Computer Assisted Mechanics and Engineering Sciences, 3: 317–326, 1996.

Space-Time Generalization of *R*-Function Method (*ST-RFM*)

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(Received November 13, 1996)

A generalized, space-time version of the *R*-function method has been presented. The general structure of the solution for the space-time problem and the algorithm for the determination of the unknown parameters of the structure have been given. The considerations are illustrated by two numerical examples: the first one concerns the cooling of a square plate, while in the second one more complex shape of domain is considered. The numerical solution of the first problem is compared with the solution obtained on the basis of FDM.

1. Introduction

The *R*-function method (*RFM*) is an analytical-numerical method, well described in the monographs of Rvachev [6, 7] and others. Its application in the solutions of the problems of heat transfer has been discussed in e.g., [2, 3, 5, 8, 9]. The general structure of the solution (*GSS*) derived in the analytical part, is a class of functions satisfying (accurately) all the boundary conditions on the assigned parts of the boundary. The shape of the boundary (arbitrarily complex) is also taken into account in the computations. An additional advantage of *RFM* is the possibility of formulating the problem in the rectangular Cartesian system of coordinates, irrespective of the shape of the domain studied.

In the recent studies, for the problems of non-stationary heat transfer, the Laplace transformation (see [6, 7]) has been used, or the time derivative has been replaced by a suitable differential quotient (see [3, 5, 8]). In the present paper, the application of the *R*-function for the description of the space-time domain has been suggested. The derived general structure of the solution takes into consideration, apart from the boundary conditions, also the initial condition. The unknown parameters have been determined on the basis of Galerkin's method.

2. Problem statement

Let the considered object occupy in space the domain $\Omega \subset \mathbb{R}^3$ (or $\mathbb{R}^2$ or $\mathbb{R}^1$), and let its boundary $\partial\Omega$ be composed of n disjoint parts $\partial\Omega_i$ $(i = 1, 2, ..., n)$, on which various boundary conditions — of the 1st, 2nd or 3rd type — will be assigned. We assume the thermophysical parameters λ, c, ρ to be constant (a linear problem). The non-stationary temperature field $T(\mathbf{x}, t)$ in the domain $\Omega(\mathbf{x})$ is described by the Fourier equation in the form

$$\frac{\partial T(\mathbf{x},t)}{\partial t} = \frac{\lambda}{c\rho} \sum_{i=1}^{3} \frac{\partial^2 T(\mathbf{x},t)}{\partial x_i^2}, \qquad \mathbf{x} = (x_1, x_2, x_3) \in \Omega, \quad t \in (0, \infty), \tag{1}$$

to which suitable boundary and initial conditions must be added. Over $k < n$ parts of the boundary $\partial\Omega_i$, the temperature is known

$$T(\mathbf{x},t)\big|_{\partial\Omega_i} = T_{0i} = f_i\,, \quad i = 1,\ldots,k, \quad (\mathbf{x},t) \in \partial\Omega_i \otimes (0,\infty)\,. \tag{2}$$

Over the remaining parts $\partial\Omega_j (k < j \le n)$ the 3rd type of boundary conditions are prescribed

$$\left(\frac{\partial T}{\partial \nu} + h_j T\right)\bigg|_{\partial\Omega_j} = f_j\,, \quad j = k+1,\ldots,n, \quad (\mathbf{x},t) \in \partial\Omega_j \otimes (0,\infty)\,, \tag{3}$$

where: $f_j = \alpha_j T_j^{\infty}/\lambda$, $h_j = \alpha_j/\lambda$; α_j, T_j^{∞} are the heat transfer coefficient and the ambient temperature. If it is assumed that $h_j = 0$ and $f_j = q_j/\lambda$ (q_j — heat flux), then the conditions of the 2nd type are obtained. In formula (3) the operator $\partial(\,.\,)/\partial\nu$ denotes the derivative in the direction normal to the boundary of the domain Ω. The right-hand sides in the above conditions f_i, $i = 1, 2, \ldots, n$ — can, of course, be the functions of time.

