), \quad \dot{V}^{\text{p}} = \frac{\alpha}{\gamma_{\text{p}}} \dot{m} = \frac{\beta}{\gamma_{\text{Fe}}} \dot{m}, \quad \frac{\gamma_{\text{p}}}{\gamma_{\text{Fe}}} \beta = \alpha, \quad \beta = \frac{V^{\text{p}}}{V}, \quad \dot{m} = k_e I(t), \quad (25)$$

where  $m$  is the mass of ferrous ions transferred into the solution,  $V^{\text{p}}$  is the corrosion products volume,  $\gamma_{\text{p}}$  is the corrosion products density,  $\alpha$  is the proportionality coefficient of the corrosion product mass to the mass of ferrous ions transferred into the solution,  $\beta$  is the proportionality coefficient of the corrosion products volume to the volume of corrosion cavity,  $t$  is time,  $k_e$  is the electrochemical equivalent of iron,  $I(t)$  is the corrosion current intensity,  $V$  is the corrosion cavity volume,  $V_0$  is the initial volume, and  $\gamma_{\text{Fe}}$  is the density of iron. Basing on these relationships the distortional strain tensor that describes the mechanical effects caused by corrosion can be presented in the following form [9, 10]

$$\begin{aligned} \dot{\boldsymbol{\varepsilon}}^d &= \tilde{\mathbf{I}} \frac{\beta k}{2V_0 \gamma_{\text{Fe}}} I(t) \equiv \boldsymbol{\chi} \dot{I}^e, \quad \tilde{\mathbf{I}} = \begin{cases} \delta_{ij} \mathbf{e}_i \otimes \mathbf{e}_j & \text{for } i, j \neq 3, \\ \mathbf{0} & \text{otherwise,} \end{cases} \\ \boldsymbol{\varepsilon}^d &= \tilde{\mathbf{I}} \frac{\beta k}{2V_0 \gamma_{\text{Fe}}} I^e \equiv \boldsymbol{\chi} I^e, \quad I^e = \int_0^t I(t) dt, \quad I(t) = \frac{dI^e}{dt}. \end{aligned} \quad (26)$$

#### 4. THE INCREMENTAL FORMULATION OF FEM EQUATIONS

The process of reinforcement corrosion is defined by weak form of mass transport and momentum transport equation. Taking advantage of Euler scheme in the weak form of momentum balance equation the incremental formula describing the aggressive substance mass transport in the media will be obtained. The solution of weak form of momentum balance equation requires linearization of the weak form in the neighbourhood of points  $\mathbf{u}_{n+1}^k$  and  $I_{n+1}^{e,k}$  according to the relationship, cf. [9, 10, 16]

$$\begin{aligned} G &= \bar{G} + D\bar{G} d\Delta\mathbf{u}_{n+1}^{k+1} + D\bar{G} d\Delta I_{n+1}^{e,k+1}, & G &= G(\mathbf{u}_{n+1}^{k+1}, I_{n+1}^{e,k+1}, \delta\mathbf{u}), \\ \bar{G} &= \bar{G}(\mathbf{u}_{n+1}^k, I_{n+1}^{e,k}, \delta\mathbf{u}), & \mathbf{u}_{n+1}^{k+1} &= \mathbf{u}_{n+1}^k + d\Delta\mathbf{u}_{n+1}^{k+1}, \\ I_{n+1}^{e,k+1} &= I_{n+1}^{e,k} + d\Delta I_{n+1}^{e,k+1}, & t_{n+1}^{k+1} &= t_{n+1}^k + d\Delta t_{n+1}^{k+1}, \\ DG(\mathbf{x})\mathbf{u} &= \left. \frac{d}{d\alpha} \right|_{\alpha=0} G(\bar{\mathbf{x}} + \alpha\mathbf{u}), & \mathbf{x} &= \bar{\mathbf{x}} + \alpha\mathbf{u}. \end{aligned} \quad (27)$$

