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Structure–subsoil contact task – an iterative engineering realisation

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The analytical formulation of an iterative procedure applied for structure–subsoil systems is presented in the paper. A physical and engineering interpretation has been given for the presented algorithm.

1. Introduction

The boundary value task describing the equilibrium of subsoil loaded by a building structure has been presented in this paper. It is an interactive task in which the accuracy of the solution results from both:

- constitutive laws which approximate stress–strain relations in both subsystems,
- and the method of realisation of the contact zone between the subsystems in the calculation model.

The classical engineering estimation method of interaction between structure and subsoil is based on an independently carried out:

- estimation of subsoil load carrying capacity and subsidence under structure loads,
- calculation analysis of foundations (or the whole structure) resting on the subsoil which usually has the form of:
  - a parametric (analogous) calculation model,
  - or, an elastic half-plane or half-space with possible modifications (e.g. change of modulus of elasticity with depth).

Modern numerical ground models (e.g. models containing elastic-plastic constitutive relations or critical state models) are still rarely used while predicting the behaviour of ground environment interacting with the designed structure. This is mainly caused by the problems in "coupling" the structure calculation model in an interactive system together with the subsoil numerical model. Another approach to the discussed contact task is to create a global model for the whole structure–subsoil system. However, this implies a huge amount of time to elaborate the model, carry out the calculations and process the results. Therefore, it can only be applied while analysing certain scientific cases.

Consequently, it seems reasonable to put forward a proposal of the contact task solution in the interactive process between two independently modelled subsystems: structure and subsoil [4, 5, 6]. Application of the interactive process enables us to obtain the solution for the whole system by means of:

- any adequate computer programmes, providing us with the complexity grade suitable for the geometrical or physical description of each of the subsystems,
- a description of both discretised subsystems by one common or two different numerical methods.

2. Definition of the problem

The calculation model of the structure–subsoil (B)–(S) system was considered which provided for interaction between both the subsystems conforming to a certain class of real engineering problems. Vertical load that is transferred from the structure onto the subsoil is considered in the range of the so-called working loads [5]. Loss of contact between the structure and the subsoil is not considered. Two different types of modelling the contact between discretised subsystems have been considered:

- (α) – main nodes of discrete elements at the contact zone of both subsystems maintain their common vertical displacements, whereas horizontal displacements may be different (Fig. 1a),
- (β) – main nodes of discrete elements of both subsystems are joined by connecting elements (R_o^j) constituting a parameter (or parameters) for the system of equilibrium algebraic equations for the boundary value problem (Fig. 1b).

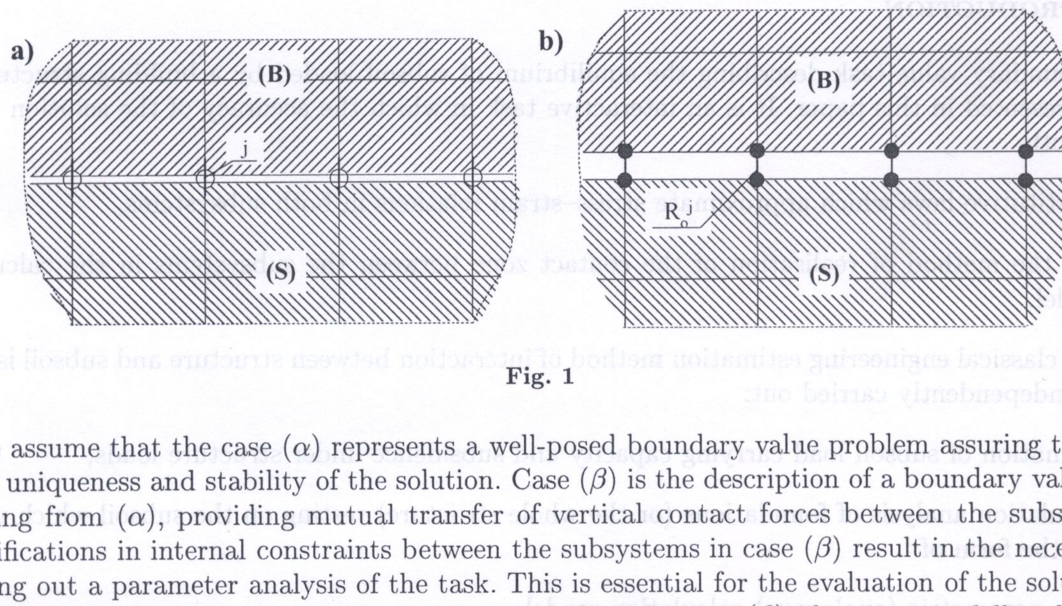

Fig. 1

We assume that the case (α) represents a well-posed boundary value problem assuring the existence, uniqueness and stability of the solution. Case (β) is the description of a boundary value task resulting from (α), providing mutual transfer of vertical contact forces between the subsystems. Simplifications in internal constraints between the subsystems in case (β) result in the necessity of carrying out a parameter analysis of the task. This is essential for the evaluation of the solution.

Description of the boundary value problem, including the type (β) of contact modelling between the subsystems enables us to give fully applicable engineering character to the considered problem, which is possible due to the easy formulation of the iterative solution procedure.

3. Analytical description of the task

The following assumptions in the presented contact task solutions were made:

- the incremental procedure will be adopted for numerical analysis within the frames of elastic-plastic constitutive theory,
- the subsystem (B) will be described by the REM – Rigid Elements Method [3–9, 11, 12] or by the FEM [1, 11]; in the REM unknown vector contains the displacements of the elements centres of discrete model,
- the subsystem (S) will be modelled by FEM.

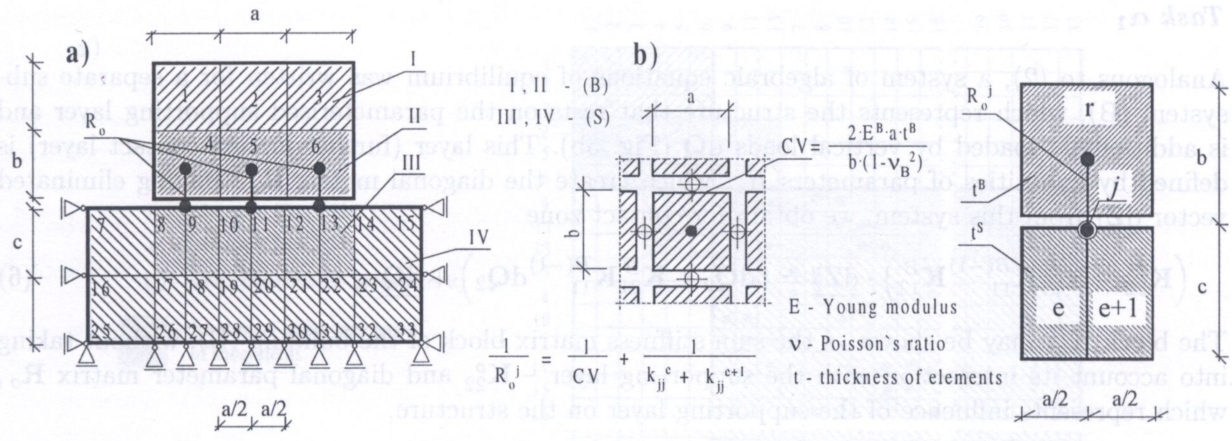

Fig. 2

A global model of (B)–(S) system was shown in Fig. 2a. Discretising was achieved by means of REM and FEM methods (Fig. 2b). In order to assure clarity of this presentation, we limited the number of elements to a minimum. Assuming that the structure acts on the subsoil by vertical forces only (acc. to par.2), a (β) type connection has been used in the contact zone of the subsystems. The solution given for 2D state does not contradict the general character of the considerations.