Finally, we assume that the function describing the temperature at the initial moment is known, i.e.,

$$T(\mathbf{x},t)\big|_{t=0} = T_0(\mathbf{x}) = f_0\,, \quad \mathbf{x} \in \Omega\,. \tag{4}$$

The Cartesian product $\Omega^* = \Omega \otimes (0,\theta)$ can be treated as a finite cylinder Ω^*; its base is the geometrical domain Ω, and its height — the duration θ of the analyzed process. The boundary of the space-time domain Ω^* is composed of the following parts (see Fig. 1):

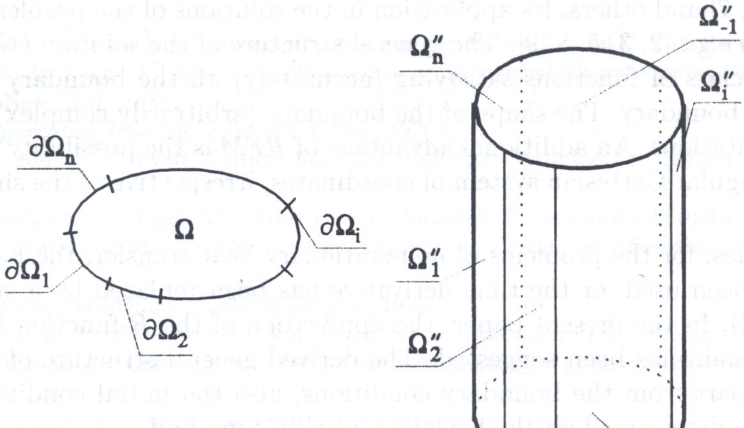

Fig. 1. Domain Ω and space-time domain Ω^*

a) two bases being the geometrical domains at the moments $t = 0$ and $t = \theta$, respectively, i.e. $\Omega''_0 = \Omega$ and $\Omega''_{-1} = \Omega'$;

b) lateral surfaces composed of n parts which are Cartesian products $\Omega'' = \partial\Omega \otimes (0,\theta) = \Omega''_1 \cup \ldots \cup \Omega''_n$, while $\Omega''_i = \partial\Omega_i \otimes (0,\theta)$ — $i = 1, 2, \ldots, n$.

Applying Rvachev's operators

$$x_1 \bigwedge\nolimits_0 x_2 = x_1 + x_2 - \sqrt{x_1{}^2 + x_2{}^2}\,,$$
$$x_1 \bigvee\nolimits_0 x_2 = x_1 + x_2 + \sqrt{x_1{}^2 + x_2{}^2}\,, \tag{5}$$
$$\bar{x} = -x$$

the analytical form of the space-time domain Ω^* defined above may be described as

$$\omega^* = \omega^*(\mathbf{x}, t) = [\omega(\mathbf{x}, t)] \bigwedge\nolimits_0 [(\theta - t)t/\theta]\,. \tag{6}$$

The function ω^* has the following properties: for the points not belonging to the domain Ω^* it is negative, for the boundary points — equal to zero, and finally, for the points belonging to the interior of Ω^* — positive,

$$\omega^*(\mathbf{x}, t) \begin{cases} < 0\,, & (\mathbf{x}, t) \notin \Omega^*\,, \\ = 0\,, & (\mathbf{x}, t) \in \partial\Omega^*\,, \\ > 0\,, & (\mathbf{x}, t) \in \Omega^*\,. \end{cases} \tag{7}$$

Now the boundary conditions (2)–(3) and initial condition (4) will be presented as follows:

$$T_{|\Omega''_i} = f_i\,, \quad i = 0, \ldots, k, \tag{8}$$

$$\left(\frac{\partial T}{\partial \nu} + h_j T\right)_{|\Omega''_j} = f_j\,, \quad j = k+1, \ldots, n\,. \tag{9}$$

In Eq. (9), the operator $\partial(\,.\,)/\partial\nu$ is a derivative in the direction normal to the boundary of the space-time domain Ω^*. Since these conditions are given on the lateral surface of the cylinder, then

$$\frac{\partial(\,.\,)}{\partial \nu}_{|\Omega''_j} = \frac{\partial(\,.\,)}{\partial \nu}_{|\partial\Omega_j}\,, \quad j = 1, \ldots, n\,. \tag{10}$$

The initial condition (4) is now a condition of the 1st type assigned to the boundary Ω''_0, or it is included in formula (8) for $i = 0$.

