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Minimization of the energy in metal forming process of the cylindric shape tool through punch shape changes

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In this paper the sensitivity based optimization problem is considered. The shape of the first of two contacting bodies is optimized on the basis of sensitivities calculated for the second body i.e. workpiece. The finite element simulation of sheet metal forming process and direct differentiation method of sensitivity analysis is used. Some energy measure of deforming sheet metal is chosen as a cost functional. Its gradients with respect to the tool (punch) shape parameters are evaluated. Tool shape optimization based on 'exact' sensitivity results is performed. Calculated sensitivities with respect to the tool shape parameters are the input for the optimization algorithm. The cost functional is minimized, yielding the optimal shape of the tool.

The theory is illustrated by numerical example. Shape optimization of the compressor cover produced in one of sheet stamping factories is performed.

Keywords: sheet metal forming, sensitivity, optimization, tools design

1. INTRODUCTION

In the last years the sensitivity analysis has been found the most effective (exact, reliable and computationally efficient) tool in optimal design. The subject of this paper is 'exact sensitivity' -based optimization of transient problems of sheet metal forming.

Finite element simulation of industrial sheet metal forming processes is now widely used in design. Tool shape optimization is still performed, however, by classical trial and error design procedures and appropriate adjustments in stamping factories.

Classical mathematical methods of optimization based on sensitivity analysis have been developed and mainly applied in structural engineering. A short state of art in this field and interesting sensitivity based shape optimization of heat conduction systems are presented in [1]. Application of these methods in the area of metal forming was limited to stationary problems [3].

The analytical (exact) calculations of the problem functions sensitivities with respect to the design parameters are relatively complex. This is the reason that the majority of known solutions are based on the finite difference sensitivity techniques [6, 7].

Possible gains can be achieved during the process by minimization of forming energy consumption, tool wear, number of operations, friction forces or maximization of admissible tool velocity.

Also the better product quality can be expected with proper surface characteristics, without wrinkling or other geometrical defects. Uniform blank thickness, strain and stress distribution, smaller residual stresses can be obtained.

In sheet metal forming various and sometimes contradictory criteria must be satisfied. So, many different objective functions are necessary in order to obtain proper quality and product cost. Very often traditional trial and error procedures should be used as complementary to minimization process.

Possible design variables are initial sheet thickness, blankholder forces, drawbead profiles and location, friction law coefficients and parameters defining tool shape.

In this paper the minimization of some measure of the global dissipation energy is performed

$$\Psi = \int_\Omega \int_0^{t^e} (\boldsymbol{\sigma}^T \dot{\boldsymbol{\varepsilon}}) \, d\Omega dt. \tag{1}$$

The configuration of the body Ω varies in time t. t^e is the time of the end of deformation process. The stress and strain rate fields are denoted by $\boldsymbol{\sigma}$ and $\dot{\boldsymbol{\varepsilon}}$ respectively.

The expression (1) must be calculated for the whole deformation process by accumulation of all incremental values.

Sometimes we must admit more than one objective function to be considered simultaneously, so many functions can be used at the same time for the multicriterion optimization.

After calculation of sensitivity gradients $\frac{d\Psi}{dh_i}$ of the objective function $\Psi(h_i)$ with respect to the design parameters vector $\mathbf{h} = \{h_1, \ldots, h_n\}$ at the end of the process (which is crucial for time dependent plasticity problems) the minimization of the function (1) with respect to the design parameter $\mathbf{h}$ must be performed. Next the design variable must be updated and all calculations are followed by the next step of the iterative optimization procedure. It means that it is necessary to repeat the calculations of the whole deformation process in order to get the new values of the function Ψ, its gradients and problem constraints. So the optimization procedure is relatively expensive as the number of optimization iterations is equal to the number of numerical simulation of the sheet metal forming process, including necessary sensitivity calculations with respect to the basic design variables during all these simulations.

In this paper exact sensitivities are obtained by direct differentiation of all functions entering the problem. Sensitivity calculations are included into the nonlinear code for finite element simulation of sheet metal forming.

As an example the axisymmetrical part of the compressor cover produced in one of sheet stamping factories is considered. For the time being only total energy consumption is minimized but any other objective function can easily be included into the algorithm.

The design variable vector $\mathbf{h}$ is assumed to depend on the tool shape. Tool master nodes with coordinates $\mathbf{X}$, which participate in design variation are specified. The proper choice of tool surface representation and design variable selection are shortly discussed in Section 4.1.

2. Basic formulation of sheet metal forming

The flow approach to metal forming problems with the rigid-viscoplastic material model is used as the basis in this paper [8, 13].

The virtual work expression (equilibrium equation in the weak form) to be solved reads

$$\int_\Omega \boldsymbol{\sigma}^{\mathrm{T}} \, \delta\boldsymbol{\varepsilon} \, d\Omega = \int_\Omega \mathbf{f}^T \delta\mathbf{v} \, d\Omega + \int_{\partial\Omega_t} \mathbf{t}^T \delta\mathbf{v} \, d(\partial\Omega) \tag{2}$$

where $\mathbf{v}$ denotes the velocity field, $\mathbf{f}$ is the distributed volumetric load, $\mathbf{t}$ is the traction on the boundary and integrals are taken over the actual body volume element $d\Omega$ or its surface element $d(\partial\Omega)$, respectively.

