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Computer-assisted hybrid reasoning in simulation and analysis of physical systems

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The aim of the paper is to advocate the use of hybrid reasoning systems for computer-assisted analysis of physical systems. The paper starts from a critical assessment of classic numerical techniques, with the problem of sensitivity analysis of fuel rod support spring in a nuclear reactor used as an example. Then, the significance and some basic issues concerning *qualitative physics* methods of analysis of physical systems are discussed. Using the example of the so-called "snap-through" mechanism, the basic principles, advantages and limitations of qualitative simulation technique are shown. Certain future development possibilities are indicated, especially the necessity to formalise the *order-of-magnitude* reasoning. The recently developing techniques of diagrammatic reasoning are also introduced, with another mechanical example illustrating sources of their advantages for certain kinds of problems. The significant role of logical (expert-system-like) reasoning techniques and constraint-satisfaction systems is shown as well. Finally, the hybrid reasoning system concept is sketched. Such hybrid systems should integrate *quantitative (numerical)* analysis, various methods of *qualitative* analysis as well as *diagrammatic* and *logical* reasoning techniques.

1. Introduction

Lots of widely used science and engineering software systems are now capable of solving quantitatively problems that once occupied researchers and engineers. They address a significant portion of current research and engineering needs, are applicable to a wide variety of realistic problems in a wide variety of computing environments, and are based on frequently very robust and reliable numerical algorithms. A sophisticated computer-aided sensitivity analysis of fuel rod support spring in a nuclear reactor will serve us as a good example of such a problem. The discrete model used contained nearly 5000 primary unknowns.

Even though such numerical (or quantitative) simulation techniques may provide crucial information about the system behaviour, they as a rule prove insufficient for addressing problems involving incomplete knowledge about system parameters. If extensive data on parameter variability are available, then a stochastic process theory could be applied. Otherwise, as is the case in most practical situations, a new methodology is needed. A promising new developments emerge in the field of so-called *Qualitative Physics* which introduced the techniques of qualitative simulation, order-of-magnitude reasoning, and the like. These techniques, and also other new developments like diagrammatic reasoning and constraint satisfaction systems, together with knowledge-based approaches, are opening new possibilities for analysis of complex systems with incomplete information or for simplification of analysis when exact numerical solution is not needed. Another important aim of these new methodologies, usually not addressed by traditional numerical techniques, is automating the intelligent selection of appropriate technique and mathematical model to the given problem, as well as controlling the solution process in a fashion transparent to the human user.

The purpose of this paper is to demonstrate that no single approach is able to solve all needs of computer simulation and analysis of practical, complex systems. To support the thesis, we first explore the basic principles of major approaches, concentrating on their advantages and limitations. To facilitate this task, we use a series of examples, some of them taken from the literature (often considerably modified and extended), some our own. Gathering together the examples, otherwise scattered among disparate literature sources, provides new insights into relationships between various approaches and techniques, highlighting their potential advantages and limitations. As a result, we come to the conclusion that the future belongs to *hybrid reasoning systems*, combining various techniques, especially quantitative and qualitative analysis, diagrammatic reasoning, expert-system-like logical reasoning and constraint satisfaction systems into integrated, versatile analysis tools for analysis of complex physical systems.

The paper is a considerably extended and reworked version of the earlier short article presented at the *Japanese-Polish Joint Seminar on Advanced Computer Simulation* in Tokyo [18].

2. Numerical simulation

For all the unquestionable successes and significance of the quantitative computational methods of solving engineering problems, some limitations of them recently became obvious. Let us start the discussion from an example of a typical numerical simulation which will clearly illustrate the power of the classic approach and will serve as a model problem for discussing its limitations.

The example, taken from [5], concerns sensitivity analysis of a grid spring. Figure 1 shows one of two identical grid springs from a supporting assembly of a fuel rod in a Pressurised Water nuclear Reactor (PWR). There are hundreds of such springs in a fuel assembly of a typical PWR. To simulate the insertion of the fuel rod between the supporting springs, a prescribed displacement history of the spring's centre A is given. The problem is to evaluate first-order sensitivities (i.e. gradients) of the reaction R at the point A with respect to changes of the spring thickness along the spring. The problem is of great practical importance, as the spring dimensions will always vary because of unavoidable manufacturing imperfections while the constancy of the contact force between the spring and the fuel rod is essential for reliable performance of the reactor.

Using the terminology of traditional computational mechanics, the problem can be, and has been, analysed within the framework of the so-called *shape structural sensitivity analysis* [15]. Many complicating factors influence the numerical analysis, making it highly complex. Non-linear elastic-plastic material behaviour and potentially large deformations imply highly non-linear behaviour of the system. Varying thickness results in a necessity to formulate the appropriate partial differential equations boundary-value problem in accordingly modified spatial domains. That in turn hampers systematic solution of comparisons required by any sensitivity study. Solving the problem involves looking for values of the gradients $\frac{dR}{dh_i}$, where h_i, $i = 1, 2, \ldots$ is the spring thickness at subsequent spring cross-sections. The finite element method was used with a discretization mesh of 164 three-dimensional elements resulting in 4473 degrees of freedom, see Fig. 1. The material data for the spring are given by Young modulus (186.2 GPa in this case), Poisson's ratio (equal to 0.27 here), and a multi-linear constitutive law $\varepsilon_p = f(\sigma_y)$.

Numerical programs typically employed for solving problems of this type are very complex. Due to complicated geometry of real-life specimens, the use of automatic input data generators is of limited assistance. Assuring accuracy of huge amounts of numerical input data and confidence in correctness of the program execution and output is also a serious problem, to a large extent because the (intuitive) assessment of the physical feasibility of the results is seriously hampered by great amounts of unstructured numerical data produced and deceptive nature of physical intuition for highly non-linear, novel problems like that presented here. The amount of resulting data is quite overwhelming — for every time step (of which typically there may be as many as 10^3 to 10^5), the programs may produce also tens of thousands real numbers. Sorting them out effectively requires a lot of experience and tedious, error-prone work, even with the help of graphical data visualisation tools. Not surprisingly, the total cost of the analysis — which should include the costs of the development of the analysis package, formulating the FEM model, preparing and checking large amounts of input data, running the program and analysing the results — is quite substantial.

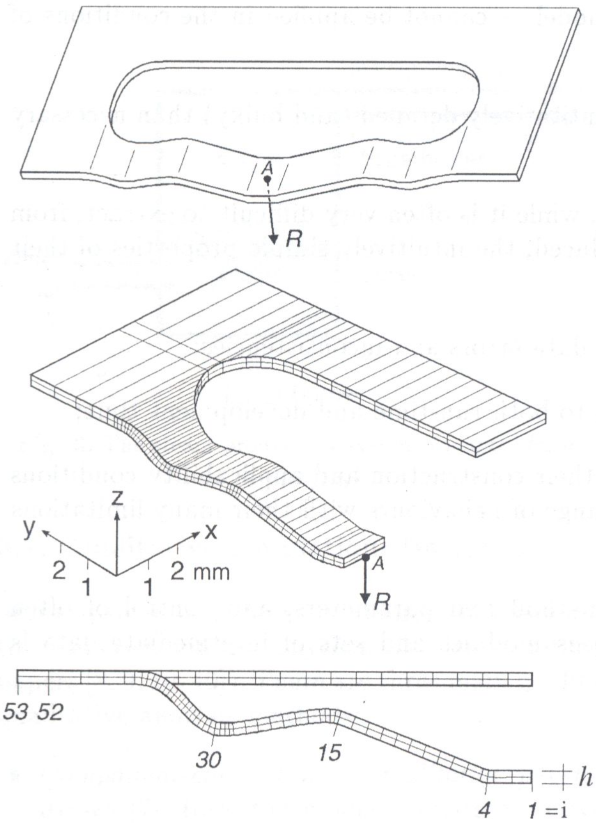

Fig. 1. The fuel rod support spring and its finite element model *(adapted from [5])*

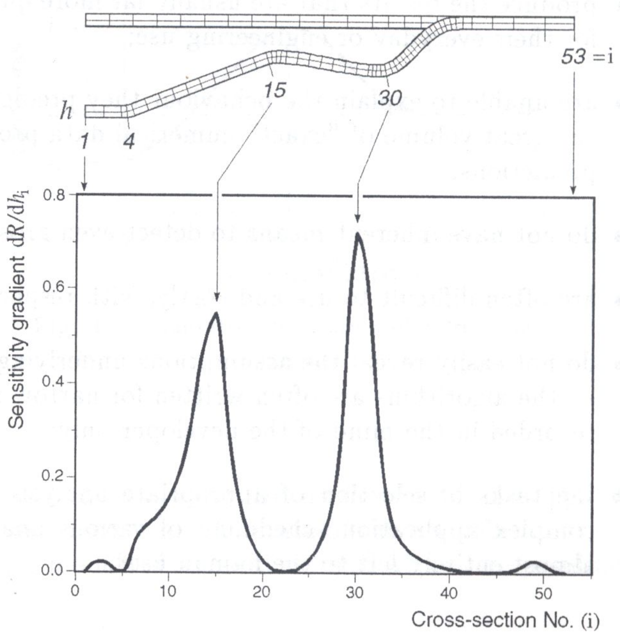

Fig. 2. Some results of analysis of the spring: sensitivity gradients at different cross-sections

The analysis, briefly reported here, has been done with the in-house finite element analysis package at the University of Tokyo [5], by a team of researchers including one of the authors of this paper (M. Kleiber). About 5000 real numbers had to be supplied as the program input. Some of the significant results of the analysis are shown in Fig. 2 which displays values of $\frac{dR}{dh_i}$ at different cross-sections for a certain value of the prescribed displacement. Other results obtained concerned variations of the sensitivity gradients $\frac{dR}{dh_i}$ (at the series of particular cross-sections i) with varying prescribed displacement. From the practical point of view, the most important general result of all this complex analysis is the conclusion that the critical areas in the spring (with high sensitivity to manufacturing imperfections, see Fig. 2) occur around sections no. 15 and 30, that is at the two of several bends in the spring. For certain purposes such a qualitative answer would be fully sufficient; at least knowing it in advance may be of great help in formulation, checking, running and assessment of results of the full quantitative numerical analysis. Unfortunately, the new problem appears: how to find methods of obtaining such qualitative answers quickly, cheaply, and automatically?

