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Effect of panelling, application of the Kutta condition and the way of positioning the control points on the numerical solution of panel methods. Part II

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An efficient, accurate, and simple numerical method is necessary for analysis and design of an incompressible potential flow around multi-element airfoils. In this paper, which is the subject of the second part of the study, the mathematical model is built utilizing the local coordinate system, while in the first part of the study [2] only the global coordinate system is used. Mathematical model, by the vortex panel method with the use of the stream function, is written for the analysis of potential flow over multi-element airfoils. The computational model is built for both uniform and linear vortex distributions with utilizing the constant stream function boundary condition. From the fact that any study which does not consider deeply and precisely, all the parameters relevant to the computational model, might make it fragmentary. Hence, the following parameters are tested to investigate their effect on the accuracy of the method. They are: both types of the vortex distribution, two types of panelling, different ways of applying the Kutta condition, and two ways of positioning the control points. For the purpose of easier comparison, the study cases performed using the present model are restricted only to single-element airfoils; NACA 0012 airfoil at an angle of attack $\alpha = 8.3°$, and a cusped trailing edge 15 percent thick Van de Vooren airfoil at $\alpha = 5°$.

1. Introduction

An efficient, accurate, and simple numerical method is necessary for the process of calculating the velocity distribution over multi-component airfoils at a given angle of attack. Then this direct method can easily be converted to an inverse method that can be used as a method of producing an airfoil shape which matches the desired velocity distribution. Kennedy and Marsden [4] method utilizes uniform vortex distribution with constant stream function boundary condition and off-trailing-edge control point. Soinne and Laine [7] method uses linear vortex distribution with constant stream function boundary condition and off-trailing-edge control point. Moreover, the control points are placed exactly on the airfoil surface.

This study paid attention to the influence of different parameters on the numerical solution of the panel method based on the stream function. These parameters are: the type of the function that represents the singularity distribution over the panels, two types of panelling, four ways of applying the Kutta condition, and two ways of positioning the control points. Part I of the study [2] is restricted to the use of the global coordinate system for building the computational model. Part II of the study, which is the subject of this paper, utilizes the local coordinate system in building the computational model for the analysis of potential flow over multi-element airfoils. Both the uniform and linear vortex distributions are used. When the linear vortex distribution is utilized, both the boundary points and the control points are placed exactly on the airfoil surface.

2. Theme of work

The surface panel method philosophy for solving arbitrary potential flow problems consists of mating the classical potential theory with contemporary numerical techniques, cf. [1, 6]. Classical potential theory is utilized to reduce an arbitrary flow problem to a surface integral equation relating boundary conditions to an unknown singularity distribution. The contemporary numerical techniques are then used to calculate an approximate solution to the integral equation [5].

The theme of work is as follows: identification of the surface on which singularity is required to be distributed, panelling the surface, the choice of singularity, selection of the function for approximating the singularity distribution on the panelled surface, positioning the control points, writing the mathematical expression relevant to the problem, satisfying the boundary condition at every control point, satisfying the Kutta condition, generation of the simultaneous equations, solving the simultaneous equations to evaluate the singularity strengths; once the singularity strengths are known, the tangential velocity and the pressure coefficient distributions over the airfoil surface and the attendant lift coefficient, can be calculated.

3. Mathematical model

The calculation of the velocity induced by the vortex distribution on the panels is facilitated by introducing a local coordinate system. The origin of the local frame of reference for j-th panel is placed at its centroid as it is shown in Fig. 1. The local ξ-axis runs along the panel, and the local η-axis points outward from the airfoil into the flow field. For j-th panel the transformation between the global and local frames is

$$\left\{ \begin{array}{c} \xi_i + \frac{1}{2}S_j \\ \eta_i \end{array} \right\} = \left[\begin{array}{cc} b_1 & b_2 \\ -b_2 & b_1 \end{array} \right] \left\{ \begin{array}{c} x_i - X_j \\ y_i - Y_j \end{array} \right\} \tag{1}$$

where (ξ_i, η_i) are coordinates of the i-th control point in the local frame and (x_i, y_i) are the corresponding coordinates in the global frame. Also

$$b_1 = \frac{X_{j+1} - X_j}{S_j}, \qquad b_2 = \frac{Y_{j+1} - Y_j}{S_j}, \tag{2}$$

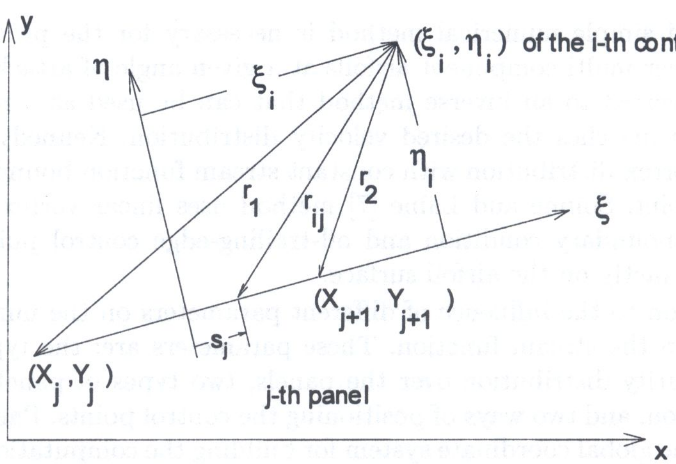

Fig. 1. Geometry required for the evaluation of the integrals

and

$$S_j = \left[(X_{j+1} - X_j)^2 + (Y_{j+1} - Y_j)^2\right]^{\frac{1}{2}} \tag{3}$$