Stresses are calculated from the constitutive equations

$$\sigma_{ij} = s_{ij} + p\delta_{ij}, \tag{3}$$

$$s_{ij} = 2\mu^* \dot{\varepsilon}_{ij}, \tag{4}$$

where s_{ij} is the Cauchy stress deviator, p denotes the mean stress and δ_{ij} is the Kronecker delta. The constitutive function μ^* is defined in the flow problem as [13]

$$\mu^* = \frac{\bar{\sigma}}{3\dot{\bar{\varepsilon}}} = \frac{\sigma_y + \left(\frac{\dot{\bar{\varepsilon}}}{\gamma}\right)^{\frac{1}{n}}}{3\dot{\bar{\varepsilon}}}. \tag{5}$$

Here, σ_y is the current static uniaxial tensile yield stress of the material, $\bar{\sigma}$ is the equivalent stress

$$\bar{\sigma} = \left(\frac{3}{2}\, s_{ij} s_{ij}\right)^{\frac{1}{2}}, \tag{6}$$

$\dot{\bar{\varepsilon}}$ is the effective inelastic strain rate,

$$\dot{\bar{\varepsilon}} = \left(\frac{2}{3}\, \dot{\varepsilon}_{ij} \dot{\varepsilon}_{ij}\right)^{\frac{1}{2}}, \tag{7}$$

and γ, n are physical parameters of the rigid-viscoplastic model used.

For pure plasticity assumed later in this paper we set $\gamma \to \infty$ and Eq. (5) yields simply

$$\mu^* = \frac{\sigma_y}{3\dot{\bar{\varepsilon}}}. \tag{8}$$

For strain hardening plastic materials the yield limit σ_y is a function of the effective inelastic strain $\bar{\varepsilon}$,

$$\sigma_y = \sigma_y(\bar{\varepsilon}) \tag{9}$$

where $\bar{\varepsilon}$ has to be computed as the time integral of $\dot{\bar{\varepsilon}}$.

The analogy between plastic flow and incompressible elasticity allows the treatment of pure plastic flow problem using a numerical code developed for linear elasticity. The incompressibility condition must be satisfied.

In sheet metal forming the shell theory as the simplification of 3D problems is used and large plastic deformations of thin sheets of metal are treated as elastic incompressible shell deformations. Plane stress assumptions are used in shell theory so the incompressibility can be easily achieved by adjusting the shell thickness during consecutive steps of the solution to ensure the constant volume.

Velocities $\mathbf{v}$ of Eq. (2) are approximated by nodal velocities vector $\dot{\mathbf{q}}$, $\mathbf{v} = \mathbf{N} \cdot \dot{\mathbf{q}}$, where $\mathbf{N}$ denotes matrix of standard shape functions. After spatial finite element discretization the 'secant' stiffness matrix $\mathbf{K}$ depends on the nodal velocities $\dot{\mathbf{q}}$ through the parameter μ^* so that an iterative process is needed to find the solution vector $\dot{\mathbf{q}}$

$$\mathbf{K}^{(i)} \dot{\mathbf{q}}^{(i+1)} = \mathbf{Q}, \qquad i = 0, 1, 2, \ldots, \tag{10}$$

in which $\mathbf{Q}$ denotes the external force and

$$\mathbf{K}^{(i)} = \mathbf{K}[\mu^*(\dot{\mathbf{q}}^{(i)})]. \tag{11}$$

The element contributions to the stiffness matrix $\mathbf{K}$ and the nodal forces vector $\mathbf{Q}$ are

$$\mathbf{K}_{(ij)} = 2\pi \int_l \mathbf{B}_i^T \mathbf{D}\, \mathbf{B}_j\, r ds, \tag{12}$$

$$\mathbf{Q}_{(i)} \;= 2\pi \int_l \mathbf{N}_i \mathbf{t}\, r ds + 2\pi r_i \mathbf{p}_i\,, \tag{13}$$

where l is the element length, r the radial distance from the symmetry axis and $\mathbf{t}$ and $\mathbf{p}$ surface and point load vectors, respectively. Details of the generalized strain rate–velocity relationships for the axisymmetric viscous shell are given in Appendix 1 and in [8].

Please, note that in this approach $\mathbf{K}^{(i)}$ is not the function of $\mathbf{q}$ because $\mathbf{q}$ is not the main unknown. $\mathbf{q}$ is calculated for a rigid plastic material as the product of velocity $\dot{\mathbf{q}}$ and pseudotime increment.