A properly stated problem requires the definition of the boundary condition on the boundary $\Omega^*(\mathbf{x}, \theta) = \Omega''_{-1}$. This condition may be the solution of a steady-state problem with an assumption that time θ is large (see Crank [1]). Another, less accurate condition is the assumption that in the final stage of the analyzed process, the rate of the changes in the temperature field $\partial T/\partial t$, or

$$\frac{\partial T}{\partial t}_{|\Omega''_{-1}} = w(\mathbf{x})\,, \tag{11}$$

is small and, in particular, equal to zero. The solution of the problem with assumption (11) can be treated as the first step of an iterative algorithm in which the function $w = w(\mathbf{x})$ is corrected.

The general structure of the solution of a space-time problem, determined in the domain Ω^*, is of the form

$$T(\mathbf{x}, t) = F_0 - \omega \mathrm{D}_1(F_0) + \omega F_1 + \Phi F_2 - \omega \mathrm{D}_1(\Phi F_2) - \omega \Phi H_1\,, \tag{12}$$

where

$$F_0 = \frac{\sum\limits_{i=0}^{k} \dfrac{f_i}{\omega_i}}{\sum\limits_{i=0}^{n} \dfrac{1}{\omega_i}}\,, \quad F_1 = \frac{\sum\limits_{j=k+1}^{n} \dfrac{f_j}{\omega_j}}{\sum\limits_{j=k+1}^{n} \dfrac{1}{\omega_j}}\,, \quad F_2 = \frac{\sum\limits_{j=k+1}^{n} \dfrac{1}{\omega_j}}{\sum\limits_{j=0}^{n} \dfrac{1}{\omega_j}}\,, \quad H_1 = \frac{\sum\limits_{j=k+1}^{n} \dfrac{h_j}{\omega_j}}{\sum\limits_{j=k+1}^{n} \dfrac{1}{\omega_j}}\,. \tag{13}$$

In Eq. (12) D_1 is the differential operator of the form

$$D_1(\,.\,) = \sum_{j=1}^{3} \frac{\partial\omega}{\partial x_i}\frac{\partial(\,.\,)}{\partial x_i} + \frac{\partial\omega}{\partial t}\frac{\partial(\,.\,)}{\partial t} \tag{14}$$

satisfying the following conditions

$$D_1(\omega)\big|_{\partial\Omega^*} = 1\,,$$
$$D_1(\,.\,)\big|_{\Omega''_j} = \sum_{j=1}^{3} \frac{\partial\omega}{\partial x_i}\frac{\partial(\,.\,)}{\partial x_i} = \frac{\partial(\,.\,)}{\partial\nu}\,, \quad j = 1,\ldots,n\,. \tag{15}$$

It means that the boundary of area Ω^* is normalized, and this operator on lateral surface of the cylinder passes into a derivative normal to the boundary. At the same time, function $\Phi = \Phi(\mathbf{x},t)$ occurring in *GSS* can have the form

$$\Phi(\mathbf{x},t) = \sum a_{ij}\varphi_{ij}(\mathbf{x},t)\,, \tag{16}$$

where a_{ij} are the unknown parameters, and φ_{ij} are the given base functions (see [3, 5, 8, 9]).

We shall now demonstrate that *GSS* determined by Eq. (12) actually meets all the conditions (8)–(9).

Since $\omega = 0$ for $(\mathbf{x},t) \in \partial\Omega^*$, then only the first and third components of the structure are different from zero on the boundary, or

$$T(\mathbf{x},t)\big|_{\partial\Omega^*} = F_1 + \Phi F_2\,. \tag{17}$$

Hence we obtain the following relationships

$$T(\mathbf{x},t)\big|_{\Omega''_i} = f_i\,, \quad i = 0,1,\ldots,k\,,$$
$$T(\mathbf{x},t)\big|_{\Omega''_j} = \Phi\,, \quad j = k+1,\ldots,n\,. \tag{18}$$

The operator $D_1(\,.\,)$ applied to function (12) on the boundary of the domain Ω^*, satisfies the following conditions

$$D_1(t)\big|_{\partial\Omega^*} = F_1 - \Phi H_1\,, \tag{19}$$

or

$$D_1(t)\big|_{\Omega''_j} = f_j - \Phi h_j\,, \quad j = k+1,\ldots,n \tag{20}$$

and this shows that all the boundary conditions are satisfied.