Defined at every step of the incremental procedure the system load vector (external and mass forces) as $\mathbf{dQ}$ the system of algebraic equations of equilibrium shall be written as

$$\mathbf{K}\cdot\mathbf{dZ}=\mathbf{dQ} \tag{1}$$

(For linear elastic problem $\mathbf{dQ}=\mathbf{Q}$, $\mathbf{dZ}=\mathbf{Z}$).

The considered system (B)–(S) shall be divided into four zones I–IV (Fig. 2a). In the contact of type (β) the interaction of subsystems (B) and (S) is expressed by the difference in displacements of elements REM and FEM of zones II and III which are bound together by connecting elements R_o. After the regrouping of stiffness matrix $\mathbf{K}$ and vectors of displacement $\mathbf{dZ}$ and load $\mathbf{dQ}$, in order to obtain equations describing vertical displacements of the contact elements in zones II and III only, and after realising that matrix $\mathbf{K}$ is a strip matrix, the system of equations (1) shall be written in the form of

$$\begin{aligned}
\mathbf{K}_{11}\cdot\mathbf{dZ}_1 &+ \mathbf{K}_{12}\cdot\mathbf{dZ}_2 && && &&= \mathbf{dQ}_1\,,\\
\mathbf{K}_{21}\cdot\mathbf{dZ}_1 &+ \mathbf{K}_{22}\cdot\mathbf{dZ}_2 &&+ \mathbf{K}_{23}\cdot\mathbf{dZ}_3 && &&= \mathbf{dQ}_2\,,\\
& \quad\;\, \mathbf{K}_{32}\cdot\mathbf{dZ}_2 &&+ \mathbf{K}_{33}\cdot\mathbf{dZ}_3 &&+ \mathbf{K}_{34}\cdot\mathbf{dZ}_4 &&= \mathbf{dQ}_3\,,\\
& && \quad\;\, \mathbf{K}_{43}\cdot\mathbf{dZ}_3 &&+ \mathbf{K}_{44}\cdot\mathbf{dZ}_4 &&= \mathbf{dQ}_4\,.
\end{aligned} \tag{2}$$

The stiffness matrix of the system shown in Fig. 3a corresponds with this record. Vectors of displacement and load increase are of the form

$$\mathbf{dZ}=\left\{\ \mathbf{dZ}_1^T\quad \mathbf{dZ}_2^T\quad \mathbf{dZ}_3^T\quad \mathbf{dZ}_4^T\ \right\}^T, \tag{3}$$

$$\mathbf{dQ}=\left\{\ \mathbf{dQ}_1^T\quad \mathbf{dQ}_2^T\quad \mathbf{dQ}_3^T\quad \mathbf{dQ}_4^T\ \right\}^T. \tag{4}$$

Having eliminated vectors $\mathbf{dZ}_1$ and $\mathbf{dZ}_4$ from Eqs. (2) we obtain equations of equilibrium of the global system, reduced to the contact zones II and III of both subsystems:

$$\begin{aligned}
&\left(\mathbf{K}_{22}-\mathbf{K}_{21}\mathbf{K}_{11}^{(-1)}\mathbf{K}_{12}\right)\cdot\mathbf{dZ}_2+\mathbf{K}_{23}\cdot\mathbf{dZ}_3=\mathbf{dQ}_2+\mathbf{K}_{21}\mathbf{K}_{11}^{(-1)}\mathbf{dQ}_1\,,\\
&\mathbf{K}_{32}\cdot\mathbf{dZ}_2+\left(\mathbf{K}_{33}-\mathbf{K}_{34}\mathbf{K}_{44}^{(-1)}\mathbf{K}_{43}\right)\cdot\mathbf{dZ}_3=\mathbf{dQ}_3+\mathbf{K}_{34}\mathbf{K}_{44}^{(-1)}\mathbf{dQ}_4\,.
\end{aligned} \tag{5}$$

The above notation allows observations presented below. In order to do this we will consider two auxiliary tasks $\boldsymbol{\alpha}_1$ and $\boldsymbol{\alpha}_2$, describing the separate subsystems (B) and (S).

***Task* $\boldsymbol{\alpha}_1$**

Analogous to (2), a system of algebraic equations of equilibrium was written for a separate subsystem (B), which represents the structure that rests on the parameterised supporting layer and is additionally loaded by vertical loads $\mathrm{d}\mathbf{Q}$ (Fig. 3b). This layer (further named contact layer) is defined by quantities of parameters R_o, which create the diagonal matrix $\mathbf{R}_o$. Having eliminated vector $\underline{\mathrm{d}\mathbf{Z}_1}$ from this system, we obtain for contact zone

$$\left(\mathbf{K}_{22}^{B}-\mathbf{K}_{21}^{B}\mathbf{K}_{11}^{B(-1)}\mathbf{K}_{12}^{B}\right)\cdot\underline{\mathrm{d}\mathbf{Z}_2}=\left(\mathrm{d}\mathbf{Q}_2+\mathbf{K}_{21}^{B}\mathbf{K}_{11}^{B(-1)}\mathrm{d}\mathbf{Q}_2\right)+\mathrm{d}\mathbf{Q}. \tag{6}$$

The block $\mathbf{K}_{22}^{B}$ may be shown as the sum stiffness matrix block of the building (B), without taking into account its interaction with the supporting layer – $\mathbf{K}_{22}^{o}$ and diagonal parameter matrix $\mathbf{R}_o$, which represents influence of the supporting layer on the structure,

$$\mathbf{K}_{22}^{B}=\mathbf{K}_{22}^{o}+\mathbf{R}_o. \tag{7}$$

***Task* $\boldsymbol{\alpha}_2$**

For the separated subsystem (S) that represents the subsoil, loaded by the vertical forces $\mathrm{d}\mathbf{F}$ (Fig. 3c) at the contact of subsystems, a system of algebraic equations of equilibrium has been written, analogous to (2). Having eliminated vector $\underline{\mathrm{d}\mathbf{Z}_4}$ from this system, we obtain for the contact zone

$$\left(\mathbf{K}_{33}^{S}-\mathbf{K}_{34}^{S}\mathbf{K}_{44}^{S(-1)}\mathbf{K}_{43}^{S}\right)\cdot\underline{\mathrm{d}\mathbf{Z}_3}=\left(\mathrm{d}\mathbf{Q}_3+\mathbf{K}_{34}^{S}\mathbf{K}_{44}^{S(-1)}\mathrm{d}\mathbf{Q}_4\right)+\mathrm{d}\mathbf{F}. \tag{8}$$

Now we achieve a comparison between the global system (B)–(S) and the separated subsystems of tasks $\boldsymbol{\alpha}_1$ and $\boldsymbol{\alpha}_2$.