Because of computer application the matrix notation will be used in the further part of the paper. The approximation of concentration and displacement field in the finite element will be assumed in the following form

$$\mathbf{c} \approx \mathbf{c}^h(\mathbf{x}) = \bar{\mathbf{N}}(\mathbf{x})\bar{\mathbf{c}}, \quad \mathbf{u} \approx \mathbf{u}^h(\mathbf{x}) = \bar{\mathbf{N}}(\mathbf{x})\bar{\mathbf{u}}, \quad (28)$$

where  $\bar{\mathbf{N}}(\mathbf{x})$  is the shape function matrix, whereas  $\bar{\mathbf{c}}$  and  $\bar{\mathbf{u}}$  are the vectors of nodal concentration and displacements, respectively. Using the formula (28) for the approximation of aggressive substance concentration field the FEM equation of mass transport in concrete will have the following form

$$\begin{aligned} \frac{1}{\Delta t} (\bar{\mathbf{C}}_{n+1} - \bar{\mathbf{K}}_{n+1}) \bar{\mathbf{c}}_{n+1} &= \bar{\mathbf{R}}_{n+1} + \frac{1}{\Delta t} \bar{\mathbf{C}}_n \bar{\mathbf{c}}_n, \\ \bar{\mathbf{C}} &= \int_{B^h} \bar{\mathbf{N}}^T \bar{\mathbf{N}} dB, \quad \bar{\mathbf{K}} = \int_{B^h} \frac{\partial \bar{\mathbf{N}}^T}{\partial \mathbf{x}} \mathbf{k}_D \frac{\partial \bar{\mathbf{N}}}{\partial \mathbf{x}} dB, \quad \bar{\mathbf{R}} = \int_{B^h} \bar{\mathbf{N}}^T \bar{\mathbf{j}} dB, \end{aligned} \quad (29)$$

where  $\bar{\mathbf{j}}$  is the mass flux on the boundary,  $\mathbf{k}_D$  is the diffusion coefficient matrix. Using the formula (28) in the linearized weak form of momentum balance equations we will obtain the following incremental FEM equations

$$\bar{\mathbf{K}}_{n+1}^k d\Delta\bar{\mathbf{u}}_{n+1}^{k+1} - \bar{\mathbf{Q}}_{n+1}^k d\Delta I_{n+1}^{e,k+1} = \bar{\mathbf{F}}_{n+1}^{\text{ext}} - \bar{\mathbf{F}}_{n+1}^{\text{int}^k}, \quad d\Delta I_{n+1}^{e,k+1} = I(t_{n+1}^{k+1}) d\Delta t_{n+1}^{k+1}, \quad (30)$$

$$\bar{\mathbf{K}}_{n+1}^k = \int_{B^h} \bar{\mathbf{B}}^T \cdot \bar{\mathbf{C}}^{\text{ep}}|_{n+1}^k \cdot \bar{\mathbf{B}} dB, \quad \bar{\mathbf{Q}}_{n+1}^k = \int_{B^h} \bar{\mathbf{B}}^T \cdot \bar{\mathbf{C}}^{\text{ep}}|_{n+1}^k \cdot \bar{\chi} dB, \quad (31)$$

$$\bar{\mathbf{F}}_{n+1}^{\text{ext}} = \int_{B^h} \bar{\mathbf{B}}^T \cdot \rho \bar{\mathbf{b}}_{n+1} dB + \int_{B^h} \bar{\mathbf{B}}^T \cdot \bar{\mathbf{p}}_{n+1} dB, \quad \bar{\mathbf{F}}_{n+1}^{\text{int}^k} = \int_{B^h} \bar{\mathbf{B}}^T \cdot \bar{\boldsymbol{\sigma}}_{n+1}^k dB, \quad (32)$$

$$\bar{\mathbf{C}}^{\text{ep}}|_{n+1}^k = \left. \frac{\partial \bar{\boldsymbol{\sigma}}}{\partial \bar{\boldsymbol{\varepsilon}}} \right|_{n+1}^k = - \left. \frac{\partial \bar{\boldsymbol{\sigma}}}{\partial \bar{\boldsymbol{\varepsilon}}^d} \right|_{n+1}^k, \quad \bar{\mathbf{u}}_{n+1}^{k+1} = \bar{\mathbf{u}}_{n+1}^k + d\Delta\bar{\mathbf{u}}_{n+1}^{k+1}, \quad \bar{\mathbf{u}}_{n+1}^0 = \bar{\mathbf{u}}_n. \quad (33)$$

where  $\bar{\mathbf{B}}$  is strain displacement matrix. In Eqs. (30)-(33)  $n$  means the loading step and  $k$  refers to the correction within the loading step. The application of the foregoing algorithm in to the analysis of corrosion can be approached by using the schematic drawing showing corrosion degradation problem in the advanced state.

Figure 1 presents the sketch of the algorithm application, where the following regions are distinguished: 1 – concrete, 2 – steel, 3 – no active transition region elements (the region where the corrosion products are created and the elements expand their volume), 4 – the region where the concentration of aggressive substance is deferent then zero  $c^1 \neq 0$ , 5 – active transition region elements (the distortional strains are present),  $I \neq 0$ .