Note that (X_j, Y, j) and (X_{j+1}, Y_{j+1}) are the coordinates of the boundary points of the j-th panel in the global frame and S_j is the corresponding panel length. Role of panelling consists in selecting a number of points (boundary points) on the surface of the airfoil. The airfoil surface is modelled by connecting each two neighbouring boundary points by a straight line (panel). Figure 2 illustrates such a procedure for a single-element airfoil. The airfoil contour is divided into N number of panels,

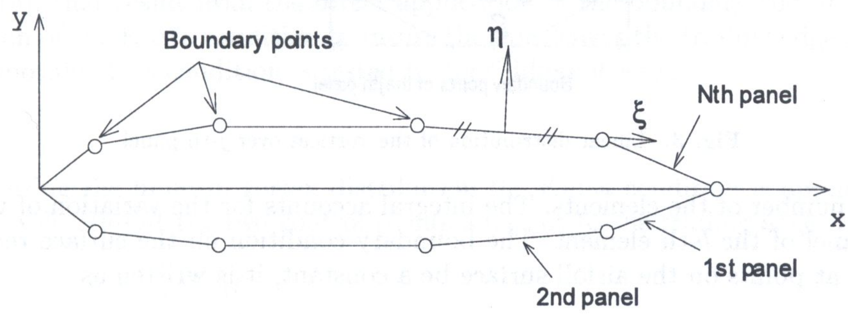

Fig. 2. Panelling the surface of a single-element airfoil with global coordinate system at the L.E. of the airfoil and the local coordinate system at the centroid of the panel

starting with the 1st panel on the lower surface at the trailing edge and ending with the N-th panel on the upper surface at the trailing edge. The x-coordinates of the boundary points of the N (even number) panels can be defined from either one of the following equations

$$\frac{X_k}{C} = \frac{1}{2}(1 + \cos\Theta_k) \qquad \text{or} \qquad \frac{X_k}{C} = \frac{1}{2}(1 - \cos\Theta_{N/2-k}) \tag{4}$$

where

$$\Theta_k = \frac{\pi k}{N/2}, \qquad \Theta_{N/2-k} = \frac{\pi(N/2 - k)}{N}, \qquad k = 0, 1, \ldots, N/2.$$

Then the y-coordinates of these boundary points are calculated from the airfoil geometry. Joining each two successive points generates the panels, that furnish a continuous broken line over the airfoil surface. At the centre of these panels control points are chosen, while with the use of the linear vortex distribution the control points can be positioned at the boundary points; exactly at the airfoil surface. At these control points the boundary condition is satisfied. With the use of the linear vortex distribution, the vortex strength per unit length, γ, varies linearly from one boundary point to the next as shown in Fig. 3. Hence, the vortex strength at any point along the panel can be written as

$$\gamma(s_j) = \frac{1}{2}(\gamma_j + \gamma_{j+1}) + \frac{\gamma_{j+1} - \gamma_j}{S_j}\, s_j \tag{5}$$

while using the uniform vortex distribution, the uniform function that is used to approximate the vortex distribution on each panel is written as:

$$\gamma(s_j) = \gamma_j\,. \tag{6}$$

Consider multi-element airfoil placed in a uniform flow of speed V_∞. The stream function $\Psi_i^{(c)}$ at any control point on every airfoil element is influenced by the vortex panels of all the elements and the undisturbed free stream, is written in a general form of vortex distribution as:

$$\Psi_i^{(c)} = V_\infty(y_i^{(c)} \cos\alpha - x_i^{(c)} \sin\alpha) + \frac{1}{2\pi} \sum_{b=1}^{M} \sum_{j=1}^{N_b} \int_{b_j} \gamma\big(s_j^{(b)}\big) \ln\Big[\big(\eta_i^{(c)}\big)^2 + \big(\xi_i^{(c)} - s_j^{(b)}\big)^2\Big]^{\frac{1}{2}} \, ds_j^{(b)}, \tag{7}$$

$$c = 1, \ldots, M, \quad i = i^{(c)} = 1, \ldots, N_c,$$

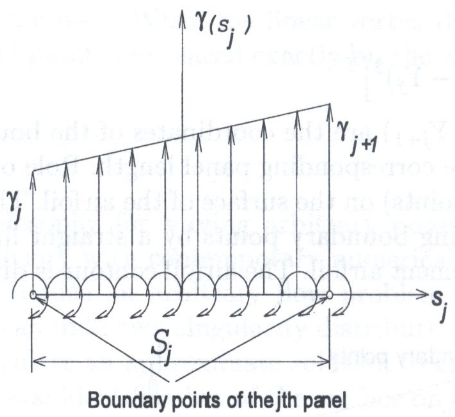

Fig. 3. Linear distribution of the vortices over j-th panel

where M is the number of the elements. The integral accounts for the variation of vortex strength over the j-th panel of the b-th element. The boundary condition on the surface requires that the stream function at points on the airfoil surface be a constant, it is written as