In summary, we may reasonably say that complex and costly numerical analysis may not always be just *the* thing (or *the only* thing) we need — in many cases, less precise, qualitative answers might suffice, or at least they might help us in effective planning and controlling of the more precise numerical analysis. Moreover, by their nature the numerical analysis methods address only that part of engineering which involves numerical manipulation and data management. Engineering knowledge is, however, not mere numbers and most of it cannot be modelled adequately using purely numerical means. Configuring aeroplanes, predicting avalanches and automating factories requires intuitive judgement and qualitative assessment, i.e., processes of reasoning that cannot be precisely quantified. But we would like to see computers capable of helping us with this non-numerical analysis to a similar extent as they are good at intensive number-crunching.

Thus, while current numerical techniques and their computer implementations are powerful and useful, they have, nevertheless, certain limitations:

- require complete numerical specification of the model — cannot be applied in the conditions of incomplete or imprecise knowledge;
- produce the results that are usually far more quantitatively detailed (and bulky) than necessary for their everyday or engineering use;
- are unable to explain the behaviour they predict, while it is often very difficult to extract, from the great volume of "exact" numerical data produced, the intuitively simple properties of their predictions;
- do not have inherent means to detect even gross data errors and inconsistencies;
- are often difficult to use and costly, with respect to both run-time and development time;
- do not easily reveal the assumptions underlying their construction and applicability conditions — the algorithms are often written for narrow range of behaviours with their many limitations recorded in the mind of the developer only;
- the tasks of selection of appropriate analysis method and parameters, and control of often complex application scheduling of various analysis modules and sets of intermediate data is almost entirely left to the human user.

3. Qualitative analysis

Let us start our introduction to methods of *qualitative analysis and reasoning* with the already classic example introduced by Forbus [12]. It is a steam production subsystem of a propulsion system used, e.g., on warships, see Fig. 3.

Water is fed into the boiler, heated by oil-fired burners, and turned to steam. The steam is additionally heated in the superheater, and leaves it through the steam outlet. The question is: *what will happen to the temperature* T_{out} *of the produced steam when the water temperature* T_{in} *increases*?

Note that neither numerical values nor exact quantitative equations describing the behaviour of the system were given. Also, the solution is not straightforward — e.g., the immediate answer that *'when inlet temperature rises, so will the outlet temperature'*, though sounds plausible, yet somehow feels in need of more thorough justification. But an engineering student is quite able to answer it, after some consideration. The reasoning goes as follows. The boiling occurs at the same temperature, so when the water coming into the boiler becomes hotter, the amount of heat that must be added to boil a piece of water is reduced. That is, the water will boil sooner which means the rate of steam production increases. Larger amounts of produced steam must now flow through the superheater, that is, the steam must flow faster, thus spending less time in the superheater. Less time spent in the superheater means less heat is transferred to the steam, and as the starting temperature of the steam is the same, the final temperature at the outlet will fall when the feedwater temperature increases!

This was an example of the kind of reasoning called *qualitative reasoning* — reasoning that often suffices to solve quite sophisticated physical problems, and in many cases is the only method we have (when quantitative data are missing or unreliable) or can afford (when time presses or formulating and solving complex numerical model is too costly). Engineers and scientists are quite apt in such kind of "inexact" reasoning — in fact, it is an indispensable ingredient underlying the quantitative knowledge needed to build, understand and solve the complex analytical and numerical models of real physical systems. The challenge before the field is thus how to formulate the more or less formalised descriptions of qualitative reasoning processes so as to be able to program computers capable to demonstrate the human ability to reason in this way.

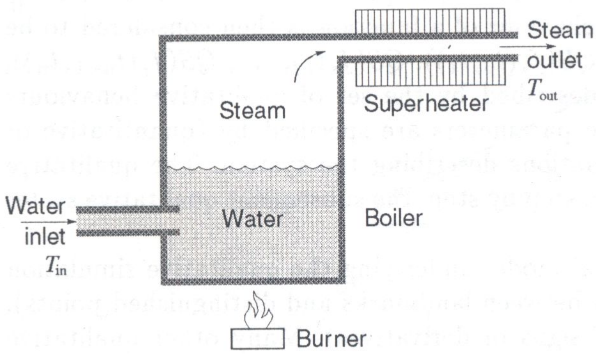

Fig. 3. The steam production system *(adapted from Forbus [12])*

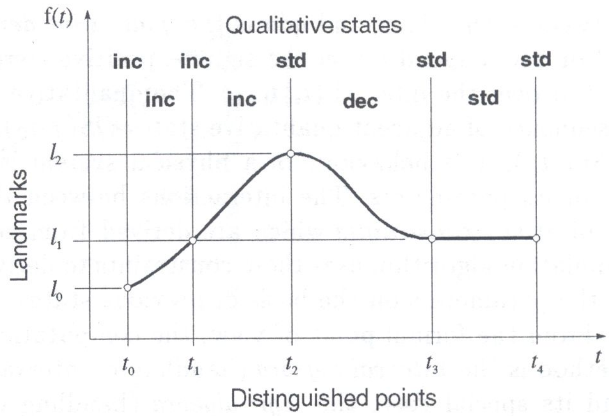

Fig. 4. A qualitative description of a "reasonable" function

3.1. Qualitative simulation: Basics

Many specific techniques of qualitative reasoning and analysis were already developed and applied to diverse exemplary physical problems and systems (see, e.g., the state-of-the-art collections of papers [8, 9, 10, 40]). There are three main qualitative models of physical systems with corresponding qualitative analysis methods:

- *Component-connection* or *qualitative physics based on confluences* approach of de Kleer and Brown [7]. Here the model consists of active objects (components) transforming attributes of matter, energy and information that are exchanged between objects along their passive connections.
- *Qualitative Process Theory* (QPT) of Forbus [11]. The QPT model consists of a set of passive objects and relations between their attributes, and a set of active processes describing possible changes of the attributes of objects.
- *Qualitative simulation* (QSIM) of Kuipers [20–22]. The model resembles traditional mathematical modelling techniques, as it describes the system as a set of qualitative variables subject to constraints defined by a set of qualitative differential equations.

The third of the above approaches seems now to be the most popular. It is considered to be well understood as to its formal properties [21, 36, 37] and comparatively easy to implement. Translators from descriptions using the other two models into QSIM were constructed as well. To spare the reader the trouble of searching for appropriate literature in order to understand the next two examples and further discussion, below we will briefly introduce the basic principles of the qualitative simulation technique, as described in main papers [20, 21] and the recent book [22] by Kuipers.

In QSIM, the parameters of a physical system are modelled by a set of functions $\{f(t)\}$ over the extended real number domain $\mathcal{R}^* = [-\infty, +\infty]$. It is assumed that these functions are "reasonable" (which means, briefly, that they are continuous, continuously differentiable, have finite number of critical points within any bounded interval, and values of their derivatives at the ends of any such interval are equal to appropriate limits). Such functions can be described qualitatively by dividing them into monotonic segments, see Fig. 4. The values of f at the joints of the segments are called *landmark values*; the (possibly partially) ordered set of such values is called a *quantity space* of the function. A set of *distinguished points* $t_0 < t_1 < \ldots < t_n$ marks points at which something interesting happens to the value of f, such as passing a landmark value or reaching an extremum. A *qualitative state* $QS(f, t)$ or $QS(f, (t_i, t_{i+1}))$ of a function f at a point t or in an interval (t_i, t_{i+1}) is a pair $\langle \mathsf{qval}, \mathsf{qdir} \rangle$, where qval is a landmark value l or an interval (l_j, l_{j+1})

between such values, and **qdir** is the *qualitative derivative*, taking on one of the symbolic values **inc**, **std** or **dec**, depending on the sign — positive, zero or negative, respectively — of the value of $\frac{df}{dt}$ at t or over the interval (t_i, t_{i+1}). The qualitative behaviour of a function is then considered to be a sequence of adjacent qualitative states $QS(f,t_0)$, $QS(f,(t_0,t_1))$, $QS(f,t_1)$, ..., $QS(f,(t_{n-1},t_n))$, $QS(f,t_n)$. The behaviour of a physical system is described by the set of qualitative behaviours of all its parameters. The interactions between the parameters are specified by (quantitative or qualitative) *constraints* which are derived from equations describing the system. The qualitative simulation algorithm uses these constraints to derive, step by step, the subsequent qualitative states of the parameters on the basis of previous states.