Using the Newton–Raphson scheme the i-th residual is defined as

$$\mathbf{R}^{(i)} = \mathbf{Q} - \mathbf{K}^{(i)} \dot{\mathbf{q}}^{(i)} \tag{14}$$

while the iterative correction $\delta\dot{\mathbf{q}}^{(i+1)}$ such that

$$\dot{\mathbf{q}}^{(i+1)} = \dot{\mathbf{q}}^{(i)} + \delta\dot{\mathbf{q}}^{(i+1)} \qquad i = 0, 1, 2, \ldots, \tag{15}$$

is computed from

$$\mathbf{K}_T^{(i)}\,\delta\dot{\mathbf{q}}^{(i+1)} = \mathbf{R}^{(i)} \tag{16}$$

where

$$\mathbf{K}_T = \mathbf{K} + \frac{\partial \mathbf{K}}{\partial \mu^*}\frac{\partial \mu^*}{\partial \dot{\mathbf{q}}}\dot{\mathbf{q}} = -\frac{\partial \mathbf{R}^{(i)}}{\partial \dot{\mathbf{q}}} \tag{17}$$

is the tangent stiffness matrix.

3. Tool shape sensitivity in sheet metal forming

For the design of the new sheet metal forming process it is useful to know the sensitivities of many different functions describing stress, strain, plastic strain, thickness distribution inside deformed blank with respect to the shape parameters of the tools (punch, die, blankholder).

If the shape parameter changes are not large, the first gradient of the function with respect to shape parameter can be treated as the proper sensitivity measure.

In [11] direct differentiation method was used in order to calculate thickness sensitivity with respect to friction. The extension of this algorithm to the shape sensitivity is based on domain parametrization (control volumes) approach (DPA) [3].

The response functional (1) must be mapped to the reference configuration, $\mathbf{x} = \mathbf{x}(\xi, \mathbf{h})$ using the determinant J of the Jacobian matrix

$$(\mathbf{J})_{ij} = \frac{\partial \mathbf{x}_i}{\partial \xi_j}. \tag{18}$$

Isoparametric element concept, typical for many standard finite element codes was used in this mapping.

Equation (2) in the reference configuration is as follows,

$$\int_{\Omega_0} (\boldsymbol{\sigma}^T\,\delta\dot{\boldsymbol{\varepsilon}} - \mathbf{f}^T\,\delta\mathbf{v})\,J\,\mathrm{d}\Omega_0 - \int_{\partial\Omega_0\mathbf{t}} \mathbf{t}^T\,\delta\mathbf{v}\,\partial J\,\mathrm{d}(\partial\Omega_0) = 0, \tag{19}$$

where ∂J defines the surface transformation into the reference configuration, $\partial\mathbf{J} = \mathbf{J}|\mathbf{J}^{-T}\mathbf{n}|$ and $\mathbf{n}$ is the normal vector field in the reference configuration.

3.1. Sensitivity of velocity fields

The gradients of velocity field with respect to shape parameter $\mathbf{h}$ can be calculated after linearization of (19) as follows

$$\begin{aligned}
\mathbf{K}_T\frac{\mathrm{d}\dot{\mathbf{q}}}{\mathrm{d}\mathbf{h}} = &-\int_{\Omega_0}\left(2\mathbf{B}^T\left(\frac{\mathrm{d}\mu^*}{\mathrm{d}\mathbf{h}}\right)\mathbf{B} + \mu^*\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}\mathbf{h}}\,\dot{\mathbf{q}}\,J\,\mathrm{d}\Omega_0\right) \\
&-\int_{\Omega_0} 2\mu^*\left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}\mathbf{h}}\right)^T\mathbf{B}\dot{\mathbf{q}}\,J\,\mathrm{d}\Omega_0 - \int_{\Omega_0} 2\mu^*\,\mathbf{B}^T\mathbf{B}\dot{\mathbf{q}}\,\frac{\mathrm{d}J}{\mathrm{d}\mathbf{h}}\,\mathrm{d}\Omega_0 \\
&-\int_{\Omega_0}\left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}\mathbf{h}}\right)^T\mathbf{pI}\,J\,\mathrm{d}\Omega_0 - \int_{\Omega_0}\mathbf{B}^T\mathbf{pI}\,\frac{\mathrm{d}\,\partial J}{\mathrm{d}\mathbf{h}}\,\mathrm{d}\Omega_0,
\end{aligned} \tag{20}$$

where $\dot{\mathbf{q}}$ is a typical nodal velocity affected by the perturbation of the design variable $\mathbf{h}$, $\mathbf{B}$ is the velocity–strain rate matrix, J is the determinant of the Jacobian matrix, μ^* is the viscosity. The matrix $\mathbf{B}$ and its derivative with respect to the one of two independent design parameter sets selected as described in Section 4.1 are given in Appendix 1.

The effect of the pressure field $\mathbf{p}$ on velocity can be neglected in plasticity, so the last two terms in Eq. (20) vanish.

The right hand side of the above expression was calculated by direct differentiation (i.e. without finite difference scheme) coupled with control volume approach which is relatively easy to implement because of its full analogy to isoparametric element concept. Errors typical for semianalytical finite difference methods are avoided. We can observe that on the left hand side of Eqs. (16) and (20), for both fundamental and sensitivity problems the same stiffness matrix appears. This fact makes the algorithm efficiency much higher.