$$\Psi_i^{(c)} = \Psi_c\,, \qquad c = 1, \ldots, M. \tag{8}$$

After rewriting this boundary condition for each control point, the following dimensionless equation, derived from Eq. (7), will be obtained as

$$\sum_{b=1}^{M}\sum_{j=1}^{N_b}\left[\left(L_{i,j}^{(c)(b)} - L_{i,j_L}^{(c)(b)}\right)\gamma'^{(b)}_j + \left(L_{i,j}^{(c)(b)} + L_{i,j_L}^{(c)(b)}\right)\gamma'^{(b)}_{j+1}\right] + \Psi_c = y_i^{(c)}\cos\alpha - x_i^{(c)}\sin\alpha\,, \tag{9}$$

$$c = 1, \ldots, M, \qquad i = i^{(c)} = 1, \ldots, N_c\,.$$

The form of Eq. (9) is written for the case of using the linear vortex distribution. While for the case of using the uniform vortex distribution, Eq. (9) should be rewritten in a form with L_{i,j_L} and the second term between the parentheses equal to zero. The coefficients $L_{i,j}^{(c)(b)}$ and $L_{i,j_L}^{(c)(b)}$ of Eq. (9) take the forms of:

$$L_{i,j}^{(c)(b)} = S_j^{(b)} + \left(\xi_j^{(c)} - \frac{1}{2}S_j^{(b)}\right)\ln r_2 - \left(\xi_j^{(c)} + \frac{1}{2}S_j^{(b)}\right)\ln r_1$$
$$- \eta_i^{(c)}\left[\tan^{-1}\left(\frac{\xi_j^{(c)} + \frac{1}{2}S_j^{(b)}}{\eta_i^{(c)}}\right) - \tan^{-1}\left(\frac{\xi_j^{(c)} - \frac{1}{2}S_j^{(b)}}{\eta_i^{(c)}}\right)\right] \tag{10}$$

and

$$L_{i,j_L}^{(c)(b)} = -\frac{1}{S_j^{(b)}}\left\{\left[\frac{1}{4}\left(S_j^{(b)}\right)^2 + \left(\eta_i^{(c)}\right)^2 - \left(\xi_i^{(c)}\right)^2\right](\ln r_2 - \ln r_1)\right.$$
$$\left. + 2\eta_i^{(c)}\xi_i^{(c)}\left[\tan^{-1}\left(\frac{\xi_j^{(c)} + \frac{1}{2}S_j^{(b)}}{\eta_i^{(c)}}\right) - \tan^{-1}\left(\frac{\xi_j^{(c)} - \frac{1}{2}S_j^{(b)}}{\eta_i^{(c)}}\right)\right] - \xi_i^{(c)}S_j^{(b)}\right\}. \tag{11}$$

For the case of positioning the control points exactly at the airfoil surface, with utilizing the linear vortex distribution; note that for $i^{(c)} = j^{(b)}$: when $r_1 = 0.0$

$$L_{i,j}^{(c)(b)} = S_j^{(b)}\left[1 - \ln S_j^{(b)}\right] \tag{12}$$

and

$$L_{i,j_L}^{(c)(b)} = -\frac{1}{2}S_j^{(b)}\,, \tag{13}$$

and when $r_2 = 0.0$

$$L_{i,j}^{(c)(b)} = S_j^{(b)}\left[1 - \ln S_j^{(b)}\right] \tag{14}$$

and

$$L_{i,j_L}^{(c)(b)} = \frac{1}{2} S_j^{(b)} \,. \tag{15}$$

The Kutta condition states that airflow should leave the trailing edge of the airfoil smoothly. Therefore, Eq. (9), that result from the direct application of the boundary condition, is remedied by the application of the Kutta condition to insure the flow leaves the trailing edge of each element of the airfoil smoothly. This condition is tested by four different ways.

The first way

For the case of using the uniform vortex distribution the Kutta condition is applied by requiring equal tangential velocities at the two control points adjacent to the trailing edge. It is written as

$$\gamma'^{(c)}_N = -\gamma'^{(c)}_1 \,, \qquad c = 1, \ldots, M, \tag{16}$$

where the notations 1 and N correspond to the 1-st and N-th control points respectively.