Taking into consideration Eq. (16), *GSS* can be presented as

$$T(x,y) = X_0 + \sum a_{kl}X_{kl}\,, \tag{21}$$

where we have introduced

$$X_0 = F_0 - \omega(\omega_{,\alpha}F_{0,\alpha} + F_1)\,,$$
$$X_{kl} = \Phi_{kl}\left[F_2 - \omega(H_1 + \omega_{,\alpha}F_{2,\alpha})\right] - \Phi_{kl,\alpha}(\omega_{,\alpha}\omega F_2)\,. \tag{22}$$

In the latter equation, Einstein's convention of repeated indices α and a simplified notation of differentiation have been introduced.

It is easy to check that

$$X_0 = f_i \;\wedge\; X_{kl} = 0\,, \quad (\mathbf{x},t) \in \Omega''_i \quad i = 0,1,\ldots,k\,,$$
$$X_0 = 0 \;\wedge\; X_{kl} \neq 0\,, \quad (\mathbf{x},t) \in \Omega''_i \quad j = k+1,\ldots,n\,. \tag{23}$$

3. Solution of the problem

To solve the problem described by Eqs. (1)–(3), the unknown coefficients a_{ij} should be determined. This can be done by demanding that *GSS* — (12) — should fulfill Eq. (1), or using Galerkin's method

$$\int_{\Omega^*}\left[\frac{\partial T}{\partial t}-\frac{\lambda}{c\rho}\sum_{i=1}^{3}\frac{\partial^2 T}{\partial x_i^2}\right]\delta T\,\mathrm{d}\Omega^*=0\,. \tag{24}$$

Variation δT must satisfy the following conditions

$$\begin{aligned}
&\delta T=0 \wedge \delta T_{,\nu}\neq 0 && \text{for } (\mathbf{x},t)\in\bigcup_{i=1}^{k}\Omega_i'' \,,\\
&\delta T\neq 0 && \text{for } (\mathbf{x},t)\in\Omega^*\cup\left(\bigcup_{j=k+1}^{n}\Omega_j''\right),\\
&\delta T_{,\nu}\neq 0 && \text{for } (\mathbf{x},t)\in\bigcup_{j=k+1}^{n}\Omega_j''\,.
\end{aligned} \tag{25}$$

Since function T is determined by Eq. (21), then

$$\delta T=\sum_{ij}X_{ij}\delta a_{ij}\,,\qquad \delta T_{,\alpha}=\sum_{ij}X_{ij,\alpha}\delta a_{ij}\,, \tag{26}$$

and due to the properties (23), the conditions (25) are fulfilled.

Now we shall derive the algebraic system of equations of the problem resulting from integral (24). The integral over the domain Ω^* can be treated as a superposition of an integral over the geometrical domain Ω and time t in the interval $(0,\theta)$, or

$$\int_{\Omega^*}\left[\frac{\partial T}{\partial t}-\frac{\lambda}{c\rho}\sum_{i=1}^{3}\frac{\partial^2 T}{\partial x_i^2}\right]\delta T\,\mathrm{d}\Omega^*=\int_{\Omega^*}\left(T_{,t}-\kappa^2T_{,\alpha\alpha}\right)\delta T\,\mathrm{d}\Omega^*=$$
$$\int_{\Omega^*}T_{,t}\delta T\,\mathrm{d}\Omega^*-\kappa^2\int_0^{\theta}\left(\int_{\Omega^*}T_{,\alpha\alpha}\delta T\,\mathrm{d}\Omega\right)\mathrm{d}T\,. \tag{27}$$

Making use of Green's theorem about the transformation of a surface integral into a curvilinear one, the second of the integrals in Eq. (27) will take the form

$$-\int_{\Omega^*}T_{,\alpha}\delta T_{,\alpha}\,\mathrm{d}\Omega^*+\int_{\Omega^*}T_{,\nu}\,\delta T\,\mathrm{d}\partial\Omega^*=-\int_{\Omega^*}T_{,\alpha}\delta T_{,\alpha}\,\mathrm{d}\Omega^*+\sum_{l=k+1}^{n}\int_{\Omega_j''}(f_j-h_jT)\,\delta T\,\mathrm{d}\Omega_j''\,. \tag{28}$$