As interaction of subsystems (B) and (S) is limited to vertical forces only, matrices $\mathbf{K}_{22}$ and $\mathbf{K}_{33}$ in the equations of equilibrium (5) may be shown in the form:

$$\mathbf{K}_{22}=\mathbf{K}_{22}^{o}+\mathbf{R}_o,\qquad \mathbf{K}_{33}=\mathbf{K}_{33}^{S}+\mathbf{R}_o, \tag{9}$$

where $\mathbf{R}_o$ is a diagonal parametric matrix (which contains the parameters R_o). The matrix $\mathbf{R}_o$ expresses interaction of subsystems (B) and (S). Comparing the system of equations of equilibrium for the global system (5) with similarly composed equations of equilibrium for two separate subsystems (6) and (8), taking into consideration (7) and (9) as well as the obvious relations:

$$\mathbf{K}_{32}=\mathbf{K}_{23}^{T}=-\mathbf{R}_o$$

and

$$\mathbf{K}_{21}=\mathbf{K}_{21}^{B},\qquad \mathbf{K}_{12}=\mathbf{K}_{12}^{B},\qquad \mathbf{K}_{11}=\mathbf{K}_{11}^{B},\qquad \mathbf{K}_{44}=\mathbf{K}_{44}^{S},\qquad \mathbf{K}_{43}=\mathbf{K}_{43}^{S},\qquad \mathbf{K}_{34}=\mathbf{K}_{34}^{S},$$

system (5) can be written in the following form,

$$\begin{aligned}(\mathbf{K}^{B}+\mathbf{R}_o)\cdot\mathrm{d}\mathbf{Z}_2&=\mathrm{d}\mathbf{Q}^{B}+\mathbf{R}_o\cdot\mathrm{d}\mathbf{Z}_3\\ \mathbf{K}^{S}\cdot\mathrm{d}\mathbf{Z}_3&=\mathrm{d}\mathbf{Q}^{S}+\mathbf{R}_o\cdot(\mathrm{d}\mathbf{Z}_2-\mathrm{d}\mathbf{Z}_3)\end{aligned} \tag{10a}$$

or in the symmetric form

$$\begin{aligned}(\mathbf{K}^{B}+\mathbf{R}_o)\cdot\mathrm{d}\mathbf{Z}_2&=\mathrm{d}\mathbf{Q}^{B}+\mathbf{R}_o\cdot\mathrm{d}\mathbf{Z}_3\\ (\mathbf{K}^{S}+\mathbf{R}_o)\cdot\mathrm{d}\mathbf{Z}_3&=\mathrm{d}\mathbf{Q}^{S}+\mathbf{R}_o\cdot\mathrm{d}\mathbf{Z}_2\end{aligned} \tag{10b}$$

where $\mathbf{K}^{B}$, $\mathbf{K}^{S}$, $\mathrm{d}\mathbf{Q}^{B}$, $\mathrm{d}\mathbf{Q}^{S}$ – are stiffness matrices and vectors of loads transformed in the process of eliminating unknowns which do not belong to the contact zones. These matrices and vectors, not taking into consideration the interaction of the subsystems, take following form,

$$\mathbf{K}^{B}=\mathbf{K}_{22}^{o}-\mathbf{K}_{21}^{B}\mathbf{K}_{11}^{B(-1)}\mathbf{K}_{12}^{B},\qquad \mathrm{d}\mathbf{Q}^{B}=\mathrm{d}\mathbf{Q}_2-\mathbf{K}_{21}^{B}\mathbf{K}_{11}^{B(-1)}\cdot\mathrm{d}\mathbf{Q}_1,$$

$$\mathbf{K}^{S}=\mathbf{K}_{33}^{S}-\mathbf{K}_{34}^{S}\mathbf{K}_{44}^{S(-1)}\mathbf{K}_{43}^{S},\qquad \mathrm{d}\mathbf{Q}^{S}=\mathrm{d}\mathbf{Q}_3-\mathbf{K}_{34}^{S}\mathbf{K}_{44}^{S(-1)}\cdot\mathrm{d}\mathbf{Q}_4.$$

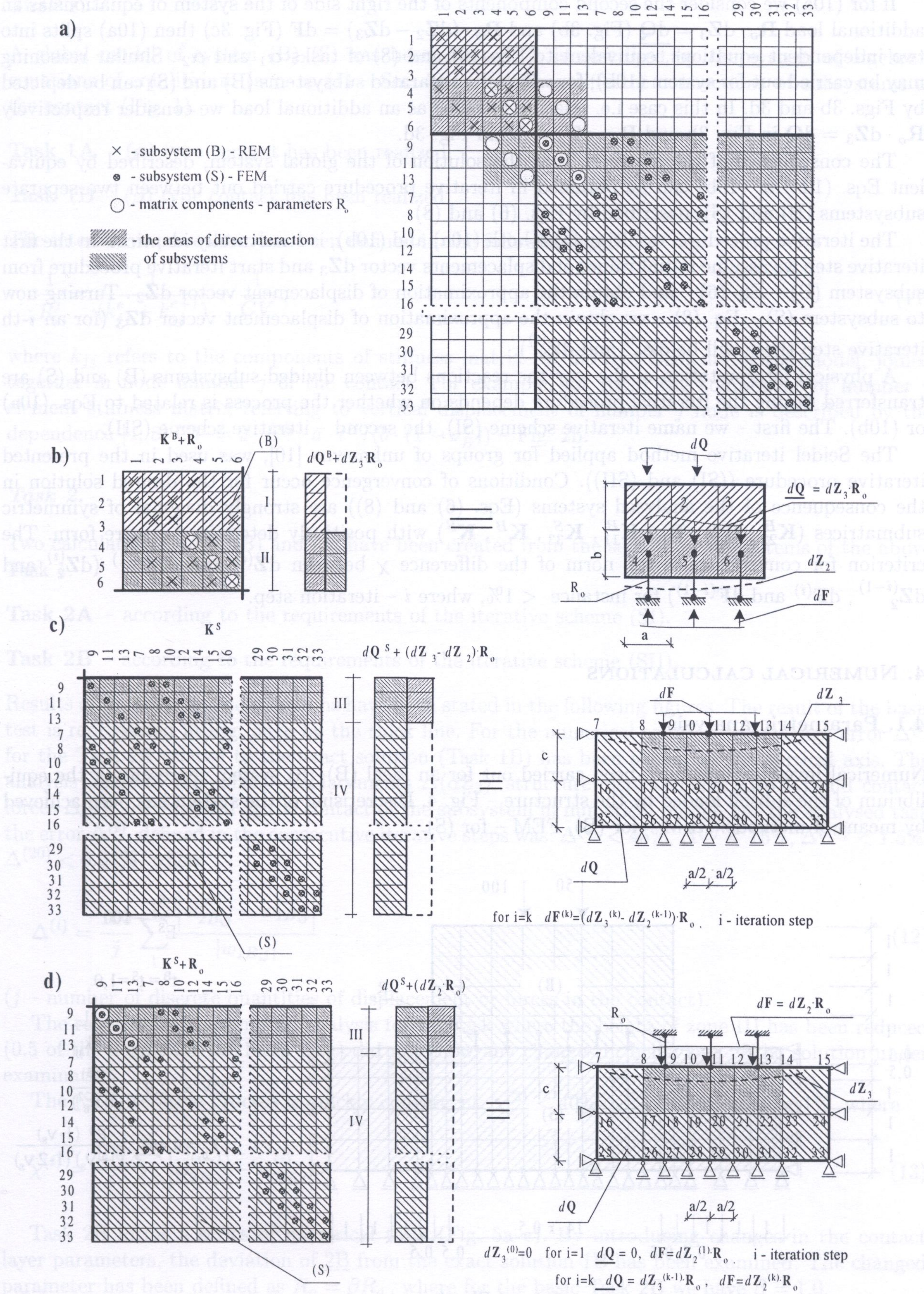
a)
× - subsystem (B) - REM
⊗ - subsystem (S) - FEM
○ - matrix components - parameters R₀
the areas of direct interaction of subsystems
b)
K^B+R₀
(B)
dQ^B+dZ₃·R₀
I
II
dQ
dQ = dZ₃·R₀
dZ₂
dF
c)
K^S
dQ^S+(dZ₃-dZ₂)·R₀
III
IV
(S)
dF
dZ₃
dQ
a/2 a/2
for i=k dF^(k)=(dZ₃^(k)- dZ₂^(k-1))·R₀ . i - iteration step
d)
K^S+R₀
dQ^S+(dZ₂·R₀)
III
IV
(S)
R₀
dF = dZ₂·R₀
dZ₃
dQ
a/2 a/2
dZ₃^(0)=0 for i=1 dQ = 0, dF=dZ₂^(1)·R₀ i - iteration step
for i=k dQ = dZ₃^(k-1)·R₀ , dF=dZ₂^(k)·R₀

Fig. 3

If for (10a) we consider the second components of the right side of the system of equations as an additional load $\mathbf{R}_o \cdot d\mathbf{Z}_3 = \underline{d\mathbf{Q}}$ (Fig. 3b) and $\mathbf{R}_o \cdot (d\mathbf{Z}_2 - d\mathbf{Z}_3) = d\mathbf{F}$ (Fig. 3c) then (10a) splits into two independent equations, equivalent to Eqs. (6) and (8) of tasks $\boldsymbol{\alpha}_1$ and $\boldsymbol{\alpha}_2$. Similar reasoning may be carried out for system (10b), for which the separated subsystems (B) and (S) can be depicted by Figs. 3b and 3d. In this case i.e. relating to (10b) as an additional load we consider respectively $\mathbf{R}_o \cdot d\mathbf{Z}_3 = \underline{d\mathbf{Q}}$ in Fig. 3b and $\mathbf{R}_o \cdot d\mathbf{Z}_2 = d\mathbf{F}$ in Fig. 3d.