From the formal point of view, the computational model underlying the qualitative simulation method is the *interval algebra* (handling of intervals between landmarks and distinguished points), and its special case, the *sign algebra* (handling of signs of derivatives). Many other qualitative reasoning approaches are based on these algebras [36,37]. The consequences of this fact will be discussed in Section 3.3.

3.2. The snap-through problem

Although the meaning of the (control) variable t in the above exposition of the qualitative simulation technique is typically considered to be *time*, it has not necessarily to be so. Let us illustrate the technique with an (untypical in this respect) example from structural mechanics, adapted from Kleiber [17].

It is a so-called snap-through mechanism with geometric nonlinearity and one degree of freedom. The mechanism, see Fig. 5a, consists of one bar of cross-sectional area A and Young modulus E which is subject to a load P enforcing its tip to move a distance u. We consider u as a control variable and ask for the (quasi-static) behaviour of other system parameters (especially the force P) as a function of u.

From the geometry of the mechanism (see Fig. 5a), we have $l^2 - h^2 = (l + \Delta l)^2 - (h - u)^2$. Solving the above equation for $\varepsilon = \frac{\Delta l}{l}$ (the axial strain in the bar), we get

$$\varepsilon = -1 + \sqrt{1 - \frac{u(2h-u)}{l^2}}\,.$$

For small ϑ, we have $u \ll l$ and $h \ll l$, and thus $\frac{u(2h-u)}{l^2}$ is small; therefore the square root can be eliminated according to the standard approximation formula $\sqrt{1-x} \approx 1 - \frac{x}{2}$. As a result, we get the approximate formula $\varepsilon \approx \frac{u(u-2h)}{2l^2}$.

From the equilibrium condition, the axial force N is related to the load P as $P = -N \sin\vartheta$. Again, for small values of ϑ, it can be adequately approximated as $P \approx -N\vartheta \approx N\frac{u-h}{l}$.

In a linear case, the force N and strain ε would be related by the equation $N = AE\varepsilon$. Let us assume, however, the more loose, possibly non-linear, qualitative relation — that N can be any monotonically increasing function of ε, i.e. $N = M_0^+(\varepsilon)$. The subscript 0 means here that $M_0^+(0) = 0$. In summary, we have the following set of (partially qualitative) constraints relating the set $\{u, P, \varepsilon, N\}$ of parameters describing our system:

$$P \approx N\frac{u-h}{l}\,, \tag{1}$$

$$\varepsilon \approx \frac{u(u-2h)}{2l^2}\,, \tag{2}$$

$$N = M_0^+(\varepsilon)\,. \tag{3}$$

There is no general, quantitative method of solving such kind of equations. How can we find the qualitative behaviour of the mechanism on the basis of such a description?

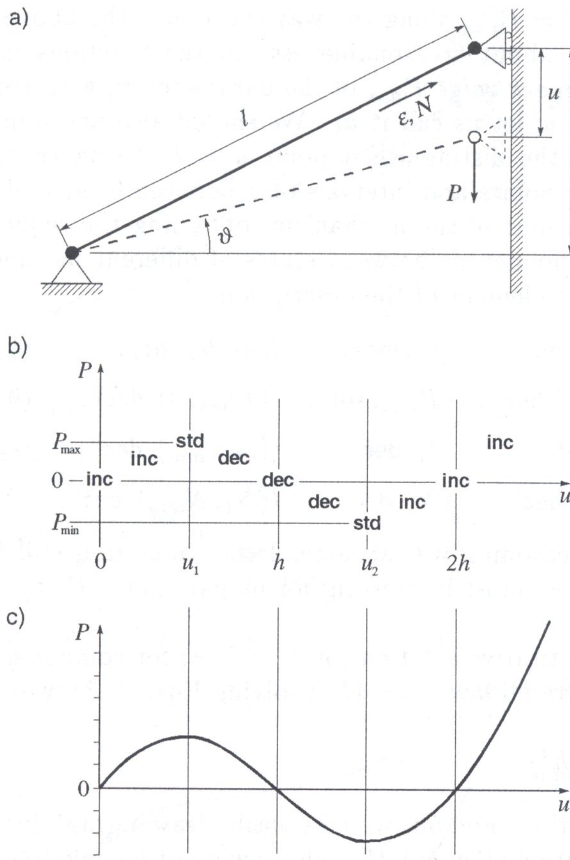

Fig. 5. The snap-through mechanism (a) and its qualitative (b) and quantitative (c) solutions

The qualitative simulation algorithm starts from establishing the first approximation to quantity spaces of the parameters. Without going into details (see [17] for them), we may safely assume the following ordered sets of landmarks for our parameters:

$u: \quad 0 < +\infty ,$

$P: \quad P_{\min} < 0 < +\infty ,$

$\varepsilon: \quad \varepsilon_{\min} < 0 < +\infty ,$

$N: \quad N_{\min} < 0 < +\infty .$

In the above we do not assume yet any correspondence between (landmark) values taken on by different parameters.

The next step is to establish the qualitative state of the system at the start of the simulation. As the displacement u is the control variable, with values between 0 and $+\infty$, its initial state should obviously be $u : \langle 0, \mathsf{inc} \rangle$. From Eq. (2) we have $\frac{\mathrm{d}\varepsilon}{\mathrm{d}u} \approx \frac{u-h}{l^2}$, so that at $u = 0$ we get $\frac{\mathrm{d}\varepsilon}{\mathrm{d}u} < 0$, thus the initial state of $\varepsilon : \langle 0, \mathsf{dec} \rangle$. Similarly, using Eqs. (3) and (1), we may complete the description of initial state, getting:

$$u : \langle 0, \mathsf{inc} \rangle ; \qquad P : \langle 0, \mathsf{inc} \rangle ; \qquad \varepsilon : \langle 0, \mathsf{dec} \rangle ; \qquad N : \langle 0, \mathsf{dec} \rangle .$$

What will be the next state of the mechanism? The displacement u simply grows indefinitely — nothing interesting here. The load P depends on both N and u, changing in opposite directions — hard to predict now which prevails. Let us consider ε, then. It decreases from 0, and its quantity space is bounded from below by $\varepsilon_{\min}$, so there should be a situation when $\varepsilon = \varepsilon_{\min}$. This is possible only if $\frac{\mathrm{d}\varepsilon}{\mathrm{d}u} = \frac{u-2}{l^2} = 0$ which implies $u = h$ and by Eqs. (1) and (3), $P = 0$ and $N < 0$. Since, however, $P : \langle 0, \mathsf{inc} \rangle$ initially, the just discovered condition $P = 0$ at a later time means that P must attain a local maximum at some $P = P_{\max}$ along the way (note how the above reasoning steps crucially depend on the assumptions about "reasonableness" of the functions involved). As a result, we have established a new *landmark value* $P_{\max}$ of the parameter P, with corresponding *distinguished point* of the control variable u; let us call it u_1. We should also add appropriate landmark values N_1 and ε_1 at $u = u_1$, and the distinguished point $u = h$. Using that, and checking again the constraints of Eqs. (1–3) for points and intervals of u between 0, u_1 and h, we are able to extend further the qualitative behaviour of the mechanism, obtaining the sequences of qualitative states as shown below. The correspondences between states of different parameters are now intentional and constitute the significant element of the description:

$$\begin{array}{rlllll} u: & \langle 0, \mathsf{inc}\rangle, & \langle (0,u_1), \mathsf{inc}\rangle, & \langle u_1, \mathsf{inc}\rangle, & \langle (u_1,h), \mathsf{inc}\rangle, & \langle h, \mathsf{inc}\rangle; \\ P: & \langle 0, \mathsf{inc}\rangle, & \langle (0,P_{\max}), \mathsf{inc}\rangle, & \langle P_{\max}, \mathsf{std}\rangle, & \langle (P_{\max},0), \mathsf{dec}\rangle, & \langle 0, \mathsf{dec}\rangle; \\ \varepsilon: & \langle 0, \mathsf{dec}\rangle, & \langle (0,\varepsilon_1), \mathsf{dec}\rangle, & \langle \varepsilon_1, \mathsf{dec}\rangle, & \langle (\varepsilon_1,\varepsilon_{\min}), \mathsf{dec}\rangle, & \langle \varepsilon_{\min}, \mathsf{std}\rangle; \\ N: & \langle 0, \mathsf{dec}\rangle, & \langle (0,N_1), \mathsf{dec}\rangle, & \langle N_1, \mathsf{dec}\rangle, & \langle (N_1,N_{\min}), \mathsf{dec}\rangle, & \langle N_{\min}, \mathsf{std}\rangle. \end{array}$$

With the same kind of reasoning we can extend the simulation still further. The whole range of qualitative behaviour of the most interesting for us parameter P can be seen in semi-graphical form in Fig. 5b.