In the last transformation it was taken into account that variation of the function T is equal to zero on those parts of the boundary at which temperature is assigned, or for $i=0,2,...,k$. Since the variations δa_{ij} $(i,j=1,2,...,N)$ are arbitrary and independent, then assuming successively that only one of them is different from zero, we shall obtain a system of equations of the form

$$\delta a_{ij}\left\{\int_{\Omega^*}\left(T_{,\alpha}X_{ij}-\kappa^2T_{,\alpha}X_{ij,\alpha}\right)\mathrm{d}\Omega^*-\kappa^2\sum_{l=k+1}^{n}\int_{\Omega_l''}(f_l-h_lT)X_{ij}\,\mathrm{d}\Omega_l''\right\}=0\,. \tag{29}$$

Using the formula (21), we obtain an algebraic system of equations with the unknowns a_{ij}

$$\sum_{kl}C_{ijkl}a_{kl}=B_{ij}\,, \tag{30}$$

where

$$\begin{aligned}
C_{ijkl}&=\int_{\Omega^*}\left(X_{kl,t}X_{ij}-\kappa^2X_{kl,\alpha}X_{ij,\alpha}\right)\mathrm{d}\Omega^*+\kappa^2\sum_{l=k+1}^{n}\int_{\Omega_l''}(f_l-h_lX_{kl})X_{ij}\,\mathrm{d}\Omega_l''\,,\\
B_{ij}&=\int_{\Omega^*}\left(X_{0,t}X_{ij}-\kappa^2X_{,\alpha}X_{ij,\alpha}\right)\mathrm{d}\Omega^*+\kappa^2\sum_{l=k+1}\int_{\Omega_l''}(f_l-h_lX_0)X_{ij}\,\mathrm{d}\Omega_l''\,.
\end{aligned} \tag{31}$$

4. Numerical examples

4.1. Square plate

We present here a solution in which the curves of cooling were determined in selected points of a square plate of the dimensions 0.6 × 0.6 m with the following physical parameters: $\lambda = 30$ W/mK, $\rho = 7000$ kg/m^3, $c = 700$ J/kgK. The 3rd type of condition was assumed with the coefficient $\alpha = 100$ W/m^{2}K and ambient temperature $T^\infty = 30$ °C. The duration of the process analyzed is $\theta = 15000$ s. Figure 2 shows the differences between the reduced temperatures (curves of cooling in the selected points of the plate) obtained by the *RFM* i *FDM* methods, with the least accurate condition on the boundary Ω''_{-1}, namely $w = 0$. The maximum difference does not exceed 7%.

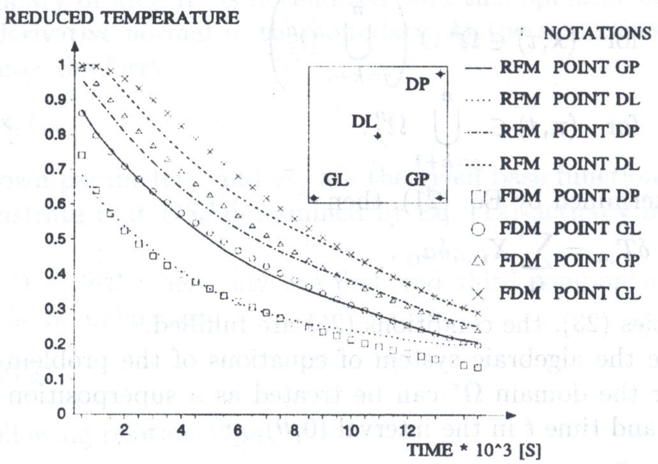

Fig. 2. Cooling curves for selected points

Tables 1–2 present the reduced temperature distributions at the beginning and at the end of the process.

Table 1. Distribution of reduced temperature at the moment $t = 13500$ s

a) *R*-Function Method

0.210	0.210	0.209	0.207	0.204	0.203	0.202	0.201	0.195	0.190
0.217	0.218	0.218	0.218	0.216	0.213	0.207	0.201	0.199	0.195
0.234	0.234	0.234	0.232	0.229	0.223	0.216	0.207	0.201	0.201
0.252	0.251	0.250	0.248	0.243	0.236	0.226	0.216	0.207	0.202
0.268	0.267	0.265	0.262	0.256	0.248	0.236	0.223	0.213	0.203
0.280	0.279	0.277	0.273	0.266	0.256	0.243	0.229	0.216	0.204
0.289	0.288	0.285	0.281	0.273	0.262	0.248	0.232	0.218	0.207
0.295	0.293	0.291	0.285	0.277	0.265	0.250	0.234	0.218	0.209
0.298	0.297	0.293	0.288	0.279	0.267	0.251	0.234	0.218	0.210
0.299	0.298	0.295	0.289	0.280	0.268	0.252	0.234	0.217	0.210