The consequence of the above is, that the solution of the global system, described by equivalent Eqs. (10a) or (10b) can be obtained in iterative procedure carried out between two separate subsystems (B) and (S), described by Eqs. (6) and (8).

The iterative procedure, resulting from both (10a) and (10b), is structurally identical. In the first iterative step we assume initial values of displacements vector $d\mathbf{Z}_3$ and start iterative procedure from subsystem (B) – Eq. (6) – and obtain the approximation of displacement vector $d\mathbf{Z}_2$. Turning now to subsystem (S) – Eq. (8) – we obtain the approximation of displacement vector $d\mathbf{Z}_3$ (for an i-th iterative step these will be $d\mathbf{Z}_2^{(i)}$ and $d\mathbf{Z}_3^{(i)}$).

A physical interpretation of the way the reactions between divided subsystems (B) and (S) are transferred during the iterative procedure depends on whether the process is related to Eqs. (10a) or (10b). The first – we name iterative scheme (SI), the second – iterative scheme (SII).

The Seidel iterative method applied for groups of unknowns [10], was used in the presented iterative procedure ((SI) and (SII)). Conditions of convergence occur for the desired solution in the consequence of the analysed systems (Eqs. (6) and (8)) are strongly diagonal of symmetric submatrices ($\mathbf{K}_{22}^B$, $\mathbf{K}_{22}^o$, $\mathbf{R}_o$, $\mathbf{K}_{11}^B$, $\mathbf{K}_{33}^S$, $\mathbf{K}^B$, $\mathbf{K}^S$) with positively determined square form. The criterion for convergence is the norm of the difference χ between $d\mathbf{Z}^{(i)}$ and $d\mathbf{Z}^{(i-1)}$ ($d\mathbf{Z}_2^{(i)}$ and $d\mathbf{Z}_2^{(i-1)}$, $d\mathbf{F}^{(i)}$ and $d\mathbf{F}^{(i-1)}$) for instance $< 1\%$, where i – iteration step.

4. Numerical calculations

4.1. Parametric analysis

Numerical investigations have been carried out for an ideal (B)–(S) system representing the equilibrium of the subsoil loaded by the structure – Fig. 4. Discretising of the system has been achieved by means of methods: REM – for (B), FEM – for (S).

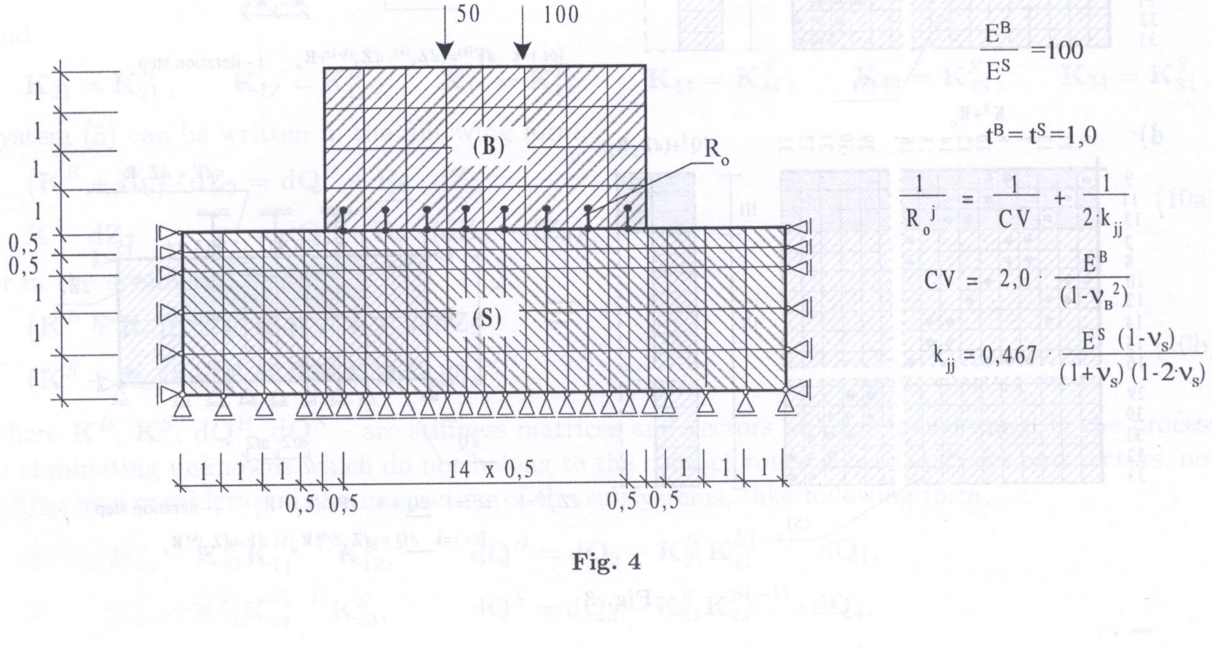

Fig. 4

Task 1.

A global model of system (B)–(S) has been created. It is described by the system of algebraic equations of equilibrium (acc. to (1) – Sec. 3) and realizes different connections of subsystems at the contact (Fig. 1).

Task 1A – (α) type contact has been realized.

Task 1B – (β) type contact has been realized.

The interactive parameter has been defined in the following way,

$$\frac{1}{R_o^j} = \frac{1}{(k_{jj}^e + k_{jj}^{e+1})} + \frac{1}{CV}, \tag{11}$$

where k_{jj} refers to the components of stiffness matrix for discrete elements of the model, joined together in node number j of the contact. For example, for REM the component of number r element stiffness matrix referring to vertical displacement of number j node is described by the dependence [4, 8] $CV = 2 \cdot E^B \cdot a \cdot t^B / (b \cdot (1 - \nu_B^2))$ – Fig. 2b.

Task 2.

Two calculation models (B) and (S) have been created from the separated subsystems of the above Task 1.

Task 2A – according to the requirements of the iterative scheme (SI).

Task 2B – according to the requirements of the iterative scheme (SII).

Results of the tests and comparisons have been stated in the following figures. The result of the basis test is represented in Fig. 5a–c by the thick line. For the numerical solution the average error $\Delta^{(i)}$ for the Task 2B related to the exact solution (Task 1B) has been shown on the vertical axis. The analysis refers to the subsoil displacements $\Delta(\mathbf{dZ}_3)$, structure displacements $\Delta(\mathbf{dZ}_2)$ and contact forces $\Delta(\mathbf{dF})$ determined at the contact of the subsystem in numerical model. For the analysed task the error $\Delta^{(i)}$, defined in the consecutive iterative steps was: $\Delta^{(3)} < 45\%$, $\Delta^{(5)} < 15\%$, $\Delta^{(10)} < 1.5\%$, $\Delta^{(20)} < 0.01\%$, where

$$\Delta^{(i)} = \frac{100}{j} \sum_1^j \frac{\left| w_{2B,j}^{(i)} - w_{1B,j} \right|}{|w_{1B,j}|} \tag{12}$$

(j – number of discrete quantities of displacement or forces in the contact).