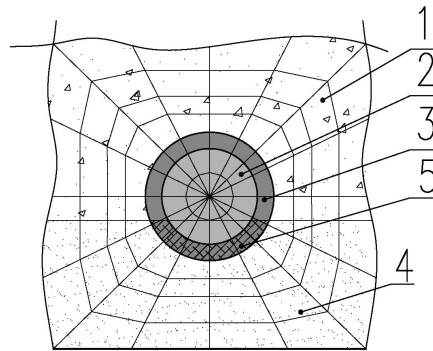


Fig. 1. The sketch of the algorithm application

## 5. NUMERICAL EXAMPLE

In order to show the practical application of foregoing considerations the numerical example will be presented. The time necessary for concrete cover fracturing in the reinforced concrete element having the static scheme presented in Fig. 1 has been analyzed.

The analysis has been performed in the system for the FEM analysis. Because there was no possibility of interfering in the finite element code the analysis has been simplified. Two types of analysis have been performed. The analysis of aggressive substance mass transport and the mechanical analysis. The analysis is made by using a separated part of a reinforced concrete element that contains rebar with diameter  $\phi_0 = 20$  mm and 40 mm thick concrete cover, Fig. 2. The computational model is  $200 \times 100 \times 10$  mm part, virtually cut from the structure, containing a rebar ideally connected with the concrete. It has been assumed that the virtually cut part of the structure satisfies plain strain conditions. Moreover, it has been assumed that for nodes laying in the cutting plain displacements in normal directions are fixed. Distortional strains caused by increase of corrosion products volume have been specified taking advantage of thermal strains tensor analogy (the constitutive equation for a solid body with thermal distortions and corrosion product distortions after some assumptions are identical). The active elements (see Fig. 2) will start to grow when the chloride ions concentration on the rebar surface overcomes 0.4% of cement mass and the corrosion process will be activated. For the assumed concrete B25/30; according to [6, 15] the mean value of compressive strength is  $f_{cm} = 33$  MPa, the mean value of tensile strength is  $f_{ctm} = 3.3$  MPa, elasticity modulus is  $E_{cm} = 31$  GPa. The Drucker–Prager plasticity model is used to describe the concrete. The input data cohesion coefficient  $c = 4,33 \cdot 10^6$  N/m<sup>2</sup> and the angle of internal friction

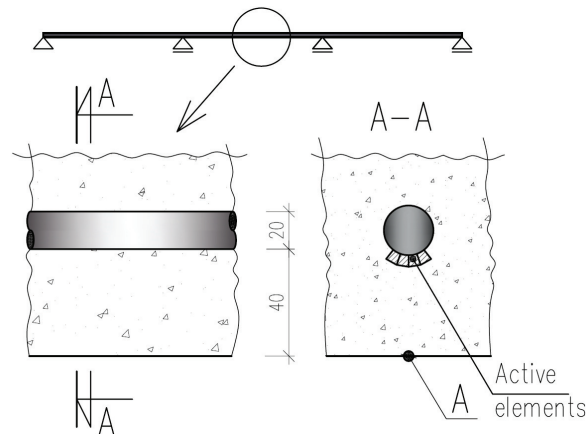


Fig. 2. Analysed numerical example

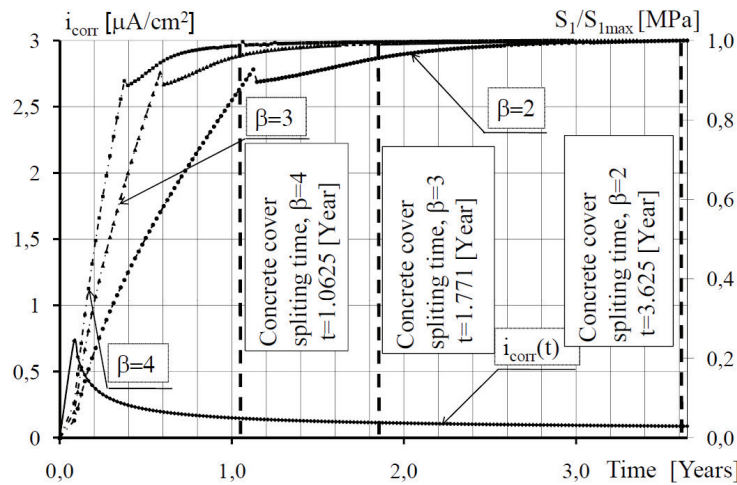
$\Phi = 60.57$  required by system have been calculated using the following formulas [4]

$$\begin{aligned} a &= \frac{m-1}{\sqrt{3}(m+1)}, & b &= \frac{2f_{cm}}{\sqrt{3}(m+1)}, & m &= \frac{f_{cm}}{f_{ctm}}, \\ c &= \frac{\sqrt{3}b(3-\sin\phi)}{6\cos\phi}, & \phi &= \arcsin\left(\frac{3\sqrt{3}a}{\sqrt{3}a+2}\right). \end{aligned} \quad (34)$$