b) Finite Differences Method

0.220	0.218	0.215	0.210	0.204	0.196	0.187	0.176	0.164	0.151
0.239	0.238	0.234	0.229	0.222	0.213	0.203	0.192	0.178	0.164
0.257	0.255	0.251	0.246	0.238	0.229	0.218	0.206	0.192	0.176
0.273	0.271	0.267	0.261	0.253	0.243	0.231	0.218	0.203	0.187
0.286	0.284	0.280	0.274	0.265	0.255	0.243	0.229	0.213	0.196
0.298	0.296	0.291	0.285	0.276	0.265	0.253	0.238	0.222	0.204
0.307	0.305	0.300	0.294	0.285	0.274	0.261	0.246	0.229	0.210
0.314	0.312	0.307	0.300	0.291	0.280	0.267	0.251	0.234	0.215
0.319	0.317	0.312	0.305	0.296	0.284	0.271	0.255	0.238	0.218
0.321	0.319	0.314	0.307	0.298	0.286	0.273	0.257	0.239	0.220

Table 2. Distribution of reduced temperature at the moment $t = 1500$ s

a) R-Function Method									
0.811	0.806	0.798	0.785	0.768	0.743	0.713	0.679	0.643	0.600
0.862	0.857	0.851	0.838	0.821	0.800	0.770	0.736	0.696	0.643
0.906	0.902	0.896	0.883	0.866	0.845	0.815	0.779	0.736	0.679
0.943	0.938	0.932	0.919	0.902	0.881	0.851	0.815	0.770	0.713
0.972	0.968	0.962	0.949	0.932	0.909	0.881	0.845	0.800	0.743
0.996	0.991	0.985	0.972	0.955	0.932	0.902	0.866	0.821	0.768
1.000	1.000	1.000	0.989	0.972	0.949	0.919	0.883	0.838	0.785
1.005	1.003	1.001	1.000	0.985	0.962	0.932	0.896	0.851	0.798
1.007	1.030	1.003	1.000	0.991	0.968	0.938	0.902	0.857	0.806
1.010	1.007	1.005	1.000	0.996	0.972	0.943	0.906	0.862	0.811
b) Finite Differences Method									
0.751	0.749	0.744	0.737	0.725	0.709	0.687	0.657	0.619	0.573
0.811	0.809	0.804	0.796	0.784	0.767	0.742	0.710	0.669	0.619
0.861	0.859	0.854	0.845	0.832	0.813	0.788	0.753	0.710	0.657
0.900	0.898	0.892	0.883	0.870	0.850	0.823	0.788	0.742	0.687
0.929	0.927	0.922	0.912	0.898	0.878	0.850	0.813	0.767	0.709
0.951	0.948	0.943	0.933	0.919	0.898	0.870	0.832	0.784	0.725
0.966	0.963	0.957	0.948	0.933	0.912	0.883	0.845	0.796	0.737
0.975	0.973	0.967	0.957	0.943	0.922	0.892	0.854	0.804	0.744
0.981	0.978	0.973	0.963	0.948	0.927	0.898	0.859	0.809	0.749
0.984	0.981	0.975	0.966	0.951	0.929	0.900	0.861	0.811	0.751

4.2. Heated element

We shall determine the temperature distribution over the section shown in Fig. 3a. On account of symmetry we shall examine only a part of this domain contained in the first quadrant of the system of co-ordinates — Fig. 3b. This section is heated by a heat stream of value $q = 60000$ W/m^2 on