The sensitivity of viscosity μ^* with respect to the design parameter can be calculated using the chain rule of differentiation, thus

$$\frac{\mathrm{d}\mu^*}{\mathrm{d}\mathbf{h}_i} = \frac{\partial \mu^*}{\partial \dot{\mathbf{q}}} \frac{\mathrm{d}\dot{\mathbf{q}}}{\mathrm{d}h_i} \tag{21}$$

In fact the viscosity does not depend explicitly on displacements $\mathbf{q} = \dot{\mathbf{q}}\Delta t$,

$$\frac{\partial \mu^*}{\partial \dot{\mathbf{q}}^{(i)}} = \frac{\partial \mu^*}{\partial \dot{\bar{\varepsilon}}} \frac{\partial \dot{\bar{\varepsilon}}}{\partial \dot{\boldsymbol{\varepsilon}}} \frac{\mathrm{d}\dot{\boldsymbol{\varepsilon}}}{\mathrm{d}\dot{\mathbf{q}}^{(i)}}\,, \tag{22}$$

where from Eq. (5) we have

$$\frac{\partial \mu^*}{\partial \dot{\bar{\varepsilon}}} = \frac{\left[-\sigma_y + \left(\frac{1}{n} - 1\right)\left(\frac{\dot{\bar{\varepsilon}}}{\gamma}\right)^{\frac{1}{n}}\right]}{3\dot{\bar{\varepsilon}}^2}\,, \tag{23}$$

$$\left(\frac{\partial \dot{\bar{\varepsilon}}}{\partial \dot{\boldsymbol{\varepsilon}}}\right)_{1\times 6} = \frac{2\dot{\boldsymbol{\varepsilon}}^{\mathrm{T}}}{3\dot{\bar{\varepsilon}}} = \frac{2\mathbf{B}^{\mathrm{T}}\dot{\mathbf{q}}^{\mathrm{T}}}{3\dot{\bar{\varepsilon}}}\,. \tag{24}$$

$\mathbf{B} = \mathbf{B}(\dot{\mathbf{q}})$, so

$$\left(\frac{\partial \dot{\boldsymbol{\varepsilon}}}{\partial \dot{\mathbf{q}}}\right)_{6\times 5} = \frac{\mathrm{d}(\mathbf{B}\dot{\mathbf{q}})}{\mathrm{d}\dot{\mathbf{q}}} = \mathbf{B}(\dot{\mathbf{q}}) + \frac{\mathrm{d}\mathbf{B}}{\mathrm{d}\dot{\mathbf{q}}}\,\dot{\mathbf{q}}. \tag{25}$$

3.2. Sensitivities of the energy function with respect to the design parameters

Total strains are calculated incrementally during deformation process,

$$\boldsymbol{\varepsilon}^{t+\Delta t} = \boldsymbol{\varepsilon}^t + \Delta\boldsymbol{\varepsilon}^t \tag{26}$$

Strain increment $\Delta\boldsymbol{\varepsilon}^t$ depends on strain rate and time increment

$$\Delta\boldsymbol{\varepsilon}^t = \int_t^{t+\Delta t} \dot{\boldsymbol{\varepsilon}}^t\, \mathrm{d}t. \tag{27}$$

For typical stamping process and for relatively small time increments Eq. (27) can be rewritten as follows,

$$\Delta\boldsymbol{\varepsilon}^t = \dot{\boldsymbol{\varepsilon}}^t\, \Delta t. \tag{28}$$

The energy dissipation given by formula (1) has to be calculated incrementally,

$$\Psi^{t+\Delta t} = \Psi^t + \Delta\Psi^t, \tag{29}$$

where the energy dissipation increment $\Delta\Psi^t$ equals

$$\Delta\Psi^t = \sum_{e=1,\dots,E} \int_{\Omega_e} (\boldsymbol{\sigma}^t)^T \Delta\boldsymbol{\varepsilon}^t \, d\Omega_e, \tag{30}$$

where E is the number of finite elements in the system.

In practical applications some energy measure can be minimized yielding optimal design close to exact solution. In this paper such energy measure was calculated as the function of stress and strain fields calculated at the end of deformation process, $t = t^e$,

$$\Psi^* = \sum_{e=1,\dots,E} \int_{\Omega_e} (\boldsymbol{\sigma}^{t^e})^T \boldsymbol{\varepsilon}^{t^e} \, d\Omega_e. \tag{31}$$

Let us consider the energy dissipation measure given by formula (31). Its derivation with respect to design variable h_j results in

$$\frac{d\Psi^*}{dh_j} = \left(\frac{(d\boldsymbol{\sigma})^T}{dh_j} \boldsymbol{\varepsilon} + \boldsymbol{\sigma}^T \frac{d\boldsymbol{\varepsilon}}{dh_j} \right) \tag{32}$$

where, to simplify notation, the index t^e is dropped in all symbols related to the end of deformation process.