The second way

Also for the case of using the uniform vortex distribution the Kutta condition can be applied at a point just off the trailing edge. Assuming that the streamline corresponding to each element also passes through the assigned off-control-point, therefore the Kutta condition can be applied, by writing Eq. (9) at each off-trailing-edge control point, $t^{(c)}$, as

$$\sum_{b=1}^{M}\sum_{j=1}^{N_b}\left[L_{t,j}^{(c)(b)}\gamma'^{(b)}_j\right] + \Psi'_c = y_t^{(c)}\cos\alpha - x_t^{(c)}\sin\alpha\,, \qquad c = 1, \ldots, M. \tag{17}$$

The third way

For the case of using the linear vortex distribution the Kutta-condition is applied by using two equations. The first equation is written by requiring equal tangential velocities at the two control points adjacent to the trailing edge as

$$\gamma'^{(c)}_N + \gamma'^{(c)}_{N+1} = -\left(\gamma'^{(c)}_1 + \gamma'^{(c)}_2\right), \qquad c = 1, \ldots, M. \tag{18}$$

The second equation results from the application of the boundary condition at a point positioned downstream of the trailing edge. By writing Eq. (9) at each off-trailing-edge control point, $t^{(c)}$, it results in

$$\sum_{b=1}^{M}\sum_{j=1}^{N_b}\left[\left(L_{t,j}^{(c)(b)} - L_{t,j_L}^{(c)(b)}\right)\gamma'^{(b)}_j + \left(L_{t,j}^{(c)(b)} + L_{t,j_L}^{(c)(b)}\right)\gamma'^{(b)}_{j+1}\right] + \Psi'_c = y_t^{(c)}\cos\alpha - x_t^{(c)}\sin\alpha\,, \tag{19}$$

$$c = 1, \ldots, M.$$

The fourth way

Also for the case of using the linear vortex distribution the Kutta condition can be applied by using two equations. The first equation results from the application of the Kutta condition exactly at the trailing edge. It is written as

$$\gamma'^{(c)}_{N+1} = -\gamma'^{(c)}_1, \qquad c = 1, \ldots, M, \tag{20}$$

The second equation, Eq. (19), results from the application of the boundary condition at point positioned at a distance of a small percentage of the chord length, downstream of the trailing edge on the trailing edge bisector. The tangential velocity at each control point on either elements, for the case of using the linear vortex distribution, is calculated according to the following relation:

$$V_{t_i}^{(c)} = \frac{1}{2}\left(\gamma_j^{(c)} + \gamma_{j+1}^{(c)}\right), \qquad c = 1,\dots,M, \qquad i = i^{(c)} = j = j^{(c)} = 1,\dots,N_c . \tag{21}$$

Notice that for the case of using the uniform vortex distribution the tangential velocity is directly equal the vortex strength at the assigned control point. Once the tangential velocity at the control point of each panel is calculated, then by using Bernoulli's equation the pressure coefficient is evaluated according to

$$C_{p_i}^{(c)} = 1 - \left(\frac{V_{t_i}^{(c)}}{V_\infty}\right)^2, \qquad c = 1,\dots,M, \qquad i = i^{(c)} = 1,\dots,N_c . \tag{22}$$

The lift coefficient is calculated by using the Kutta-Joukowski theorem. Taking the panels sufficiently short, it can be assumed that the tangential velocity is uniform over each panel, the total lift coefficient is evaluated from the relation

$$c_l = 2\sum_{b=1}^{M}\sum_{j=1}^{N_b} \frac{V_{t_i}^{(b)}}{V_\infty}\frac{S_i^{(b)}}{C^{(b)}} \tag{23}$$

where $C^{(b)}$ is the chord length of element number b of the airfoil section.

4. Discussion and conclusions

The first selected case of study is NACA 0012 airfoil at an angle of attack $\alpha = 8.3°$, corresponding to unit lift coefficient. Zedan [9] used the method of conformal mapping to estimate the lift coefficient for the same airfoil. He obtained a lift coefficient as $c_l = 0.999$. Zedan also introduced results obtained by the Hess and Smith method for the same case airfoil. He mentioned the value of $c_l = 0.963$ using 100 panels. Figures 4 to 7 present the pressure distribution over NACA 0012 airfoil at $\alpha = 8.3°$, with the use of the uniform vortex distribution. Figure 4 presents the C_p distribution utilizing the global [2] and local coordinate systems. For the both frames, the C_p distribution and lift coefficient are the same under the same conditions. While in the case of using the local coordinate system, for the case of the uniform vortex distribution, it allows a reduction in the computation time. Figure 5 presents the C_p distribution using the first way of applying the Kutta condition with both types of panelling. It is clear that with this way of applying the Kutta condition the exact value of lift coefficient is not achieved.

Figure 6 presents the C_p distribution using the first type of panelling. A comparison is made between the influence of the first and second ways of applying the Kutta condition. It is evident that the second way of applying the Kutta condition is better than the first way of applying the Kutta condition. It gives a lift coefficient with a value of $c_l = 1.0000$. Moreover, it minimizes the number of the panels that is needed for achieving the peak value of C_p at the leading edge of the airfoil. Figure 7 presents a comparison for the C_p distribution using the two types of panelling with the second way of applying the Kutta condition. The first type of panelling achieves the lift coefficient with a value of $c_l = 1.0000$, while the second type of panelling gives it as $c_l = 1.0220$. The results of Figs. 6 and 7, under the utilized conditions, insure that correct choice of both the type of panelling and the way of applying the Kutta condition is so important for building the computational model for achieving an accurate results. Figure 8 presents a comparison for C_p distribution using linear vortex distribution with utilizing the fourth way of applying the Kutta condition ($c_l = 1.0000$) and using the uniform vortex distribution with utilizing the second way of applying the Kutta condition