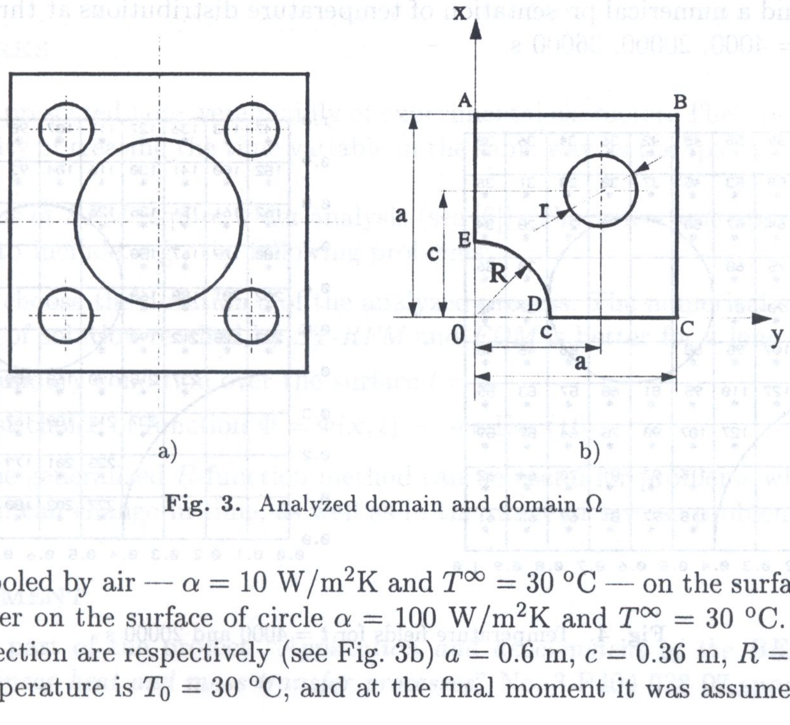

Fig. 3. Analyzed domain and domain Ω

the surface DE, cooled by air — $\alpha = 10$ W/m^{2}K and $T^{\infty} = 30$ °C — on the surfaces AB and BC, and cooled by water on the surface of circle $\alpha = 100$ W/m^{2}K and $T^{\infty} = 30$ °C. The geometrical dimension of the section are respectively (see Fig. 3b) $a = 0.6$ m, $c = 0.36$ m, $R = 0.24$ m, $r = 0.12$ m. The initial temperature is $T_0 = 30$ °C, and at the final moment it was assumed that $w = 0$.

The analyzed geometrical domain may be described as a common part of the following subregions:

$$\Omega_1 = \{(a-x)x/a > 0\}\,, \quad \Omega_2 = \{(a-y)y/a > 0\}\,,$$

$$\Omega_3 = \{(x^2+y^2-R^2)/(2R) > 0\}\,, \tag{32}$$

$$\Omega_4 = \{[(x-c)^2+(y-c)^2-r^2]/(2r) > 0\}\,,$$

or by the equation

$$\omega(x,y) = [(a-x)x/a] \bigwedge\nolimits_0 [(a-y)y/b] \bigwedge\nolimits_0$$

$$[(x^2+y^2-R^2)/(2R)] \bigwedge\nolimits_0 \left[\left((x-c)^2+(y-c)^2-r^2\right)/(2r)\right]. \tag{33}$$

The particular parts of the geometrical boundary on which suitable boundary conditions were assigned are $\partial\Omega_1 = \Gamma_{ABC}$, $\partial\Omega_2 = \Gamma_{CD} \cup \Gamma_{EA}$, $\partial\Omega_3 = \Gamma_{DE}$ and $\partial\Omega_4$ — circle of the radius r.

The space-time domain is created analogously to the one described earlier. Equations of parts of the boundary are of the form

$$\omega_i = \sqrt{(\omega^*)^2 \bigwedge\nolimits_0 f_i}\,, \tag{34}$$

where $i = -1, 0, 1, 2, 3, 4$, Δ is a very small constant (equal, e.g., 0.000000001), and

$$f_{-1} = \theta - \Delta - t\,, \quad f_0 = t + \Delta\,,$$

$$f_1 = [(a-\Delta-x)] \bigwedge\nolimits_0 [(a-\Delta-y)] \bigwedge\nolimits_0 [(\theta-t)t]\,,$$

$$f_2 = [(\Delta+x)] \bigwedge\nolimits_0 [(\Delta+y)] \bigwedge\nolimits_0 [(\theta-t)t]\,, \tag{35}$$

$$f_3 = [(x^2+y^2-(R+\Delta)^2] \bigwedge\nolimits_0 [(\theta-t)t]\,,$$

$$f_4 = [(x-c)^2+(y-c)^2-(r+\Delta)^2] \bigwedge\nolimits_0 [(\theta-t)t]\,.$$

The solution obtained will be illustrated by curves of heating of four selected points of the domain tested (Fig. 6), and a numerical presentation of temperature distributions at three instants of time ($\theta = 40000$ s): $t = 4000$, 20000, 36000 s.