Strains are calculated as product of strain rates times pseudo-time increment Δt, so strain sensitivity with respect to shape parameter h_j equals

$$\frac{d\boldsymbol{\varepsilon}}{dh_j} = \frac{d\dot{\boldsymbol{\varepsilon}}}{dh_j} \Delta t. \tag{33}$$

In case of rigid-plastic material model t is not the real time — the absence of viscosity allows to treat the time just as the integration parameter in nonlinear equations.

Strain rate derivative with respect to design variable is equal to

$$\frac{d\dot{\boldsymbol{\varepsilon}}^{(i+1)}}{dh_j} = \frac{d\mathbf{B}}{dh_j} \dot{\mathbf{q}}^{(i+1)} + \mathbf{B} \frac{d\dot{\mathbf{q}}^{(i+1)}}{dh_j} \tag{34}$$

where i denotes the step counter.

The stress $\boldsymbol{\sigma}$ in flow approach depends on strain rate and on material tangent matrix $\mathbf{D}$ only,

$$\boldsymbol{\sigma} = \mathbf{D}(\mathbf{h}, \dot{\boldsymbol{\varepsilon}}) \dot{\boldsymbol{\varepsilon}}. \tag{35}$$

The stress sensitivity with respect to shape parameter h_j equals

$$\frac{d\boldsymbol{\sigma}}{dh_j} = \frac{d\mathbf{D}}{dh_j} \dot{\boldsymbol{\varepsilon}} + \mathbf{D} \frac{d\dot{\boldsymbol{\varepsilon}}}{dh_j} \tag{36}$$

where $\mathbf{D}$ is material tangent matrix,

$$\frac{d\mathbf{D}}{dh_j} \dot{\boldsymbol{\varepsilon}} = \frac{\partial \mathbf{D}}{\partial \dot{\boldsymbol{\varepsilon}}} \frac{d\dot{\boldsymbol{\varepsilon}}}{dh_j} \dot{\boldsymbol{\varepsilon}} + \frac{\partial \mathbf{D}}{\partial h_j} \dot{\boldsymbol{\varepsilon}}, \tag{37}$$

$$\frac{d\boldsymbol{\sigma}}{dh_j} = \left[\frac{\partial \mathbf{D}}{\partial \dot{\boldsymbol{\varepsilon}}} \dot{\boldsymbol{\varepsilon}} + \mathbf{D} \right] \left(\frac{d\mathbf{B}}{dh_j} \dot{\mathbf{q}}^{(i+1)} + \mathbf{B} \frac{d\dot{\mathbf{q}}^{(i)}}{dh_j} \right) + \frac{\partial \mathbf{D}}{\partial h_j} \dot{\boldsymbol{\varepsilon}}. \tag{38}$$

Exact calculation of $\frac{d\mathbf{D}}{dh_j}$ involves thickness differentiation with respect to h_j which can be performed similary as it was done in [11] where thickness sensitivity with respect to friction was established.

Some authors, [10], have proposed some simplifications for above calculations. They neglect **D** matrix derivatives in Eq. (38) assuming they are small,

$$\frac{\mathrm{d}\boldsymbol{\sigma}^{(i+1)}}{\mathrm{d}h_j} = \frac{\mathrm{d}\boldsymbol{\sigma}}{\mathrm{d}\dot{\boldsymbol{\varepsilon}}}\frac{\mathrm{d}\dot{\boldsymbol{\varepsilon}}}{\mathrm{d}h_j} = \mathbf{D}\left(\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}h_j}\dot{\mathbf{q}}^{(i+1)} + \mathbf{B}\frac{\mathrm{d}\dot{\mathbf{q}}^{(i)}}{\mathrm{d}h_j}\right) \tag{39}$$

Sensitivities given by Eq. (32) can be used directly in optimization algorithm.

3.3. Design elements and master nodes

The derivative of any function, for instance $\mathbf{B}(x)$ with respect to the design variable can be calculated using the chain rule of differentiation, where simplified dependency of any node coordinate on the master node coordinate can be assumed for a specific problem at hand.

For the tool shape sensitivity such dependency must be established for active sets of nodes with coordinates x being in full contact with the part of the tool, where master nodes with coordinates X are defined. For these active nodes the derivative of $\mathbf{B}(x)$ with respect to the design variable h_k is equal to

$$\frac{\partial \mathbf{B}}{\partial h_k} = \frac{\partial \mathbf{B}}{\partial x_i}\frac{\partial x_i}{\partial X_j}\frac{\mathrm{d}X_j}{\mathrm{d}h_k}\,. \tag{40}$$

If a specific master node coordinate is chosen as the design parameter ($h = X_3$), as was assumed for one of design parameters set in numerical example presented in Section 5, the above equation simplifies to

$$\frac{\partial \mathbf{B}}{\partial X_3} = \frac{\partial \mathbf{B}}{\partial x_i}\frac{\mathrm{d}x_i}{\mathrm{d}X_3}\,. \tag{41}$$