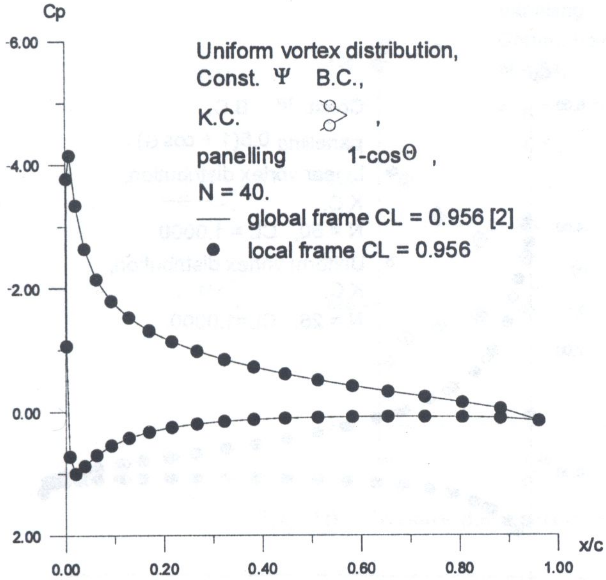

Fig. 4. Pressure distribution over NACA 0012 airfoil at $\alpha = 8.3°$

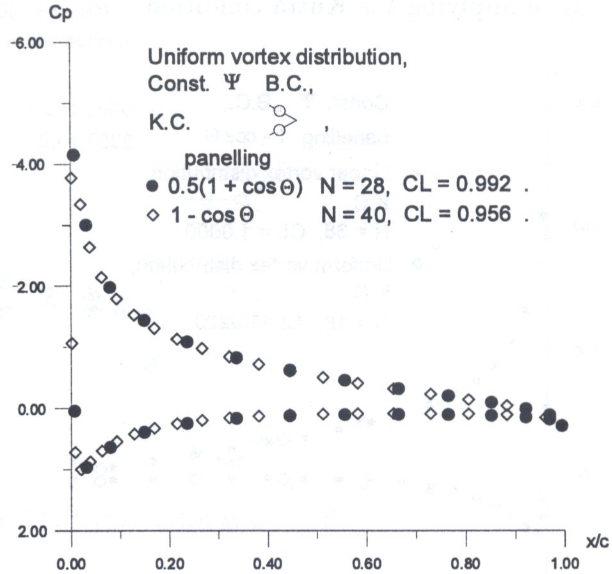

Fig. 5. Pressure distribution over NACA 0012 airfoil at $\alpha = 8.3°$

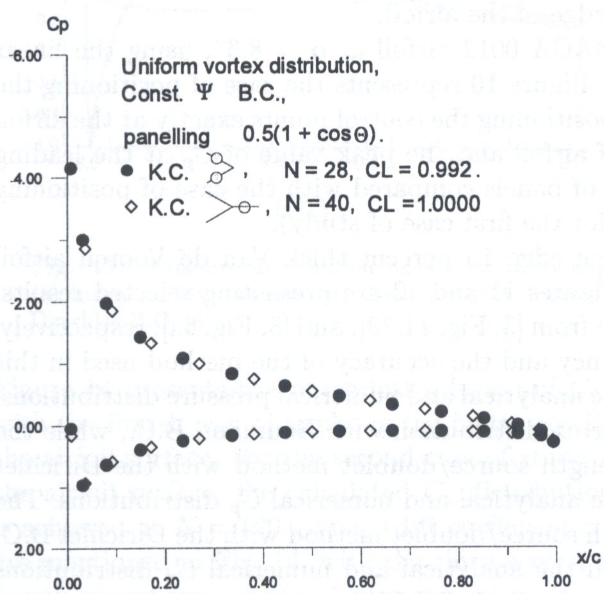

Fig. 6. Pressure distribution over NACA 0012 airfoil at $\alpha = 8.3°$

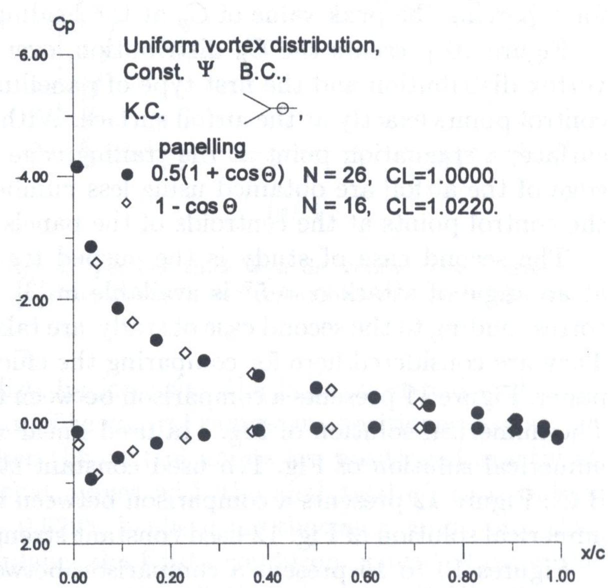

Fig. 7. Pressure distribution over NACA 0012 airfoil at $\alpha = 8.3°$

($c_l = 1.0220$). With both of them the second type of panelling is used. With this type of panelling, the linear vortex distribution utilizing the fourth way of applying the Kutta condition achieves better accuracy for the lift coefficient than using the uniform vortex distribution with the second way of applying the Kutta condition.