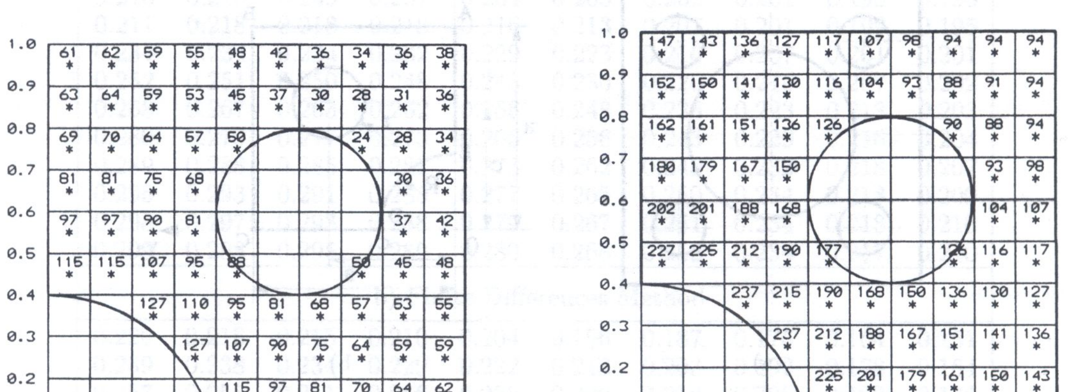

Fig. 4. Temperature fields for $t = 4000$ and 20000 s

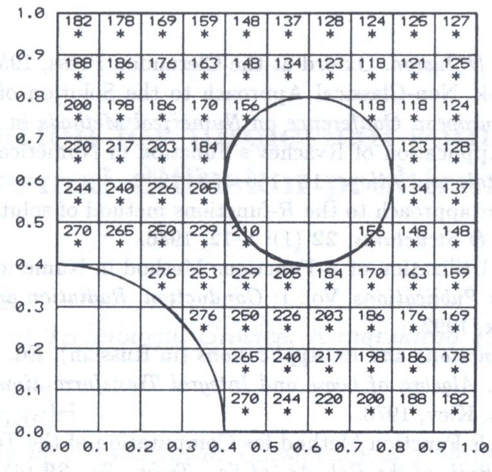

Fig. 5. Temperature field for $t = 36000$ s

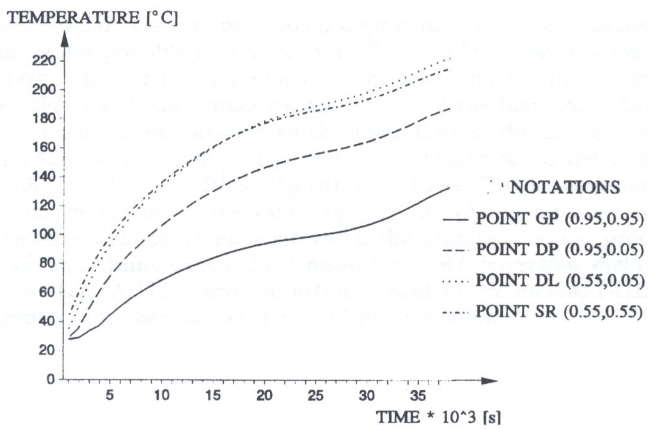

Fig. 6. Heating curves for the selected points

5. Final remarks

The investigations presented here were mainly of experimental character. Their primary object was to verify a possibility of treating the time variable in the same way as the spatial ones (in parabolic equation).

A lot of problems of *RFM* require wider analysis (see [6]). The space-time approach extends the area of this study to include, e.g., the following problems:

1) How should we choose the duration θ of the analyzed process. The numerical experiments show that agreement of solution reached by *ST-RFM* and *FDM* is better for a long period of time.
2) The form of boundary condition over the surface $t = \theta$.
3) What is the best choice of function $\Phi = \Phi(\mathbf{x}, t)$ — see Eq. (16).

It seems that the generalized *R*-function method can be useful for problems where the shape of geometrical domain can change in time, as well as in the range of inverse problems.

Acknowledgement

This research is a part of the project "*Application and development of the BEM for numerical simulation of nonlinear heat and mass transfer processes*" No. 3 P404 038 07 sponsored by KBN.

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