Figure 5c shows the quantitative solution for $P = P(u)$ for comparison. It was obtained under the assumption of linear material law $N = AE\varepsilon$; solving Eqs. (1–3), we will get in this case

$$P = \frac{AE}{2l^3}(u^3 - 3u^2h + 2uh^2).$$

From the qualitative solution (Fig. 5b) we may easily draw a graph for P very similar in appearance to the quantitative solution (Fig. 5c), though we will not be able to establish exact positions of the extrema and values $P_{\min}$ and $P_{\max}$. Note, however, that the qualitative solution is valid for the whole range of models of the mechanism — namely, all in which N is a monotonically increasing function of ε — not only for the linear material law. The qualitative solution is *less exact*, but *more general*. It may sound trivial, but note that using the qualitative analysis method we were able to establish quite precisely both the *level of precision* (by the formal qualitative behaviour description) and the *level of generality* (by the allowed class of functions relating N and ε) of the solution.

3.3. Qualitative simulation: Incompleteness

The technique of qualitative simulation and other qualitative analysis approaches, though capable to solve great many qualitative analysis problems, are not without some serious limitations. *Quantitative* techniques usually *under-abstract*, i.e., give us too specific answers, requiring at the same time to be fed with too specific data, often unavailable and in fact not necessary to produce the general, qualitative answers we are interested in. On the other hand, *qualitative* techniques which, like qualitative simulation, are based on sign and interval arithmetic, have the pronounced tendency to *over-abstract*, i.e., to give us answers too imprecise or too general. Even more alarming, that limitation has been shown to be an inherent feature of the method itself. Thus, easy remedies like, say, increasing precision of initial data or multiplying landmarks and narrowing value resolution intervals will not work (see, e.g., [36], together with Kuipers' reply in the same issue, and several papers in [40], especially [37]).

Another problem with the classic qualitative simulation method can be illustrated by the analysis of a little more complicated snap-through mechanism. Let us add a spring k to it, as shown in Fig. 6a. With this spring, the equation (1) will take the form

$$P \approx N\frac{u-h}{l} + M_0^+(u) \qquad (1')$$

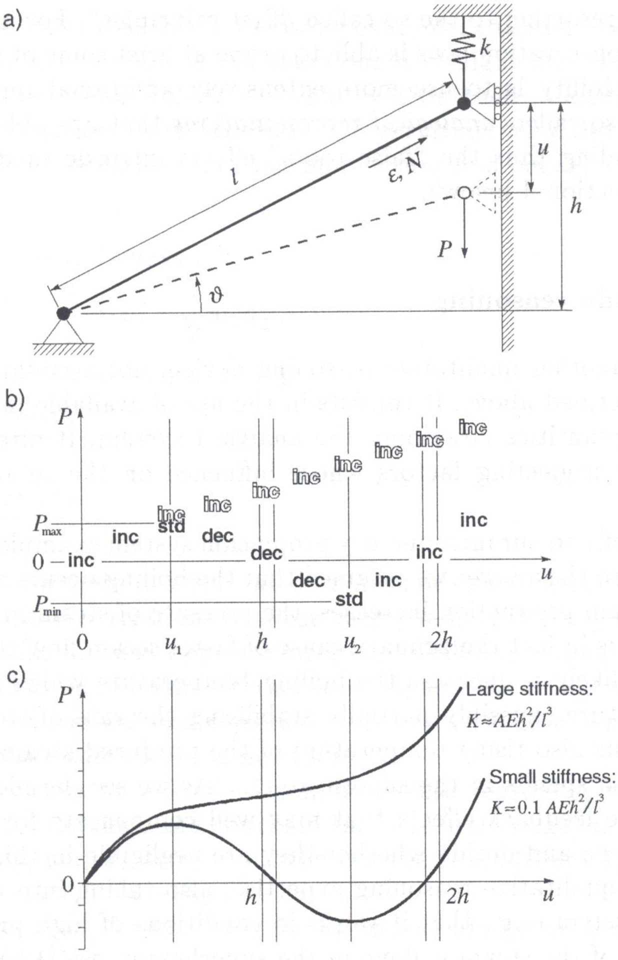

Fig. 6. The snap-through mechanism with spring (a) and its qualitative (b) and quantitative (c) solutions

where the term $M_0^+(u)$, responsible for the reaction of this spring, denotes again some monotonically increasing function of u, though generally different than the function $M_0^+(\varepsilon)$ occurring in Eq. (3). Applying the same reasoning method as before, we find that for $u = h$ we have $P = P_1 > 0$ (instead of $P = 0$), and $N < 0$ and decreasing, but we will not be able to determine the sign of the derivative of P and, as a result, also the direction of its change. There is not enough information to decide which of the additive terms in Eq. (1′) prevails. The only thing we can do is to consider several possible cases — it leads to several possible qualitative solutions, among them one very similar as in the non-spring case (compare Figs. 5b and 6b — only now the zeros of P are moved inside the interval $[h, 2h]$), and another one quite different, with P increasing all the time, see Fig. 6b.

Again, assuming linear spring with stiffness K we may calculate the quantitative solution (whose exactness, however, crucially depends on our simplifying approximations during formulation of the mathematical model of the mechanism, Eqs. (1–3)):

$$P = \frac{AE}{2l^3}\left[u^3 - 3u^2h + 2u\left(h^2 + \frac{l^3}{AE}K\right)\right].$$

and check that indeed it leads to both qualitative behaviours found above, for different relative magnitudes of the spring stiffness K as compared with the value of the term $\frac{AEh^2}{l^3}$, see Fig. 6c.

It turned out rather well in this case, but it might not in others. Indeed, it has been found that the direct qualitative history generation used in the qualitative simulation method may lead to "intractable branching", i.e., a great number of uninteresting qualitative behaviours, including also spurious, physically impossible predictions. To avoid them, additional constraints are necessary [23]. One of the ways to introduce them is to integrate the simulation with a more generic description of analysed processes by resorting to the so-called "first principles". For instance, the direct use of energy and momentum conservation laws is able to prune at least some of the physically impossible paths [13]. Another possibility is to use more extensively additional representation and reasoning tools, especially the so-called *analogical representations* that are able to *model* more directly the physical reality, avoiding thus the "false roots" effects intrinsic in descriptive, propositional formalisms ([6, 24], see Section 4 below).

3.4. Order of magnitude reasoning

Humans commonly use another qualitative reasoning device, not accounted for in the qualitative simulation technique described above. It consists in the use of available information about *relative orders of magnitude* of quantities describing the analysed system. It often leads to great simplification of the model by neglecting factors whose influence on the solution is considered to be sufficiently small.

First, let us return briefly to our introductory propulsion system example (see Fig. 3). In the qualitative reasoning leading to the answer we assumed that the boiling occurs at the same temperature. Yet, when the rate of steam production increases, the pressure of steam in the boiler also increases (this increase of pressure is in fact the primary cause of faster steam flow through the superheater). The pressure increase is likely to increase the boiling temperature which may compensate for the rising feedwater temperature (possibly partially stabilising the rate of steam production). Rising boiling temperature means also rising temperature of the produced steam which may compensate for shorter time the steam spends in the superheater... As we see, besides the linear cause-effect chain there are also some feedback effects that may well compensate for the main ones. How to estimate their contributions and decide whether they are negligible in this case? We do such estimating, as a part of our qualitative reasoning expertise, also taking into account some additional information about our system (e.g., that it works in conditions of high pressure in the boiler and that most of the heating of the steam is done in the superheater, see [12]). To conduct such kinds of reasoning, however, additional formal tools are needed, besides interval and sign arithmetic.

Second, in our snap-through example we decided that "for small ϑ" we may derive simpler, approximate formulas for ε and P, respectively. That is, we conducted some informal qualitative reasoning already in the process of formulating the model of the system, before we even started the proper qualitative analysis of it. The same remark applies also to the problem of defining proper quantity spaces for the parameters — e.g., certain amount of qualitative reasoning is already required to decide that the minimal force $P_{\min}$ can be negative. This informal qualitative reasoning should also be moved into the (formalised) qualitative analysis process if we aim at automatization of the qualitative analysis technique and want to avoid some gross errors. The errors may arise, e.g., due to improper estimation of which quantities can be safely neglected, or due to intransitivity of approximate equality that precludes indiscriminate substitution of only approximately equal terms — something we have freely done for ε and P in the example (see [34, 36, 37]).