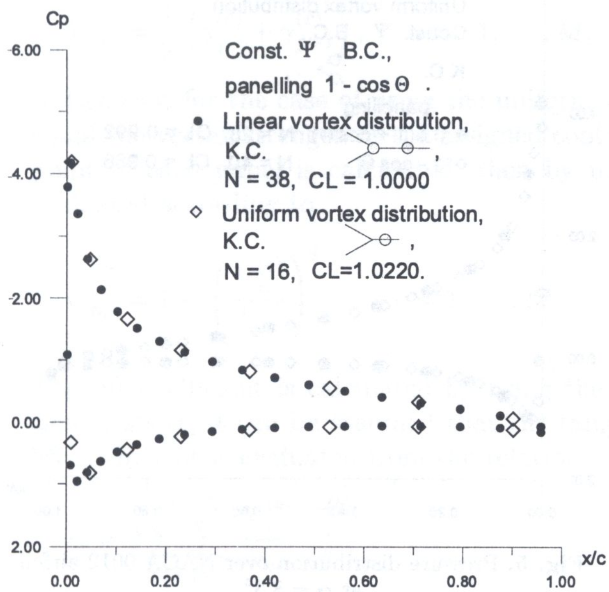

Fig. 8. Pressure distribution over NACA 0012 airfoil at $\alpha = 8.3°$

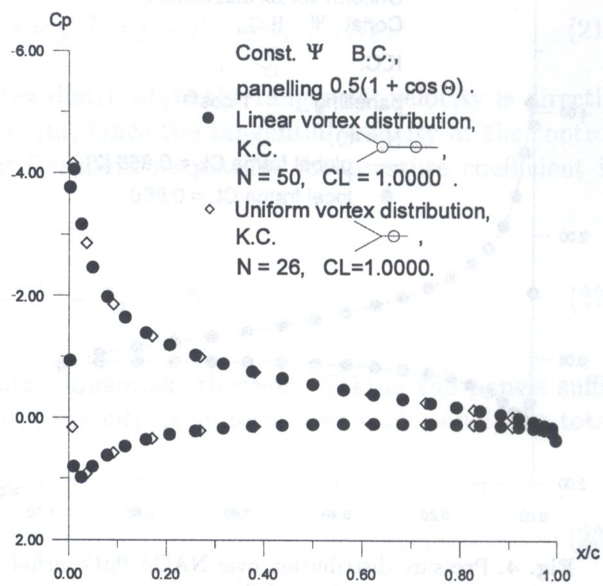

Fig. 9. Pressure distribution over NACA 0012 airfoil at $\alpha = 8.3°$

Figure 9 presents comparison for the C_p distribution, using linear vortex distribution with the fourth way of applying the Kutta condition and using the uniform vortex distribution with the second way of applying the Kutta condition. With both of them the first type of panelling is used. It is noticed that using the linear vortex distribution increases the number of panels that is needed for achieving the peak value of C_p at the leading edge of the airfoil.

Figure 10 presents the C_p distribution, over NACA 0012 airfoil at $\alpha = 8.3°$, using the linear vortex distribution and the first type of panelling. Figure 10 represents the case of positioning the control points exactly at the airfoil surface. With positioning the control points exactly at the airfoil surface, a stagnation point at the trailing edge of airfoil and the peak value of C_p at the leading edge of the airfoil are obtained using less number of panels compared with the case of positioning the control points at the centroids of the panels (for the first case of study).

The second case of study is the cusped trailing edge 15 percent thick Van de Vooren airfoil at an angle of attack $\alpha = 5°$ is available in [3]. Figures 11 and 12 are presenting selected results, corresponding to the second case of study, are taken from [3, Fig. 11.39], and [8, Fig. 6a], respectively. They are considered here for comparing the efficiency and the accuracy of the method used in this paper. Figure 11 presents a comparison between the analytical and numerical pressure distributions. The numerical solution of Fig. 11a used linear vortex distribution with Neumann B.C., while the numerical solution of Fig. 11b used constant strength source/doublet method with the Dirichlet B.C.. Figure 12 presents a comparison between the analytical and numerical C_p distributions. The numerical solution of Fig. 12 used constant strength source/doublet method with the Dirichlet B.C.

Figures 13 to 15 present a comparison between the analytical and numerical C_p distributions for the second case of study. Figure 13 presents two numerical solutions using the uniform vortex distribution with the second way of applying the Kutta condition and using the linear vortex distribution with the fourth way of applying the Kutta condition. The control points are positioned at the centroids of the panels. For this airfoil of cusped trailing edge, both of the numerical solutions achieve the C_p distribution that matches the analytical solution at the same number of panels ($N = 40$) with lift coefficient $c_l = 0.6298$ the same as the exact value of the lift coefficient.