The repetition of parametric analysis for the task where the height of zone III has been reduced (0,5 of the height for the basis task) did not show any changes in behaviour of the solution under examination.

The (SII) iterative process is quickly convergent: $\chi^{(3)} < 40\%$, $\chi^{(5)} < 10\%$, $\chi^{(10)} < 1\%$, where

$$\chi^{(i)} = \frac{100}{j} \sum_1^j \left| w_{2B,j}^{(i-1)} - w_{2B,j}^{(i)} \right|. \tag{13}$$

Task 2B forms a series of numerical tests (Fig. 5a–c). By introducing changes in the contact layer parameters, the deviation of 2B from the exact solution 1B has been examined. The changed parameter has been defined as $\underline{R_o} = \beta R_o$, where for the basic Task 2B we have $\beta = 1.0$.

The analysed iterative procedure (SII) shows considerable tolerance for error (for $\beta > 1$) in defining the quantities for the contact layer parameters. This feature is extremely important as

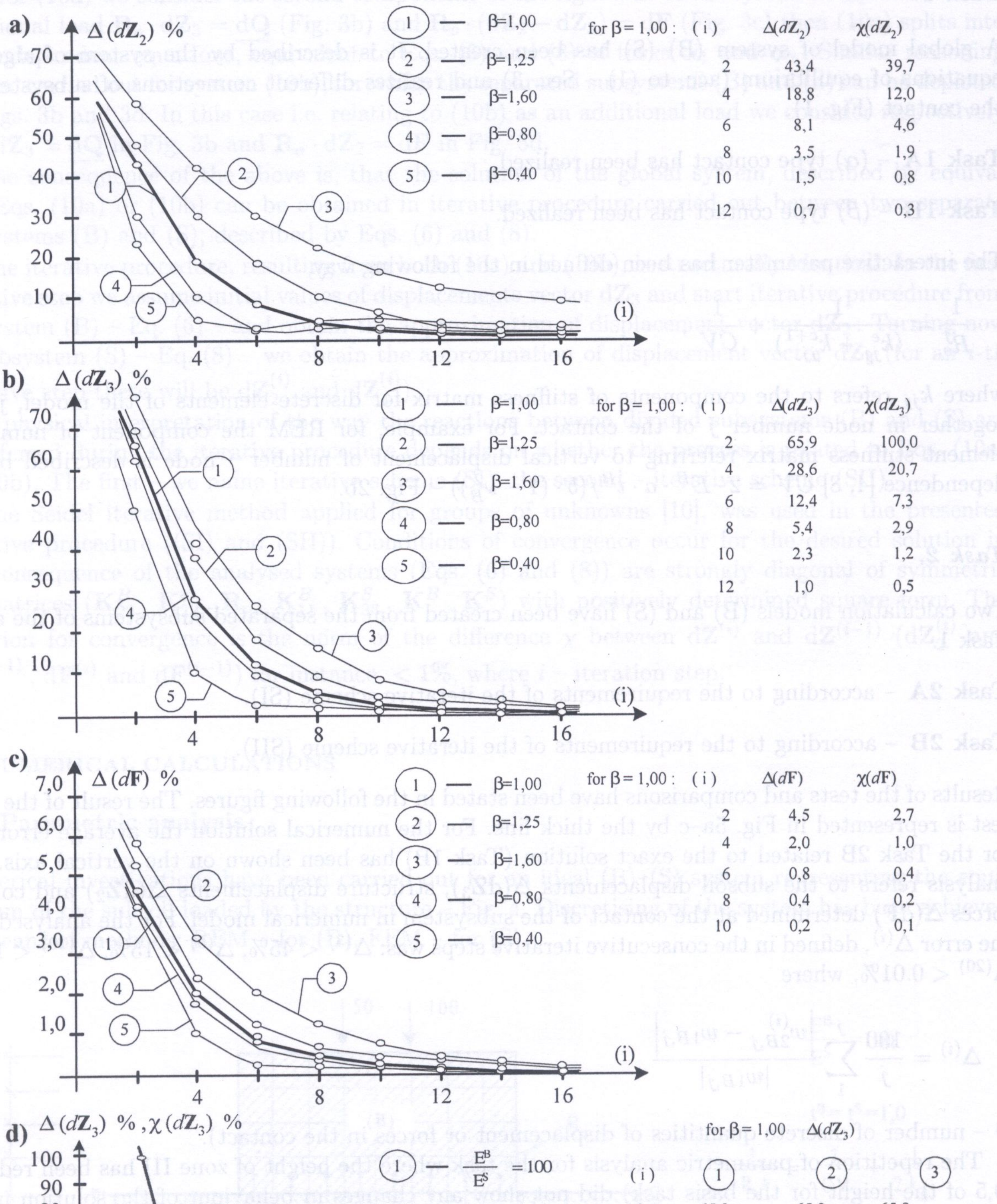
a)
Δ(dZ₂) %
70
60
50
40
30
20
10
4
8
12
16
(i)
1 — β=1,00
2 — β=1,25
3 — β=1,60
4 — β=0,80
5 — β=0,40
for β = 1,00 : (i) Δ(dZ₂) χ(dZ₂)
2 43,4 39,7
4 18,8 12,0
6 8,1 4,6
8 3,5 1,9
10 1,5 0,8
12 0,7 0,3
b)
Δ(dZ₃) %
70
60
50
40
30
20
10
4
8
12
16
(i)
1 — β=1,00
2 — β=1,25
3 — β=1,60
4 — β=0,80
5 — β=0,40
for β = 1,00 : (i) Δ(dZ₃) χ(dZ₃)
2 65,9 100,0
4 28,6 20,7
6 12,4 7,3
8 5,4 2,9
10 2,3 1,2
12 1,0 0,5
c)
Δ(dF) %
7,0
6,0
5,0
4,0
3,0
2,0
1,0
4
8
12
16
(i)
1 — β=1,00
2 — β=1,25
3 — β=1,60
4 — β=0,80
5 — β=0,40
for β = 1,00 : (i) Δ(dF) χ(dF)
2 4,5 2,7
4 2,0 1,0
6 0,8 0,4
8 0,4 0,2
10 0,2 0,1
d)
Δ(dZ₃) %, χ(dZ₃) %
100
90
80
70
60
50
40
30
20
10
4
8
12
16
(i)
1 — Δ(dZ₃)
1 — χ(dZ₃)
1 — E^B/E^S = 100
2 — E^B/E^S = 500
3 — E^B/E^S = 50
for β = 1,00 Δ(dZ₃)
(i) 1 2 3
2 65,9 65,9 65,8
4 26,6 28,7 28,5
6 12,4 12,5 12,3
8 5,4 5,4 5,3
10 2,3 2,3 2,3
12 1,0 1,0 1,0

Fig. 5

it enables us to introduce simplifications while defining the parameters R_o for actual analysis of engineering tasks:

$$\frac{1}{R_o^j} = \frac{1}{k_{jj}^e + k_{jj}^{e+1}} \,. \tag{14}$$

For the proportion of elasticity moduli $E^B/E^S = 50$–500, neglecting the influence subsystem (B) has on R_o leads to the quantity of β: $1 < \beta < 1.01$.

Similar solutions to the above type (Fig. 5a–c, $\beta = 1$) were obtained by introducing a changeable proportion of elasticity moduli $E^B/E^S = 100$, 500, 50 (Fig. 5d).

Figure 6 shows certain results of parametric analysis that referred to iterative scheme (SI). This was carried out similarly to the analysis shown in Fig. 5. While compared with procedure (SII), this procedure (SI) is much more exposed to changes in parameters R_o and it converges much more slowly.