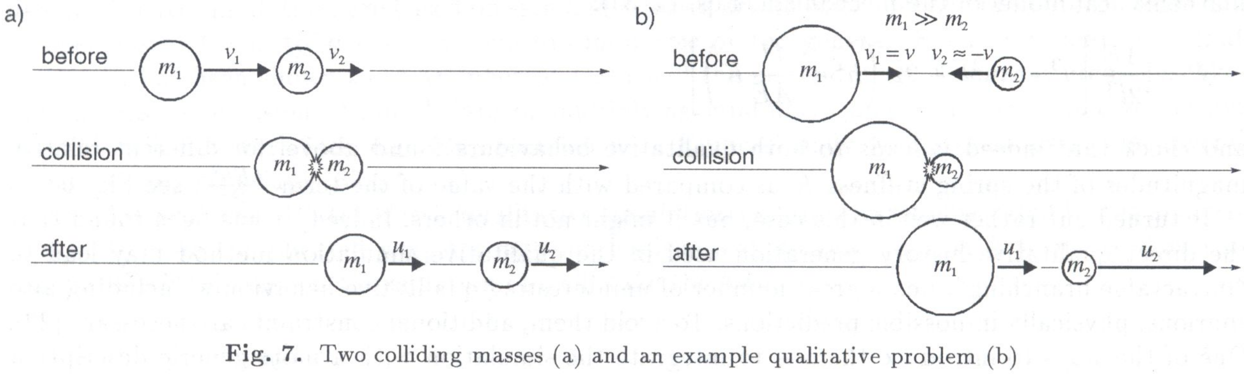

Fig. 7. Two colliding masses (a) and an example qualitative problem (b)

The point is excellently illustrated by the following example, adapted from Raiman [34]. Consider two elastic balls with masses m_1 and m_2 moving along a straight line, with velocities v_1 and v_2, respectively, see Fig. 7a. Let them collide — what will be their velocities u_1 and u_2 after the collision? Assuming no energy loss in the collision, the resulting velocities may be calculated from the following "first-principle" equations, stating momentum and energy conservation laws for the two balls:

$$\begin{aligned} M &= m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2 , \\ E &= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 . \end{aligned}$$

Solving the above equations for u_1 and u_2, we get:

$$\begin{aligned} u_1 &= \frac{(m_1 - m_2)v_1 + 2m_2 v_2}{m_1 + m_2} , \\ u_2 &= \frac{(m_2 - m_1)v_2 + 2m_1 v_1}{m_1 + m_2} . \end{aligned} \tag{4}$$

Now consider an actual problem in which we do not know exact values for masses and velocities of the balls, only some qualitative relations between them, namely that m_2 is negligible compared to m_1 ($m_2 \ll m_1$), and the balls move in opposite directions with approximately equal velocities ($v = v_1 \approx -v_2$), see Fig. 7b. What can be done in this case? Again, the primary tools of the qualitative simulation technique — *sign* and *interval algebras* — are of no use here. Another technique — some sort of an *order-of-magnitude calculus* — is necessary.

Human engineers can resort to a kind of common sense reasoning without reference to the equations above (see [34] for details), and conclude that the first (large) mass will continue moving at about the same velocity, i.e., $u_1 \approx v_1 = v$, whereas the second (small) mass will bounce back with much larger velocity $u_2 \approx 3v \approx -3v_2$.

Alternatively, when the exact formulas (4) for u_1 and u_2 are given, a human may conduct the following approximate derivation:

$$\begin{aligned} u_1 &= \frac{(m_1 - m_2)v_1 + 2m_2 v_2}{m_1 + m_2} \approx && \mid v_1 = v, \quad v_2 \approx -v \\ &\approx v\,\frac{m_1 - 3m_2}{m_1 + m_2} \approx && \mid m_2 \ll m_1 \\ &\approx v , \end{aligned}$$

$$\begin{aligned} u_2 &= \frac{(m_2 - m_1)v_2 + 2m_1 v_1}{m_1 + m_2} \approx && \mid v_1 = v, \quad v_2 \approx -v \\ &\approx v\,\frac{3m_1 - m_2}{m_1 + m_2} \approx && \mid m_2 \ll m_1 \\ &\approx 3v , \end{aligned}$$

arriving at the same values, but with the conviction that they are more reliable, being derived more formally.

However, what if someone attempts to derive the approximate result directly from the initial momentum and energy conservation laws? The derivation can be conducted as follows:

$$\begin{aligned} & m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2 , && \mid m_2 \ll m_1 , \quad v = v_1 \approx |v_2| \\ & m_1 v \approx m_1 u_1 + m_2 u_2 , \\ & v - u_1 \approx \frac{m_2}{m_1} u_2 , && \mid m_2 \ll m_1 \\ & v - u_1 \approx 0 , \\ & u_1 \approx v . \end{aligned} \tag{5}$$

So far so good, but let us try that for u_2, now starting from the energy conservation law:

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\,, \qquad | \; m_2 \ll m_1\,, \quad v = v_1 \approx |v_2|$$

$$m_1v^2 \approx m_1u_1^2 + m_2u_2^2\,,$$

$$m_1(v-u_1)(v+u_1) \approx m_2u_2^2\,, \qquad | \; m_1(v-u_1) \approx m_2u_2 \quad \text{from Eq. (5)}$$

$$m_2u_2(v+u_1) \approx m_2u_2^2\,,$$

$$v + u_1 \approx u_2\,, \qquad | \; u_1 \approx v \quad \text{as found already, thus:}$$

$$u_2 \approx 2v\,,$$

and *that* is grossly wrong!

What happened? We substituted indiscriminately approximately equal ($\approx$) terms, not taking into account that approximate equality is an intransitive relation. Note that the result is not due to some obvious error in handling approximate reasoning, as it would be, e.g., to infer from Eq. (5) that $u_2 \approx 0$ (just because $u_2 \approx \frac{m_1}{m_2}(v-u_1)$ and $v \approx u_1$).

As we see, the common and innocent-looking practice of treating approximate equality in the same way as strict equality (we did that without hesitation during formulation of the mathematical model of the snap-through mechanism above!) may produce grossly wrong results in some cases. Therefore, incorporating the approximate order-of-magnitude reasoning into the qualitative analysis system needs proper formalisation of the operations and relations involved. The formalisms like that devised in [31, 34] address this problem, leading to correct formal solutions of qualitative problems like the one above. Incorporation of such a formalism as an ingredient of practical qualitative analysis systems seems therefore essential for their proper functioning.

4. Diagrammatic reasoning

The field of *diagrammatic data and knowledge representation* and *diagrammatic reasoning* has recently become one of the most rapidly growing areas of research in artificial intelligence and related fields [24, 32]. Human problem-solvers use diagrams constantly to formulate and communicate problems and as, often indispensable, aids to solve them. Thus, it seems obvious that any computer system which aims at modelling human reasoning ability should be able to use diagrams also. Imagine how it would be like if we had not used any diagrams in exposition and solving of the examples in this paper.

Diagrammatic representation uses *diagrams* to represent data and knowledge, and diagrammatic reasoning uses direct manipulation and inspection of a diagram as primary means of inference. Diagrams are a kind of *analogical* (or *direct*) knowledge representation mechanism that is characterised by a parallel (though not necessarily isomorphic) correspondence between the structure of the representation and the structure of the represented. E.g., relative positions and distances of certain marks on a map are in direct correspondence to relative positions and distances of the cities they represent, whereas in a *propositional* representation (e.g., in a set of mathematical expressions, or formulas of predicate calculus), the parts of the representation or relationships between them need not correspond explicitly to any parts and relations within the thing denoted. The analogical representation can be said to *model* or *depict* the thing represented, whereas the propositional representation rather *describes* it. A similar distinction can be made regarding the method of retrieving information from the representation. The needed information can usually be simply *observed* (or *measured*) in the diagram, whereas it must be *inferred* from the descriptions of the facts and axioms comprising the propositional representation.

It should be added that analogical representations, including diagrams, do not provide any exceptional means for representing information that cannot be represented in other ways, e.g., by propositional schemes (say, logical formulas). They usually represent the same information, only differently organised — in the way facilitating its use for certain tasks (e.g., for modelling or reasoning). These representations permit explicit representation and direct retrieval of information that can be represented only implicitly in other types of representations and then has to be computed (or inferred), sometimes at great cost, to make it explicit for use. They also permit effective control of the reasoning process, facilitating search both in data and solution spaces, as it may be guided by explicit proximity or adjacency relations between elements of the representation. Moreover, the diagrammatic encoding of necessary information, its transformations during reasoning, and the reasoning results, are more natural and understandable to human users, especially in applications where already the use of drawings and diagrams is essential and widely practised.

Diagrammatic reasoning approach appears more and more often in recent literature on (especially qualitative) analysis of physical systems. It is used for problems involving spatial relationships, like kinematics [16] or analysis of beam structures [38]. But it may also be used to handle abstract structures of formal models, like diagrammatic analysis of phase portraits [42], or diagrammatic representations of parameter influence graphs or constraint systems to control the reasoning process [20, 21]. Diagrammatic reasoning seems especially useful for qualitative analysis because, being qualitative by its very nature, it can be nevertheless developed into a completely strict and formal method of reasoning [2, 14, 39].

A good illustration of the basic ideas and advantages of diagrammatic representation and reasoning may be provided by a simple mechanical problem (Fig. 8). Similar, though more complicated example was used by Larkin and Simon in their seminal paper [26]. Note that the diagram is an essential part of the problem statement: try to imagine how the problem description might look like without a diagram — and then see Section 5 for a continuation of this example.