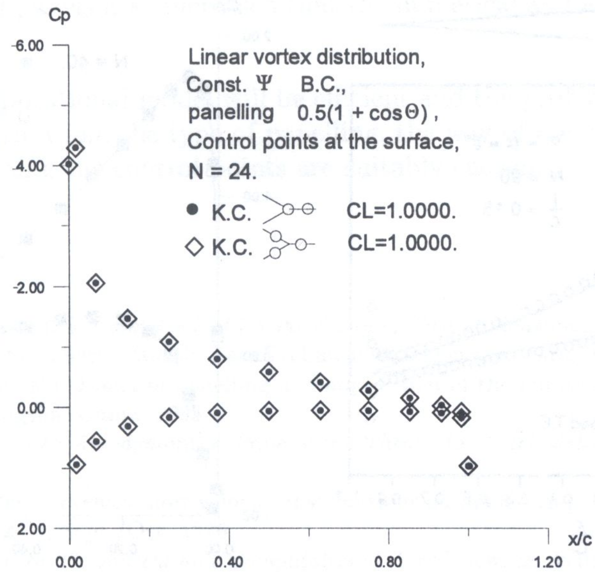

Fig. 10. Pressure distribution over NACA 0012 airfoil at $\alpha = 8.3°$

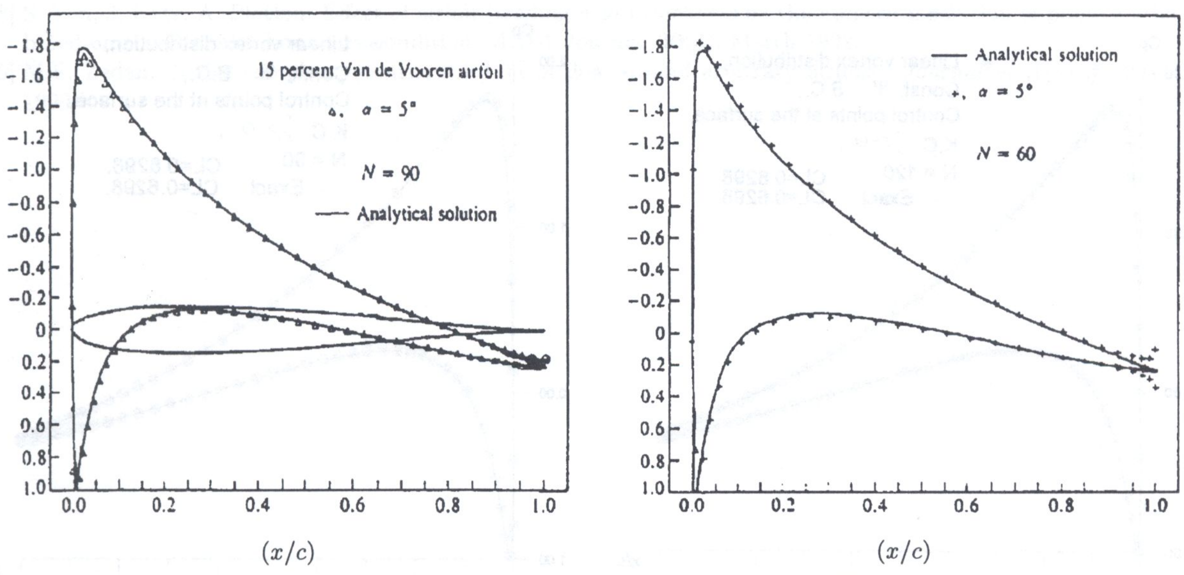

Fig. 11. Pressure distribution on a cusped trailing edge 15 percent thick Van de Vooren airfoil using: (a) linear vortex method with Neumann B.C., and (b) constant-strength source/doublet method with the Dirichlet B.C. [3]

Figure 14 presents the numerical solution of C_p distribution using the linear vortex distribution with the fourth way of applying the Kutta condition. The control points are positioned exactly at the airfoil surface. For the second case of study when the control points are positioned exactly at the airfoil surface, the calculated C_p distribution that agrees with the analytical C_p distribution is achieved at $N = 120$ giving a lift coefficient $c_l = 0.6298$. While using the same conditions, that are mentioned in Fig. 14, with the third way of applying the Kutta condition instead of using the fourth way of applying the Kutta condition, as it is shown in Fig. 15, the calculated C_p distribution matches the analytical C_p distribution at $N = 90$ giving a lift coefficient $c_l = 0.6298$.

From the above discussion the following conclusions can be drawn:

- It is recommended to use the first type of panelling.
- Using the uniform vortex distribution with the second way of applying the Kutta condition, the