4. Numerical example

An axisymmetric part of a compressor cover produced by the stamping factory HYDRAL in Wrocław (Poland), shown as a detail marked by letter F in Fig. 1 is considered. The diameter of this axisymmetric detail is denoted by ϕ and R denotes punch rounding. Some of dimensions, for instance the diameter $\phi = 13$ mm or detail height of 8 mm are fixed (can not be changed during optimization). Some dimensions defining an upper part of this detail are free to design. The deep drawing of this detail was impossible at the beginning of a production process, due to the localization effects

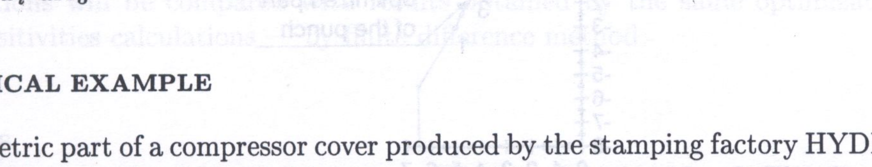

Fig. 1. Compressor cover produced by stamping factory Hydral (dimensions are given in [mm])

(failures near the bottom part of the workpiece or excessive thinning of the blank sheet). Therefore, some optimization of tool shape by simple numerical simulation and trial and error procedures were undertaken first [2] and checked next against optimal solutions obtained with finite difference sensitivities.

Now we try to do it in a more advanced manner, by exact shape sensitivity calculations and optimization by using the sequential quadratic programming method [9].

4.1. Design variable selection

The possible punch geometry representation could be expressed in terms of line segments or spline functions but the relatively large number of design variables would be needed for smooth punch surface description in the first case and wavy contour could be expected in the other case [7].

So the best choice is performed when optimized part of the punch shape is described by polynomials. In this paper the simple parabolas are taken with some design constraints imposed on their coefficients and limiting points 1 and 2 (see Fig. 2). Such boundary representation assures smoothness of the punch surface and gives possibility to reduce the number of design variables just to 3 polynomial coefficients, or even to one radial coordinate of the parabola inner point 3.

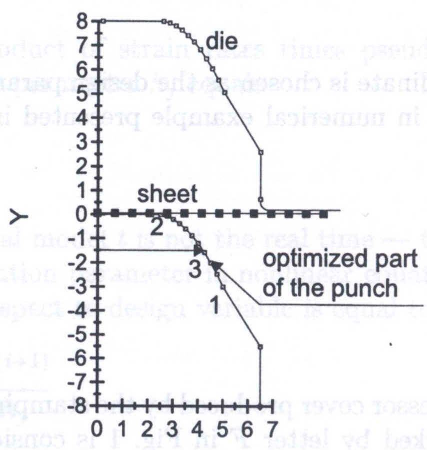

Fig. 2. Initial tools shape. Optimized part of the punch is between points 1 and 2

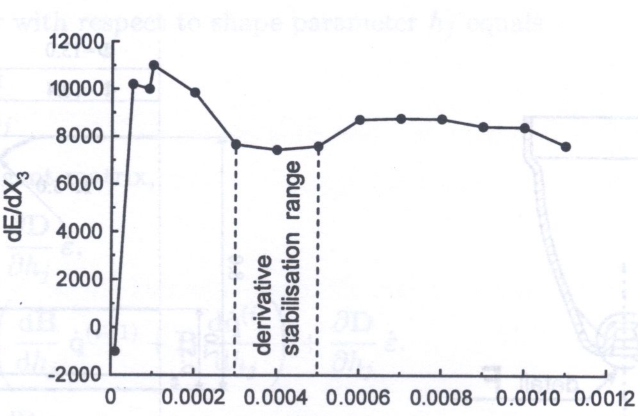

Fig. 3. Design variable perturbation choise for finite difference sensitivity calculations

The shape sensitivities are calculated analytically by direct differentiation method (DDM) with respect to two independent design variables sets.

In the first case vector **h** contains only three components namely, the parabola parameters a, b, and c. Each consecutive approximation of this vector defines a new punch shape.

In the second case only one radial coordinate of the parabola's inner point 3 is chosen as the design variable $\mathbf{h} = X_3$.

The solutions are compared with results obtained by the same optimization method but different sensitivity calculations that is by the finite difference method.

In order to calculate the sensitivities of energy function by finite difference method (FDM), the proper perturbation of design variable must be established. In Fig. 3 the results of such analysis for different perturbations of radial coordinate X_3 of the parabola inner point 3 is shown. The value of 0.0004 was chosen as some approximation of the optimal perturbation.

4.2. Summary of the algorithm in this specific example

1. Start optimization program with initial parabolic part of the punch described by parabola $\mathbf{h} = (a_o, b_o, c_o)$ or $\mathbf{h} = X_3$.
2. Calculate energy measure and its gradients with respect to the vector **h**.
3. Minimize energy by calculating its new value and new parabolic shape **h**.
4. Compute new punch shape and repeat step 2 and 3.
5. End of calculations if energy value does not differ from the previous value more than assumed tolerance in two consecutive steps.