The reinforcing steel A-I is used with the elastic material model. On the basis of [6, 15] the elasticity module  $E_s = 200$  GPa is accepted. The corrosion electric current density is determined on the basis of the regression function presented in [13]:

$$\begin{aligned} \ln(1.08 i_{\text{corr}}) &= 7.89 + 0.771 \ln(1.69 \text{Cl}) - 3006 \frac{1}{T} - \frac{1.16}{10^4} R_c + 2.24 t^{-0.215}, \\ I &= \pi \phi_0 l_a i_{\text{corr}}, \end{aligned} \quad (35)$$

where  $i_{\text{corr}}$  is the corrosion electric current density [ $\mu\text{A}/\text{cm}^2$ ], Cl is the chloride ions content in concrete [ $\text{kg}/\text{m}^3$ ],  $T$  is the steel surface temperature [K],  $R_c$  is the concrete cover electric resistance [ $\Omega$ ], and  $t$  is the corrosion process time [year]. To determine the corrosion electric current density  $i_{\text{corr}}$  the following values for parameters have been assumed: the chloride concentration in concrete  $c_{\text{Cl}} = 0.4\%$  of cement mass, the cement content in concrete  $m_{\text{cem}} = 250$   $\text{kg}/\text{m}^3$ , the rebar surface temperature  $T = 283.15$  K, the concrete electrical resistance  $R_c = 15000$   $\Omega$ , the electrochemical equivalent of iron  $k_e = 9.12 \cdot 10^{-3}$   $\text{g}/\mu\text{A} \cdot \text{year}$ , the length of section of active corrosion processes on the rebar  $l_a = 10$  mm, and the density of iron  $\gamma_{\text{Fe}} = 7.85$   $\text{g}/\text{cm}^3$ . Estimating the magnitude of distortional strains three ratios of the corrosion product volume to the corrosion cavity volume,  $\beta = V^p/V$ , have been taken into consideration, namely:  $\beta = 2$ ,  $\beta = 3$ ,  $\beta = 4$ . The results obtained for time changes of corrosion current density functions  $i_{\text{corr}}$ , the ratio of the major tension stress  $S_1(t)$  over the maximum tensile stress at the moment of concrete cover fracture  $S_{1\text{max}} \cong 3.6$  MPa (according to this model from the moment of corrosion process initiation) on the element edge (at point A according to Fig. 2) are presented graphically in Fig. 3. The distribution of equivalent plastic strain around the analyzed reinforcing bar at the moment of concrete cover splitting is shown in Fig. 4.



**Fig. 3.** The results of computer calculations of the ratio of the maximum tension stresses over the maximum stress,  $S_1(t)/S_{1\text{max}}$ , in the model at point A (cf. Fig. 2) in the function of time and electric current density



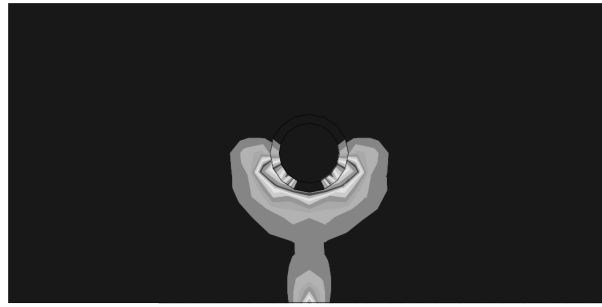


Fig. 4. Equivalent plastic strain distribution at the moment of concrete cover splitting

## 6. SUMMARY

This paper presents the model based on the theory of multicomponent media allowing the modelling of rebar's corrosion processes e.g. FEM. The presented model extends and sums up the considerations described in [8–10]. The additional internal parameter  $I^e$  describing the rebar corrosion process that depends on corrosion current intensity has been introduced in the model. The free energy function allowing formulation of constitutive relationship with mechanical distortions which depend on internal parameters has been used. FEM equations and numerical example showing application of the model for the simulation of concrete cover fracturing process have been presented. According to the author, the presented theory is in a way simplified but a complete model allowing modelling of degradation processes of concrete cover as the result of rebar's corrosion taking into account very complex physical processes.

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