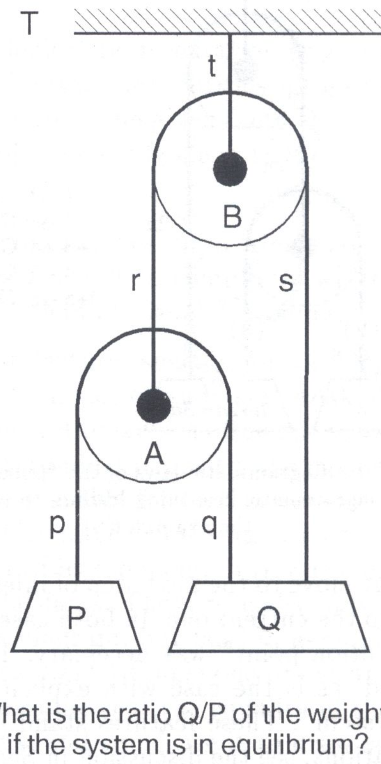

Fig. 8. The example of a simple pulley system

With such a representation, the solution is straightforward, and is guided directly by the diagram. That is, one may obtain it using *diagrammatic reasoning*. First, the "laws of nature" of the pulleys world are stated diagrammatically (Fig. 9a). There are four of them, listing the basic structural configurations occurring in pulley diagrams with the constraints they impose on values of weights and forces along ropes for every configuration. Then, one moves with a finger (real or imaginary) along the diagram (Fig. 9b), inspecting the (*local!*) configuration encountered, filling in the missing parameter value according to the matching constraint (Fig. 9a), and moving again to the next (*adjacent!*) configuration. Numbers in circles denote subsequent steps of this process, while the constraints matching appropriate diagram configurations, together with the inference conducted at each step, are listed to the right of the diagram (Fig. 9b). More detailed (and formalized) account of the diagrammatic reasoning process involved, as it may be conducted by a computer, can be found in [26] (see also [24]).

The above solution process, based on the diagrammatic representation and reasoning, has some characteristic features that are worth to enumerate here. First, it is inherently *local*. At every step, we inspected only a few adjacent diagram elements and matched their local configuration against the set of basic configurations. Second, it is directly guided and controlled by the structure of the

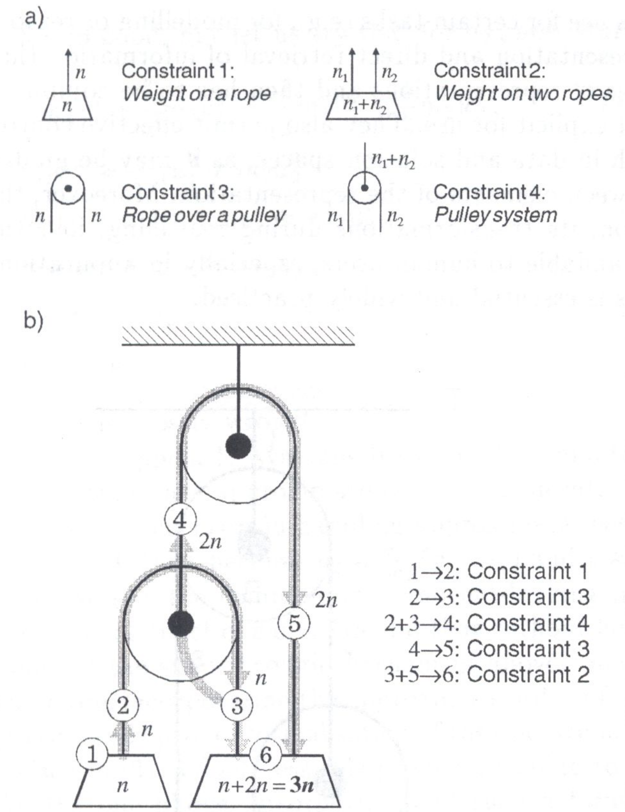

Fig. 9. The diagrammatic laws of the "pulleys world" (a) and the diagrammatic reasoning leading to the solution of the example (b)

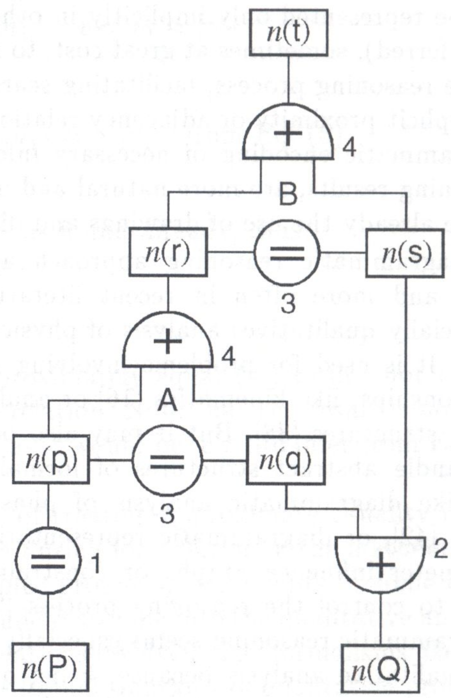

Fig. 10. A constraint-satisfaction graph of the simple pulley system (numbers denote configuration types according to Fig. 9a)

diagram. A move to the next step of inference proceeded to the element of the diagram immediately *adjacent* to the current one. In both cases, no extensive search for the matching data or appropriate new "attention point" was necessary. Third, the names (textual labels) of ropes and pulleys are not needed, as is the case with explicit naming of relations between them, like "hangs from" or "pulley system". These features stand in marked contrast to those of propositional (e.g., logical) representations, see the discussion in Section 5 of the propositional calculus version of the problem.

The above problem can be also stated as a *constraint satisfaction* problem. That is, the constraints imposed on the values of parameters by the structure of the system can be depicted using a *constraint graph* (Fig. 10), where two kinds of nodes represent parameter values and constraints imposed on them [21, 27]. Such a graph is a diagrammatic representation of a set of simultaneous equations describing the problem. The graph may then be used as a tool for solving the set of equations by some diagrammatic method(s). One of such methods, the so-called *local propagation of known states*, directly parallels the method used above for solving the pulleys problem. In this way, the constraint satisfaction formulation constitutes one of possible methods for "diagrammatisation" of problems originally not stated in a diagrammatic form. It should be noted here that the constraint satisfaction mechanism, using constraint graphs, is also used as an essential ingredient of the QSIM system introduced in Section 3.1, see [21].

What are the possible advantages of using diagrammatic representation and reasoning, as compared to traditional, propositional approaches? Larkin and Simon's paper [26] was probably the first one addressing this issue in a systematic manner, from the point of view of cognitive science. The authors compared diagrammatic and propositional formulations of two example problems — one similar to that discussed above (Fig. 8), and another one from geometry — and calculated the number of search steps needed to solve the two problems using both representations. Their main conclusion has been that the diagram, as an analogical representation of the structure of the problem, organises the problem description in a way corresponding to the problem's internal structure that should be followed in order to find the path to the solution. Taking also into account further investigations in this direction (e.g., [19], see also [24]), one may summarise the sources of these advantages as follows:

- *Diagrams are* (at least) *two-dimensional*: richer possibilities of grouping in two dimensions (by connectivity, adjacency, proximity, etc.), compared to those available in a one-dimensional (linear) strings of symbols in propositional formulations, lead to the reduction of:
  - *the size of problem search space*, i.e. the amount of data to be considered at the given inference step, and
  - *the search costs*, due to direct access to related elements eliminating the need of search for, say, matching symbolic labels.
- *Diagrams represent analogically*: construction of an analogical representation (e.g. a diagram) for the given set of facts usually causes the emergence of certain new entities, properties of the problem elements, and relations between them that follow from the given facts. These so-called *"implicit facts"* or *"emergent properties"* are a kind of ready-made inferences that can be directly read from the diagram at little or no cost.
- *Visual processing is easy*: humans possess a well developed apparatus for making *easy perceptual inferences* on a diagram. However, this does not yet apply in full to computers which are still somewhat better at brute-force number crunching, rather than at visual reasoning.

The "emergent properties" phenomenon is an interesting feature, specific for various kinds of analogical representations, and is the major source of both their power and their weakness. First, construction of a proper diagrammatic representation for some set of facts is often not too easy. The system capable to design good diagrams must, to some extent, conduct explicitly the reasoning leading to insertion into the diagram of the elements that are responsible for further emergence of the "implicit facts". Thus, these systems must often employ sophisticated, knowledge-based techniques to produce proper designs [28, 35]. Second, there is a danger of introducing, during the process, of such "implicit facts" that do not follow from the set of initial facts. These so-called *false implicatures* [30] may be true only for this particular version of the diagram, but false in general [29], see also [19, 24]. E.g., if asked to construct a diagram containing a right triangle one draws an isosceles right triangle, the inference process further on could falsely assume that the equality of the sides of this triangle is the constituent part of the problem conditions, and could use it to draw some false conclusion. But when one draws a non-isosceles right triangle instead, the inference process might draw conclusions that are invalid for isosceles triangles... This particular problem is sometimes considered as the impossibility to express "don't care" conditions in diagrams. That is not true, however — it is only the question of using proper visual language to express such situations where necessary.