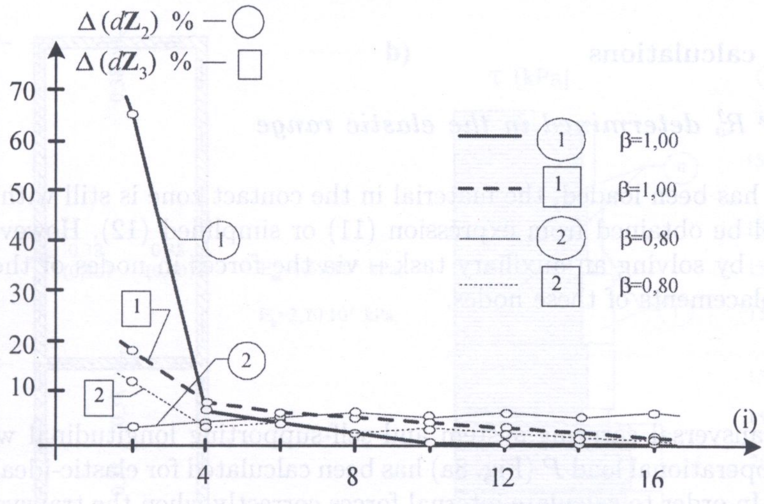

Fig. 6

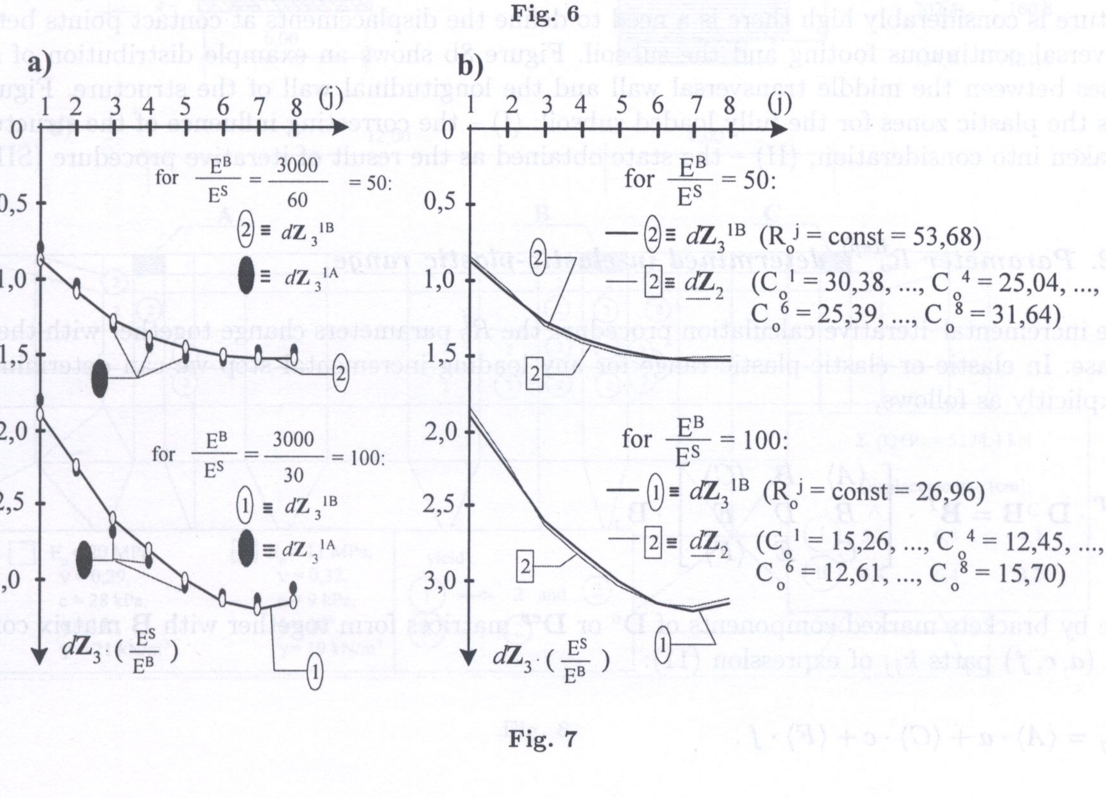

Fig. 7

Discretised displacement functions of the subsystems, obtained for two different ideas of contact modelling have been compared in Fig. 7a. From solution of Task 1B (equivalent with Task 2B) the displacements $\mathrm{d}\mathbf{Z}_3^{1B}$ have been obtained and from solution of Task 1A the displacements $\mathrm{d}\mathbf{Z}_3^{1A}$. Results of calculations carried out for $E^B/E^S = 100$ and $E^B/E^S = 50$ have been shown. In Fig. 7b the subsoil displacements function $\mathrm{d}\mathbf{Z}_3^{1B}$ has been shown together with the "translated" structure displacements function $\underline{\mathrm{d}\mathbf{Z}_2}$ that was obtained as the result on an additional solution of subsystem (B) resting on the new supporting layer which has parameters

$$C_o^j = \left| \frac{(\mathrm{d}Z_2^j - \mathrm{d}Z_3^j)\, R_o^j}{\mathrm{d}Z_3^j} \right| . \tag{15}$$

Parameters C_o defined in this way, express through the set of discrete values, the flexibility of the loaded subsoil, determined by taking into account an actual rigidity of the structure.

4.2. Examples of calculations

4.2.1. Parameter R_o^j determined in the elastic range

If, after the system has been loaded, the material in the contact zone is still within the elastic area, parameters R_o^j shall be obtained from expression (11) or simplified (12). However, R_o^j can also be expressed directly – by solving an auxiliary task – via the forces in nodes of the contact elements caused by unit displacements of these nodes.

Example 1.

The structure of transversal carrying system and self-supporting longitudinal walls, loaded by its dead weight Q and operational load P (Fig. 8a) has been calculated for elastic-ideal plastic Coulomb–Mohr subsoil [1, 2]. In order to calculate internal forces correctly when the transversal rigidity of the structure is considerably high there is a need to define the displacements at contact points between transversal continuous footing and the subsoil. Figure 8b shows an example distribution of shear stresses between the middle transversal wall and the longitudinal wall of the structure. Figure 8c shows the plastic zones for the fully loaded subsoil: (I) – the correcting influence of the structure is not taken into consideration, (II) – the state obtained as the result of iterative procedure (SII).

4.2.2. Parameter $R_o^{j(ep)}$ determined in elastic-plastic range

In the incremental–iterative calculation procedure the R_o^j parameters change together with the load increase. In elastic or elastic-plastic range for any loading incremental step we can determine the R_o^j explicitly as follows,

$$\mathbf{B}^T \cdot \mathbf{D} \cdot \mathbf{B} = \mathbf{B}^T \cdot \begin{bmatrix} \langle A \rangle & B & \langle C \rangle \\ B & D & E \\ C & E & \langle F \rangle \end{bmatrix} \cdot \mathbf{B} ,$$

where by brackets marked components of $\mathbf{D}^e$ or $\mathbf{D}^{ep}$ matrices form together with $\mathbf{B}$ matrix components (a, c, f) parts k_{jj} of expression (11):

$$k_{jj} = \langle A \rangle \cdot a + \langle C \rangle \cdot c + \langle F \rangle \cdot f . \tag{16}$$