The solutions will be compared with results obtained by the same optimization method but different sensitivities calculations — by finite difference method.

4.3. Results

The optimization path in the design space obtained with sequential programming algorithm described shortly in the paper is shown in Fig. 4. The minimal energy measure of 478252.1 Nmm corresponds to the design variable value of $X_3 = 3.9892$.

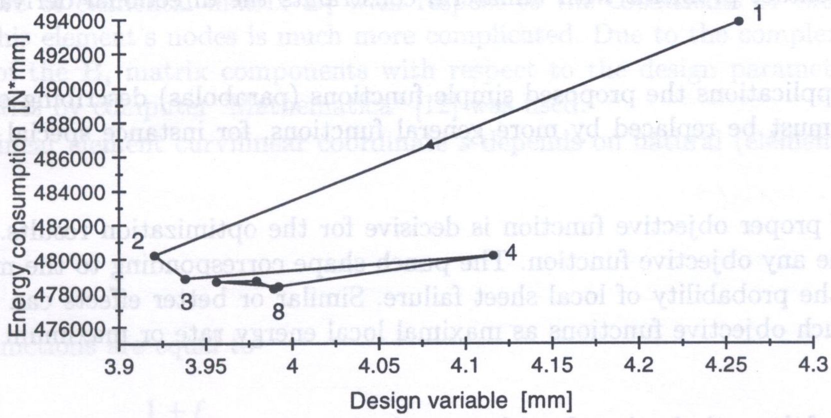

Fig. 4. Optimisation path in the design space with sequential programming algorithm

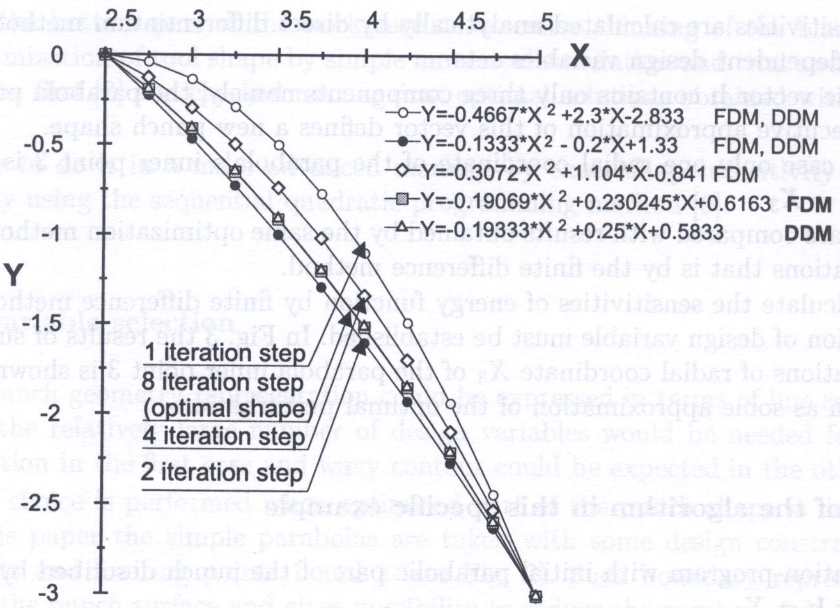

Fig. 5. Hydral test. Intermediate and optimal shapes of the optimized part of the punch

This value defines the parabola $y = -0.19069x^2 + 0.2302x + 0.616$ which describes optimal designed shape of a punch part, see Fig. 5., result by direct differentiation method (DDM).

Almost the same parabola was obtained when sensitivities were calculated by the finite difference method (FDM), as is shown in Fig. 5. Also the shape obtained with the second set of design parameters, i.e. with parabola inner point variation was almost identical. However, the calculations were about two times longer in the latter case.

The whole analysis was completed by some trial and error punch travel and die shape adjustment.

5. Conclusions

1. The parameter and shape optimization algorithm applied in practice may substantially reduce production costs in industrial sheet metal forming.
2. Sensitivity gradient evaluation based on strict analytical solutions are the most difficult task which has to be done in order to solve specific problems at hand. For effective optimization of frictional contact problems with unilateral constraints the directional derivatives should be considered.
3. In practical applications the proposed simple functions (parabolas) describing optimized shape of tool parts must be replaced by more general functions, for instance special polynomials or splines.
4. The choice of proper objective function is decisive for the optimization results. It is relatively easy to include any objective function. The punch shape corresponding to the minimal dissipation reduces the probability of local sheet failure. Similar or better effects can be achieved by introducing such objective functions as maximal local energy rate or maximum effective strain rate.
5. The selection of the optimization algorithm may be crucial for the effectiveness of the procedure. The tests presented using the sequential quadratic programming method shows its dependence of efficiency on the class of the objective functions.