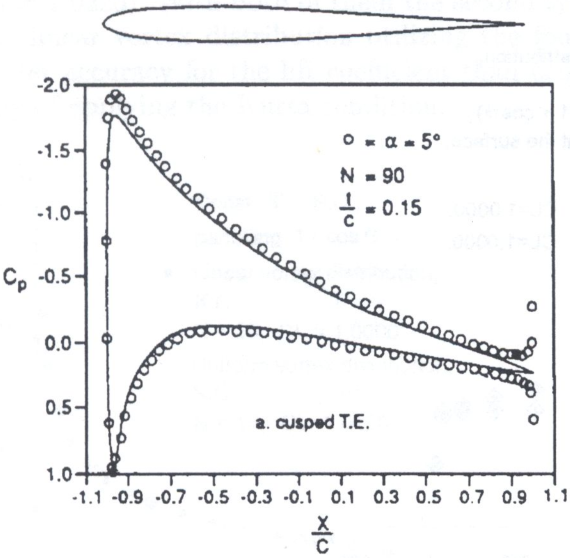

Fig. 12. Pressure distribution over Van de Vooren airfoil at $\alpha = 5°$

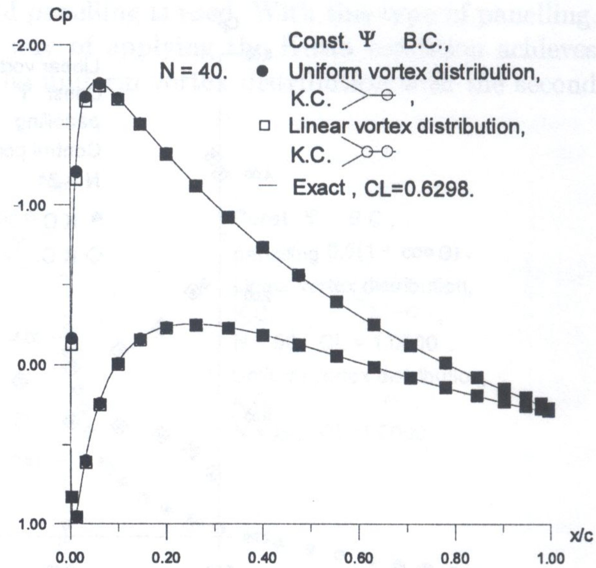

Fig. 13. Pressure distribution over Van de Vooren airfoil at $\alpha = 5°$

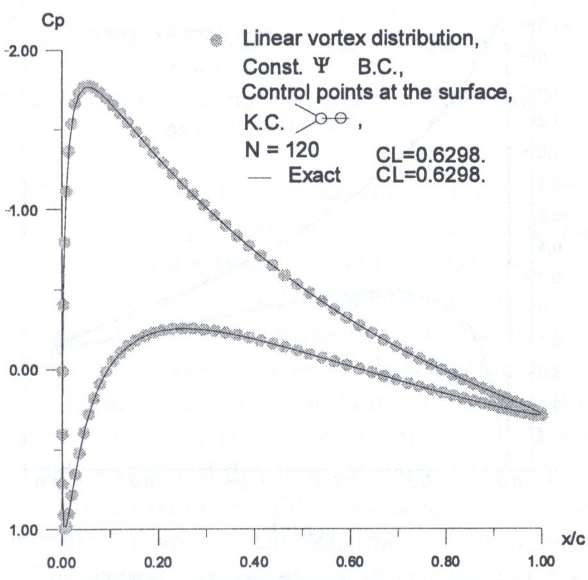

Fig. 14. Pressure distribution over Van de Vooren airfoil at $\alpha = 5°$

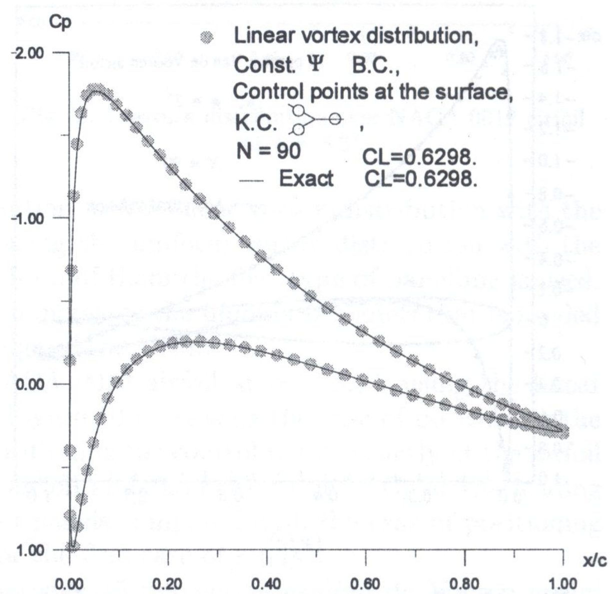

Fig. 15. Pressure distribution over Van de Vooren airfoil at $\alpha = 5°$

lift coefficient is achieved with higher accuracy at less number of panels than using the linear vortex distribution with the fourth way of applying the Kutta condition, however, utilizing the fourth way of applying the Kutta condition always ensures an equality of C_p values at the upper and lower surfaces exactly at the trailing edge of the airfoil.

- For the first case of study of the finite trailing edge, by positioning the control points exactly at the airfoil surface, with utilizing the linear vortex distribution, a stagnation point at the trailing edge of the airfoil is achieved and the number of panels corresponding to high accuracy is minimized.

For the second case of study which is an airfoil of a cusped trailing edge, with positioning the control points exactly at the airfoil surface, with utilizing the linear vortex distribution, no progress is achieved in minimizing the number of panels, however, the contrary situation occurs, in which the number of panels is so increased that the numerical and analytical C_p distributions are consistent.

- In conclusion, the computational model will be efficient and the problems can be avoided, if the type of the vortex distribution, the type of panelling, the way of applying the Kutta condition, and the way of positioning the control points are suitably chosen.

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