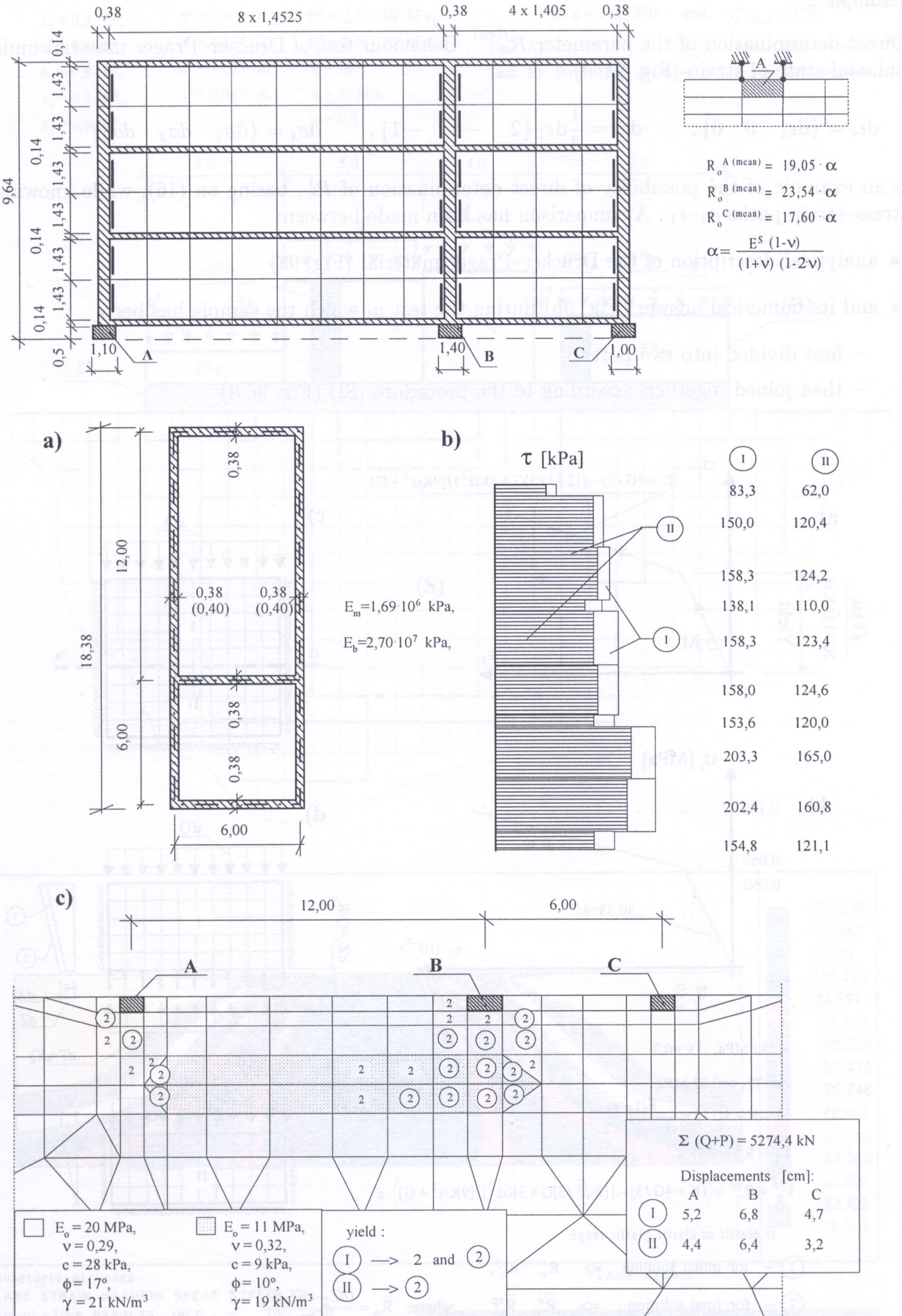
0,38
8 x 1,4525
0,38
4 x 1,405
0,38
9,64
A
B
C
1,10
1,40
1,00
R_o^A (mean) = 19,05 · α
R_o^B (mean) = 23,54 · α
R_o^C (mean) = 17,60 · α
α = E^S (1-ν) / (1+ν) (1-2ν)
a)
b)
12,00
18,38
6,00
0,38 (0,40)
E_m=1,69·10^6 kPa,
E_b=2,70·10^7 kPa,
τ [kPa]
I
II
83,3 62,0
150,0 120,4
158,3 124,2
138,1 110,0
158,3 123,4
158,0 124,6
153,6 120,0
203,3 165,0
202,4 160,8
154,8 121,1
c)
Σ (Q+P) = 5274,4 kN
Displacements [cm]:
A B C
I 5,2 6,8 4,7
II 4,4 6,4 3,2
E_o = 20 MPa, ν = 0,29, c = 28 kPa, ϕ= 17°, γ = 21 kN/m³
E_o = 11 MPa, ν = 0,32, c = 9 kPa, ϕ= 10°, γ= 19 kN/m³
yield :
I —> 2 and 2
II —> 2

Fig. 8

Example 2.

Direct determination of the parameter $R_o^{j(ep)}$. Behaviour test of Drucker–Prager material under an uniaxial state of strain (Fig. 9a–d)

$$d\epsilon_i = \{d\epsilon_1 \quad 0 \quad 0\}, \qquad de_i = \frac{1}{3}d\epsilon_1\{2 \quad -1 \quad -1\}, \qquad d\sigma_i = \{d\sigma_1 \quad d\sigma_2 \quad d\sigma_2\}$$

is an example of the possibility of direct determination of R_o^j, basing on (16), while knowing the stress–strain paths σ_1–ϵ_1. A comparison has been made between:

- analytical description of the Drucker–Prager material (Fig. 9a)
- and its numerical answer (Fig. 9b) during the test in which the sample has been
  - first divided into two parts
  - then joined together, according to the procedure (SI) (Fig. 9c,d)