APPENDIX 1

Strain rate to velocity matrix $\mathbf{B}_i$:

$$\mathbf{B}_i = \begin{bmatrix} \cos\phi\frac{\partial N_i}{\partial s} & \sin\phi\frac{\partial N_i}{\partial s} & 0 \\ \frac{N_i}{r} & 0 & 0 \\ 0 & 0 & -\frac{\partial N_i}{\partial s} \\ 0 & 0 & \frac{-N_i\cos\phi}{r} \\ -\sin\phi\frac{\partial N_i}{\partial s} & \cos\phi\frac{\partial N_i}{\partial s} & -N_i \end{bmatrix} \tag{42}$$

The derivative of this matrix with respect to the two-dimensional, axisymmetrical element node radius r_i, $i=1, 2$, see Fig. 6, is equal

$$\frac{\partial \mathbf{B}_i}{\partial r_i} = \begin{bmatrix} \sin\phi\frac{\partial N_i}{\partial s}\frac{\mathrm{d}\phi}{\mathrm{d}r} & \cos\phi\frac{\partial N_i}{\partial s}\frac{\mathrm{d}\phi}{\mathrm{d}r} & 0 \\ \frac{-N_i}{r^2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{N_i\sin\phi\frac{\mathrm{d}\phi}{\mathrm{d}r}r + N_i\cos\phi}{r^2} \\ -\cos\phi\frac{\partial N_i}{\partial s}\frac{\mathrm{d}\phi}{\mathrm{d}r} & -\sin\phi\frac{\partial N_i}{\partial s}\frac{\mathrm{d}\phi}{\mathrm{d}r} & 0 \end{bmatrix}. \tag{43}$$

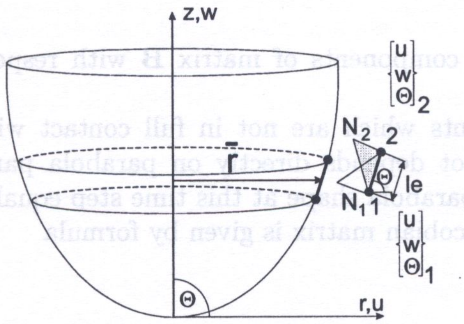

Fig. 6. Nodal displacements vectors u, w, ϕ, shape functions N_i and geometry description for 2-node, 2-D axisymmetrical element

The derivative of elemental matrix $\mathbf{B}_i$ with respect to the coefficients of the parabola which passes trough this element's nodes is much more complicated. Due to the complexity of analytical differentiation of the $\mathbf{B}_i$ matrix components with respect to the design parameters a system for doing mathematics by computer "Mathematica" [12] was used.

For 2-node linear element curvilinear coordinate s depends on natural (elemental) coordinate ξ as follows ,

$$\mathrm{d}s = \frac{l}{2}\,\mathrm{d}\xi \tag{44}$$

The shape functions are equal to

$$N_1 = \frac{1-\xi}{2}, \qquad N_2 = \frac{1+\xi}{2}, \tag{45}$$

where $-1 \leq \xi \leq 1$.

After substitution of the last two equations into (42) we can obtain matrix **B** in the form

$$\mathbf{B}_i = \begin{bmatrix} \frac{(-1)^i}{l}\cos\phi & \frac{(-1)^i}{l}\sin\phi & 0 \\ \frac{1}{2\bar{r}} & 0 & 0 \\ 0 & 0 & \frac{-(-1)^i}{l} \\ 0 & 0 & \frac{-\cos\phi}{2\bar{r}} \\ \frac{(-1)^i}{l}(-\sin\phi) & \frac{(-1)^i}{l}(-\cos\phi) & -\frac{1}{2} \end{bmatrix}. \tag{46}$$

Now $\mathbf{B}_i$ components depend on the radius of the center of the axisymmetrical, one - dimensional shell element used in our finite element code:

$$\bar{r} = \frac{(x_2 + x_1)}{2} \tag{47}$$

and on the angle between this element and horizontal coordinate axis, equals

$$\phi = \arctan\frac{(y_2 - y_1)}{(x_2 - x_1)}. \tag{48}$$

The equation of parabola which lies on coordinate points 1 and 2 of those elements which are in full contact with parabolic part of the punch reads

$$x_1 = ay_1^2 + by_1 + c \tag{49}$$

$$x_2 = ay_2^2 + by_2 + c \tag{50}$$

The differentiation of all components of matrix **B** with respect to a, $b$and $c$can be done by "Mathematica".

Matrix **B** of those elements which are not in full contact with parabolic part of the punch at specific time step does not depends directly on parabola parameters so we assume that its sensitivities with respect to parabola shape at this time step equals zero.

The determinant of the jacobian matrix is given by formula

$$J = \sqrt{[J_1^2 + J_2^2]}$$

where

$$J_1 = \frac{(x_2 - x_1)}{2} \quad \text{and} \quad J_2 = \frac{(y_2 - y_1)}{2}.$$

Gradients of this determinant with respect to the parabola coefficients for those elements which are in full contact with parabolic part of the punch can be obtained by simple differentiation.

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