Another problem with computerised diagrammatic reasoning follows from the fact that computers are not very good at handling information in visual form. Thus, implementing diagrammatic representation and reasoning seems to require, at the first sight, advanced image input and processing techniques. Fortunately, in most cases that is not necessary — current advances in graphical interfaces and graphical modeling provide means for successful solution of these problems. Diagrams are usually implemented as symbolic graph structures or geometric descriptions (e.g., in [2, 14, 19, 38]), though raster-graphics implementations are also sometimes used (see [24] for more details). With these advances, one may also expect the return (in part through a side-door of diagrammatic reasoning), of the long-neglected graphical solution methods, pushed for some time aside by an exaggerated interest in numerical methods, so seemingly well supported by computers.

5. Knowledge-based (expert) systems

Last but not least, the techniques of *expert systems*, using knowledge-based approach and logical, rule-based reasoning, are also an important ingredient of the computer systems for analysis of physical systems. Their important advantage consists in that they permit *declarative* formulation of knowledge, which is comparatively easy to formulate and to modify, especially in comparison to *procedural* (algorithmic) formulation necessary to construct a computer program solving a given problem. However, in their classic form, they epitomise the so-called *shallow knowledge* approach, characterised by representation of external, phenomenological associations between elements of the system, not taking into account the underlying laws (or "first principles") and internal structural relations governing the interaction of the system elements [3, 33]. Unfortunately, this feature constraints their use to narrow, specialised applications where as yet no sufficiently clear or formalised models of the underlying mechanisms are known. An attempt to apply them directly to handle complex mathematical models (quantitative or qualitative) leads either to an unmanageable combinatorial explosion of the number of rules, or to a necessity of augmenting their essentially declarative knowledge representation by a so-called *procedural knowledge* — that is, in effect, by calls to external procedures handling the underlying model-based calculations algorithmically.

Fortunately, recent developments suggest that these limitations of expert systems can be overcome. Such proposals as using multiple levels of rules [3], or restating rule-based inference as a constraint-satisfaction problem [27], permit also declarative representation of the *deep knowledge*, i.e., the functional model of the system, comprising its internal structure and relations governing its behaviour.

The simple pulley system example (see Fig. 8 in Section 4) will serve us again, this time as an aid to introduce basic expert-system techniques, namely the rule-based inference using predicate calculus as a knowledge representation mechanism. Our example can be stated in predicate calculus

FACTS: structural description and parameters

C(T),	[ceiling]
W(P), W(Q),	[weights]
R(p), R(q), R(r), R(s), R(t),	[ropes]
P(A), P(B),	[pulleys]
S(p, A, q), S(r, B, s),	[pulley systems]
H(P, p), H(Q, q), H(Q, s), H(A, r), H(B, t), H(t, T),	[hanging]
F(P, 1)	[force]

GOAL:

F(Q, ?n)

RULES:

Rule 1: **if** $W(w_1) \wedge R(r_1) \wedge H(w_1, r_1) \wedge \neg H(w_1, r_2) \wedge F(w_1, n_1)$
then $F(r_1, n_1)$

Rule 2: **if** $W(w_1) \wedge R(r_1) \wedge R(r_2) \wedge F(r_1, n_1) \wedge F(r_2, n_2) \wedge$
$H(w_1, r_1) \wedge H(w_1, r_2) \wedge \neg H(w_1, r_3)$
then $F(w_1, n_1 + n_2)$

Rule 3: **if** $P(p_1) \wedge R(r_1) \wedge R(r_2) \wedge S(r_1, p_1, r_2) \wedge F(r_1, n_1)$
then $F(r_2, n_1)$

Rule 4: **if** $P(p_1) \wedge R(r_1) \wedge R(r_2) \wedge R(r_3) \wedge S(r_1, p_1, r_2) \wedge$
$\{H(r_3, p_1) \vee H(p_1, r_3)\} \wedge F(r_1, n_1) \wedge F(r_2, n_2)$
then $F(r_3, n_1 + n_2)$

Fig. 11. A simple mechanical problem described with predicate calculus formulas

formulation in a manner shown in Fig. 11. First, the list of the entities involved, their properties and relations holding for them is given. It comprises the set of *(initial) facts*. Then, the solution *goal* is stated, i.e., a statement to be proven true with appropriately instantiated variable (denoted $?n$). The "physics" of the system is given by a set of *rules* for derivation of new facts from the existing (known) facts. The set of (initial) facts and rules together comprises the *knowledge base* of the problem. The rules correspond exactly to the same-numbered constraints listed in Fig. 9a.

Note that in this representation of the problem, in contrast to its diagrammatic representation (Figs. 8 and 9), one must make extensive search of *the whole set* of facts any time he/she tries to match a candidate rule with appropriate facts in the knowledge base, trying various possible variable instantiations on the way. E.g., in *Rule 3* there are $5 \cdot 5 \cdot 5 \cdot 2 = 250$ possible instantiations of the variables p_1, r_1, r_2 and r_3, with only two of them consistent with the whole store of facts. Practical problems that must be solved by expert systems involve usually thousands of facts and rules. To cope with the task, one must spend a large amount of computational effort or appropriately organise the knowledge base and reasoning procedure. Otherwise, even the simple problem domain may easily become intractable, especially when it must represent a more substantial amount of the "deep knowledge" about the underlying functional model. Moreover, this representation is hard to comprehend and operate with by human users of the system. Note that it would be very hard, if not impossible, to guess what kind of system is described by the set of formulas in Fig. 11, especially without the comments provided here (at the right side of the list of facts) for the convenience of human readers.

Use of an expert system approach to problems of the shallow-versus-deep-knowledge types may be illustrated by another example — analysis of a simple truss structure. The statically determinate 2-D truss structure is shown in Fig. 12a (bars 1–3 and 2–4, as well as 4–6 and 3–7, cross each other without touching). The example nicely illustrates some basic rules of (qualitative) analysis of truss structures, and thus is of a type commonly used as a textbook exercise or student examination problem. The problem is stated as follows:

For the bar configuration, boundary conditions and external load vector F given in Fig. 12a, determine which of the internal axial forces in the bars are compressive and which tensile.

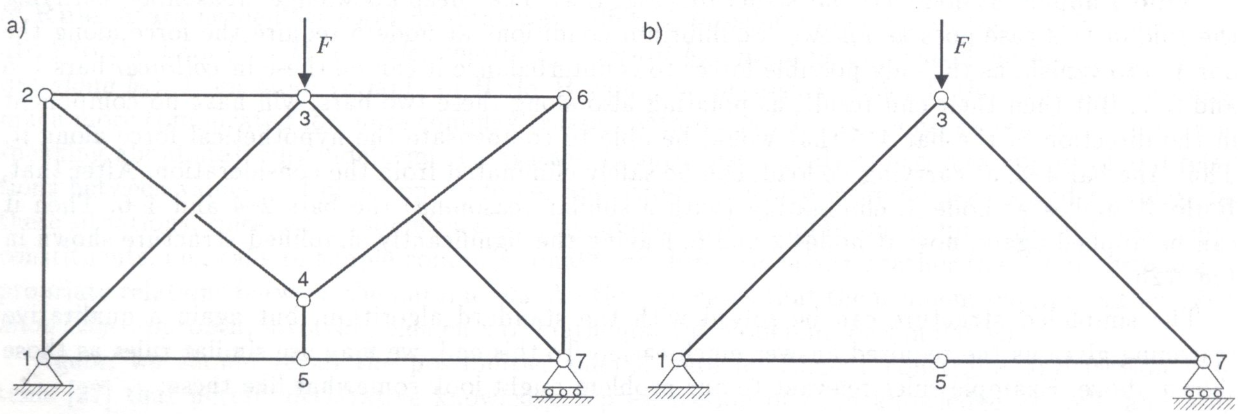

Fig. 12. A simple truss structure under load (a) and its equivalent simplified version (b)

Solving the problem with the standard algorithm, as given in every textbook on elementary structural mechanics, would require the complete numerical analysis based on establishing equilibrium conditions at each node of the structure (two equations for each node). The method would require setting up and solving a system of N linear algebraic equations with N unknowns (nodal displacements), N being equal to the number of nodes doubled minus the number of reactions at supports: in our case $N = 2 \cdot 7 - (2 + 1) = 11$ (note that there are also 11 bars in the structure, thus the structure is statically determinate). The numerical analysis might therefore be pretty cumbersome, requiring solution of 11 simultaneous algebraic equations for so simple a structure.

However, the problem can be considerably simplified through the use of some general rules concerning force distribution in truss structures at equilibrium. Specifically, they permit detection of inactive elements that may be omitted in further analysis of the structure. Two example rules of this sort may be formulated, in a quasi-natural language form fashionable in some expert system shells, as follows:

Rule 1: *Elimination of noncollinear bar*

if number of bars at node is 3
  and support is none
  and external load is none
  and bars (b_1, b_2) are collinear
  and bars (b_1, b_3) are not collinear
then eliminate bar b_3.

Rule 2: *Elimination of two bars at angle*

if number of bars at node is 2
  and support is none
  and external load is none
  and bars (b_1, b_2) are not collinear
then eliminate bars (b_1, b_2)
  and eliminate node.