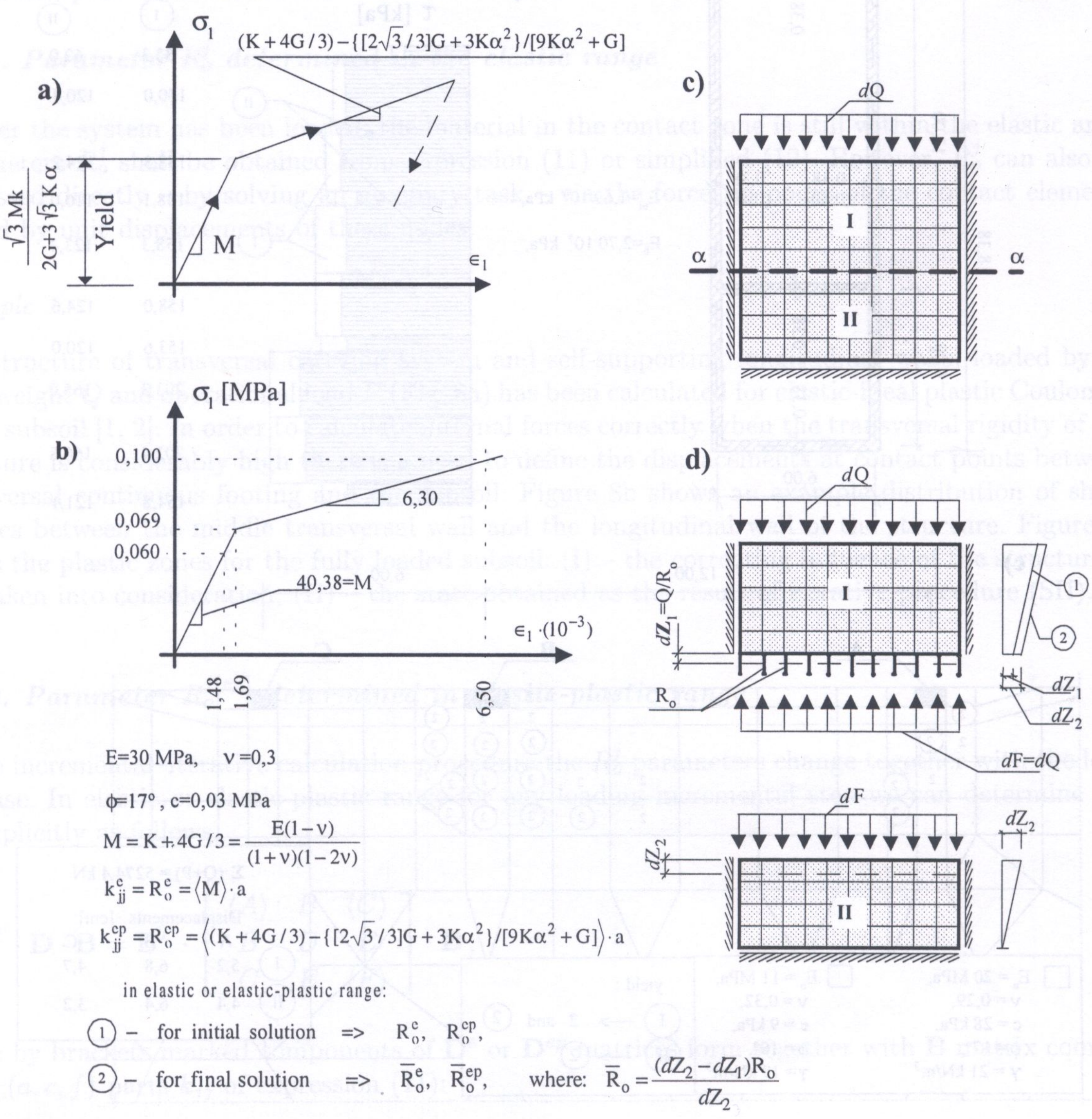

Fig. 9

$a_1 = 0{,}5 \cdot E^B$, $q^1 = 0{,}409 \cdot n$, $E^B = 2{,}53 \cdot 10^7$ kPa, for n = 1 (1) 330 and $\chi^{(n)}_{final} < 1\,\%$

$a_2 = 0{,}4 \cdot E^B$, $q^2 = 1{,}434 \cdot n$, $E^S = 2{,}50 \cdot 10^4$ kPa, Task A) for n= 330: ———

$a_3 = 0{,}8 \cdot E^B$, $q^3 = 1{,}635 \cdot n$, $\phi = 16°$, Task B) for n=270: — — — for n= 330: — — —

$a_4 = 0{,}9 \cdot E^B$, $q^4 = 0{,}449 \cdot n$, $c = 30$ kPa,

$a_5 = 0{,}6 \cdot E^B$ $\nu = 0{,}3$

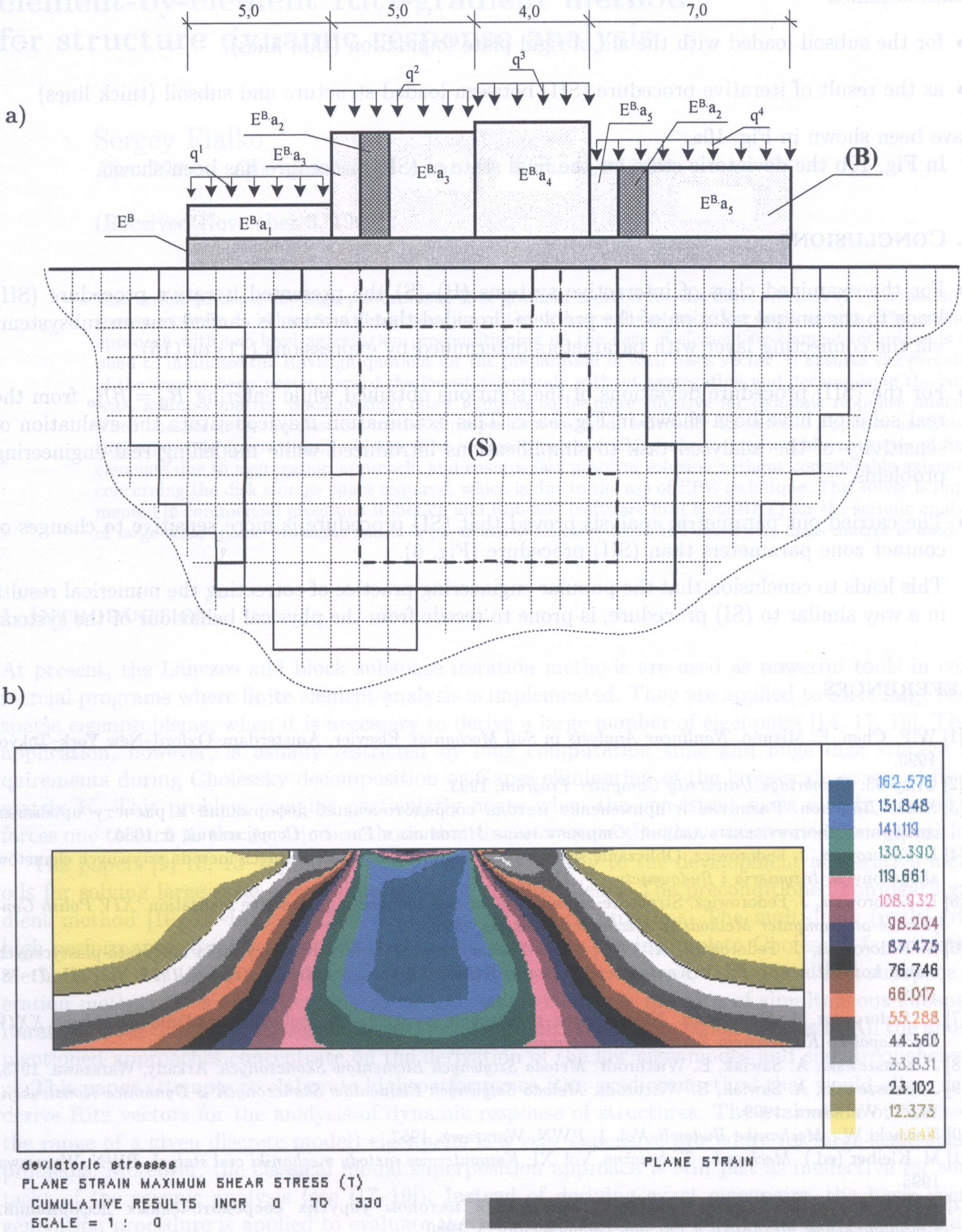

Fig. 10

Example 3.

The structure of strongly diversified rigidity that transferred a load via the foundation plate onto the subsoil has been described in state 2D – Fig. 10. The uniaxial state of strain for subsoil under the foundation plate in the central area of the contact zone can be assumed for this task. As a result of plastic zones, below the foundation, R_o^e in the contact central area receive new values R_o^{ep}. Plastic zones obtained

- for the subsoil loaded with the aid of rigid plate foundation (thin lines),
- as the result of iterative procedure (SII) between loaded structure and subsoil (thick lines)

have been shown in Fig. 10a.

In Fig. 10b the deviatoric state for the final state of (SII) procedure has been shown.

5. Conclusions

- For the examined class of interactive systems (B)–(S) the presented iterative procedure (SII) leads to the unique solution of the problem provided that iteration is carried out on subsystems via the connecting layer with parameters determined by expressions (11) or (16).
- For the (SII) procedure deviations of the solutions obtained while entering $\underline{R_o} = \beta R_o$ from the real solution have been shown in Fig. 5a–c. This examination may constitute the evaluation of sensitivity of the analysed task to simplifications introduced while modelling real engineering problems.
- The carried out parametric analysis proved that (SI) procedure is more sensitive to changes of contact zone parameters than (SII) procedure (Fig. 6).

  This leads to conclusion that the popular engineering practice of correcting the numerical results in a way similar to (SI) procedure, is prone to recede from the physical behaviour of the system.

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