Symbols b_1, b_2, b_3 denote variables that should be valuated by the appropriate objects (here, bars) such that the conditions of a rule are satisfied.

Rule 1 applies at node 5 of our structure (Fig. 12a). The "deep knowledge" reasoning justifying the rule in this case goes as follows. Equilibrium conditions at node 5 require the force along the bar 4–5 to vanish, as the only possible forces to counterbalance it can be these in *collinear* bars 1–5 and 5–7. But then their end result, as pointing also along these two bars, will have no component in the direction of the bar 4–5 that would be able to compensate the hypothetical force along it. Thus, the bar 4–5, as carrying no load, can be safely eliminated from the consideration. After that, **Rule 2** applies at node 4, eliminating (with a similar reasoning) the bars 2–4 and 4–6. Then it can be applied again, now at nodes 2 and 6, leaving the significantly simplified structure shown in Fig. 12b.

The simplified structure can be solved with the standard algorithm, but again a qualitative reasoning gives us the required answer much easier. To this end, we may use similar rules as those shown above. Example rules relevant to our problem might look somewhat like these:

Rule A: *Load at two-bar node*

if number of bars at node is 2
  and support is none
  and bars (b_1, b_2) are not collinear
  and external load is not none
  and direction of load is inside (b_1, b_2)
then bar b_1 is compressed
  and bar b_2 is compressed.

Rule B: *Equilibrium at two-bar node with moveable support*

if number of bars at node is 2
  and external load is none
  and support is moveable
  and angle of bars (b_1, b_2) is acute
  and direction of support is not inside (b_1, b_2)
  and bar b_1 is compressed
then bar b_2 is stretched.

Rule C: *Equilibrium of two collinear bars*

if number of bars at node is 2
  and support is none
  and external load is none
  and bars (b_1, b_2) are collinear
  and bar b_1 is stretched
then bar b_2 is stretched.

The phrase "direction of support" in **Rule B** means direction of the reaction force at the support (in this case, the normal to the support motion plane).

Using the above rules we may easily conclude that bars 1–3 and 3–7 are compressed (by **Rule A** at node 3) and bars 1–5 and 5–7 are stretched (by **Rule B** at node 7 and then **Rule C** at node 5).

So, it might look like the application of rule-based reasoning of this type is well suited for the qualitative analysis of truss structures. Note, however, that whereas the simplification rules (**1–2**) look rather simple and local, indicating that the structural classification of truss configuration types may be well suited for the shallow-knowledge approach, with the analysis rules (**A–C**) the situation is different. First, they are more complicated, testing many parameters, both geometrical and physical (like load distribution). As a consequence, a great number of variants of these rules, explicitly enumerating different combinations of parameter values is necessary. E.g., eight variants of **Rule A** are needed for eight qualitatively different possible directions of the external load relative to the bars (*inside, outside, toward* b_1, *toward* b_2, *in-along* b_1, *out-along* b_1, *in-along* b_2, *out-along* b_2). Even more variants of **Rule B** would be necessary. Second, the situation becomes much more complicated for more complex trusses, with many bars at nodes, several loads, etc., not speaking about statically indeterminate trusses. In such cases, analysis becomes non-local and relations between values and directions of forces and stresses become more numerous and complex. In these situations, rules can be sometimes greatly simplified with addition of "procedural knowledge" constituents, i.e., calls to simple computational procedures *calculating* rather that *enumerating* appropriate relations between the parameters. All this indicates that the problem requires rather *deep knowledge* approach, involving elements of mathematical modelling or simulation.

Again, we should recall the possibilities offered here by current constraint satisfaction systems [27] that permit declarative knowledge representation of deep knowledge as well, and also integration of both logical rules and numerical equations.

Nevertheless, the "shallow knowledge" form of an expert system can be very useful as an "intelligent encyclopaedia" for selection of proper analysis methods for a given problem [1], and as an "intelligent controller" for integration of interaction of various methods and approaches during co-operative solving of the problem [25].

6. Hybrid reasoning systems

The general conclusion from the preceding sections is that any single approach is not sufficient to cover all important cases and problems of analysis of physical systems. Especially, the promising and popular qualitative simulation technique is not as universal as it might seem at the first sight. To make it practical, one must augment it with other methods of reasoning about qualitative magnitudes, and combine it with other kinds of non-qualitative reasoning, including our old companion, quantitative (numerical) analysis, as well.

Concerning the first avenue, it seems that at least the possibilities of both exact comparison of (some) quantities and estimation of their orders of magnitude should be added to the toolbox of qualitative simulation (see [34, 41]). Especially, the formalisation of the *order-of-magnitude reasoning* seems indispensable — it is probably the most common mode of qualitative reasoning employed by humans. Other techniques, e.g. *fuzzy sets*, may also prove useful (e.g., [4]).

Many problems of analysis of physical systems, especially these involving structural and spatial relations, may benefit from application of recently developing techniques of *diagrammatic representation and reasoning.* They permit explicit representation and direct retrieval of information that can be represented only implicitly in other types of representations. Moreover, the diagrammatic encoding is more natural and understandable to human users, especially in applications where already the use of drawings and diagrams is essential and widely practised. A multitude of graphical solution methods had been developed in these fields, but then became neglected or forgotten, pushed aside by an exaggerated interest in numerical methods, so seemingly well supported by computers. They can be now revived and put on a new footing within the paradigm of diagrammatic reasoning.

Other problems may benefit from the *constraint satisfaction* formulation, also because it constitutes one of possible methods for "diagrammatisation" of problems originally not stated in a diagrammatic form. It should be noted here that the constraint satisfaction mechanism is also used as an essential ingredient of the Kuipers' qualitative simulation system QSIM introduced in Section 3.1, see [21, 22].

Expert systems, based on declarative knowledge approach and rule-based reasoning, are very useful for the "shallow knowledge" situations, like selection of proper analysis methods for a given problem [1], or integration of interaction of various methods and approaches during co-operative solving of the problem [25]. However, they are much less suited for problems requiring extensive use of "deep knowledge", i.e., the functional model of the system, comprising its internal structure and relations governing its behaviour.

Again, some of the limitations of expert systems can be compensated for by possibilities offered by current *constraint satisfaction* systems [27] that permit declarative knowledge representation of deep knowledge and integration of both logical rules and numerical equations, together with *diagrammatic reasoning* capable of handling structural and spatial models of systems.

These considerations lead us to the conclusion that the future of computer-assisted analysis of physical systems belongs to *hybrid systems*, combining quantitative and qualitative analysis, diagrams, logic and constraints into an integrated, versatile analysis tool. How to integrate such seemingly disparate paradigms into a single, united whole seems to be an interesting challenge for interdisciplinary research involving computer science, physics, and mathematics. Moreover, it seems that full automation of the analysis and design process is not possible in the near future, and in fact is neither necessary nor practical. The best results may be achieved with man-machine teams, combining advantages of both components — artificial and human. Thus, cognitive scientists may also find some challenging problems in this area of activity. With the help of effective man-machine interface, based on both textual and graphical languages adapted to the application domain and human communication characteristics, the *hybrid reasoning systems* will be able to co-operate seamlessly with the engineer or scientist in the task of analysis or design of complex physical systems [18, 25].

7. Conclusions

The paper addressed the question of finding appropriate general paradigm for effective, computer-assisted analysis of physical systems. It started from a critical assessment of classic numerical (quantitative) techniques. Certain problems with and limitations of computer implementations of these techniques were demonstrated, especially difficulty and cost of their use as well as their inability to cope with incomplete or imprecise knowledge about the system parameters.

The promising new developments in the field of the so-called qualitative physics were considered next. The basic principles and capabilities of qualitative analysis, especially the most well-known qualitative simulation technique, were shown. Again, certain limitations of these techniques, especially a tendency for over-abstraction (i.e., producing answers too imprecise or too general) and producing spurious, physically impossible system behaviours, were pointed out. Other important technique of qualitative analysis, namely order-of-magnitude reasoning, was also briefly introduced.

A recently developing technique of diagrammatic reasoning was discussed next. Diagrams are a kind of *analogical* (or *direct*) knowledge representation mechanism that is characterised by a parallel (though not necessarily isomorphic) correspondence between the structure of the representation and the structure of the represented.

The important role of logical reasoning techniques (as used in expert systems) in the computer-assisted analysis of physical systems was also pointed out. They seem indispensable as "intelligent controllers", selecting proper analysis methods and integrating their interaction during problem solving.

The importance of constraint satisfaction formulation of many problems was also indicated in various contexts discussed in the paper, namely qualitative simulation, diagrammatic reasoning, and declarative representation of the so-called "deep knowledge" in expert systems.

In conclusion, the idea of *hybrid reasoning systems* was proposed. They should combine quantitative and qualitative analysis, diagrammatic reasoning, expert-system-like logical reasoning and constraint satisfaction systems into integrated, versatile analysis tools. They also need an effective man-machine interface, based on both textual and graphical languages adapted to the application domain and human communication characteristics. Thus, their construction needs interdisciplinary research involving computer science, physics, mathematics, and cognitive science.

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