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Qualitative model-based analysis of truss structures

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The paper discusses some methods of *commonsense reasoning* applicable to analysis of truss structures. The proposed method, based on *qualitative* representation of trusses, allows to reach conclusions in the case of highly *incomplete knowledge* about the system. Two cases are considered. First, when only the general geometry of the structure is known, without the quantitative knowledge of stiffness coefficients of bars. Second, with additional assumption that all stiffness coefficients of bars are roughly equal.

1. MOTIVATION

So far most of the engineering problems have been solved by means of *numerical algorithms*. The benefits of that standard approach are unquestionable. However, it still has some disadvantages. One of them is the need of *quantitative*, as precise as possible, knowledge about the whole system, including geometrical description, material properties, etc.

In many real-life problems, such precise description of the system is not needed and sometimes even not achievable. To address such problems other methods, namely *qualitative* ones, appear to be more useful [2, 4, 5, 7, 9]. Such methods, based on *symbolic computations* rather than *numerical calculations*, are recently more and more frequently investigated and applied.

The goal of our research is to create a system with an ability to perform *commonsense reasoning* [1] about *truss structures*, without the need of knowing their exact parameters. The system thus should act like an experienced engineer, who can predict to some extent the behaviour of the system by only looking at the diagram of it, without knowing any of its parameters.

2. TRUSS STRUCTURES

A truss structure is a mechanical system built from elastic *bars* joined at *nodes* using flexible, rotary joints, and loaded by some external force(s) applied at its nodes. Some of the nodes can be constrained by *full support* (giving no degrees of freedom to the supported node) or by *partial (sliding) support* (allowing the node to move along a specified line or within a specified plane). In such a structure there is no bending of bars; hence the bars carry only axial forces. Here, for the sake of simplicity, only planar truss structures will be considered (for which all bars and loads are placed in a single plane).

The goal of the standard analysis of the truss structure is to determine the displacements of nodes and axial forces in bars under given external loads. For that purpose, well-known numerical algorithms exist. Assuming linear (elastic) material law, they lead to the problem of solving a system of linear equations.

In spite of their simplicity, the numerical methods have one disadvantage—exact knowledge of both the topology and geometry of the truss (connectivity and positions of the nodes) and the parameters of the bars (in standard cases reducible to stiffness) are required. In many real-world problems, exact values of parameters are not available, or they may be wrong, e.g., due to material faults or manufacturing imperfections. For that reason, a truss analysis system capable of taking into account *incomplete* data would be needed [4, 6–9].

In this paper we perform two types of qualitative analysis. The first one, namely *model-based*, is done under the assumption that all stiffness coefficients are unknown. The second one, with addition of *heuristic rules*, assumes that all stiffness coefficients of bars are roughly equal.

3. Model-based versus rule-based reasoning

Rule-based expert systems represent the problem to be solved with *if... then...* rules [10]. It is also the oldest approach to knowledge representation in expert systems. Typically, an expert system's rules attempt to capture human expert knowledge as it is used in practice; consequently, they are often a blend of theoretical knowledge, heuristics derived from experience, and special-purpose rules for handling odd cases and other exceptions to normal practice. In many situations, this approach has proven effective. Nonetheless, strongly heuristic systems may fail, either on encountering a problem that does not fit any available rules, or by misapplying a heuristic rule to an inappropriate situation. We may say that rule-based system represent only *shallow knowledge* about the system.

Human experts do not suffer from these problems, because they have a deeper, theoretical understanding of the problem domain that allows them to apply the heuristic rules intelligently, or resort to reasoning from "first principles" in novel situations. These are the issues that model-based approaches attempt to address. A knowledge-based reasoning system whose analysis is founded directly on the specification and functionality of a physical system is called a *model-based* system.

Let us compare rule-based reasoning versus model-based one in the field of our interest, i.e. truss structures analysis. In the rule-based setting, we create a set of "*if... then...*" rules which describe the behaviour of the system depending on the configuration of the truss. For example, using experiments or the knowledge of an expert, we can found that *if* the truss is composed from two horizontal rows of nodes, connected with bars *and* the bars don't cross *and* one of the ends (of the truss) is in full support *and* the other in sliding support *then* the behaviour of the structure (i.e., node displacements and axial forces) is such-and-such (see e.g. Fig. 6). The quality of such a rule-based system depends mostly on the number of truss structure types considered. The main advantage of this approach is the speed of running of the system. Unfortunately, such a program will fail completely for the cases which were not taken ino account before. Worse yet, when the implemented cases are not described precisely enough, the program can arrive at wrong conclusions due to similarity of otherwise different cases.

For those reasons more robust method of truss analysis is needed. The model-based systems philosophy consist in implementing directly the *physical laws* which are responsible for truss behaviour. In such a system, once the rules describing the principles of truss element behaviour are implemented, the program can work robustly for all possible truss configurations. Such an approach gives the expert system much more power and flexibility, as it implements the *deep-knowledge* about the problem domain.

4. Basic qualitative calculus

In the theory of qualitative physics three representations of approximate (qualitative) values were proven useful so far [2, 5]: *signs*, *inequalities*, and *orders of magnitude* [11]. For the purpose of model-based truss analysis described here, we will use *signs*, being the simplest qualitative representation.

The set of values which the qualitative variable can take will be denoted by $\mathbf{Q}$:

$$\mathbf{Q} = \{-, 0, +\}. \tag{1}$$

The symbol '$-$' indicates the value lower then zero, and '$+$' the value greater then zero. The symbol '?', which will be used in the sequel, denotes indeterminate value, i.e. any of $-$, 0 or $+$.

The basic algebraic operations on the set **Q** are analogical to operations on real numbers. Below we will give definitions of the operations and their properties needed in the sequel.

Definition 1 (Addition) Qualitative addition *of two variables* $[x], [y] \in \mathbf{Q}$ *is defined in Table 1.*

Definition 2 (Negation) Qualitative negation *of the variable* $[x]$ *is defined in Table 2.*

Table 1. Definition of qualitative addition: $[x] + [y]$

$[x]$ \ $[y]$	−	0	+
−	−	−	?
0	−	0	+
+	?	+	+

Table 2. Definition of qualitative negation

$[x]$	$-[x]$
−	+
0	0
+	−

Definition 3 (Nondecreasing sequence) *The sequence of qualitative values* $[x_i] \in \mathbf{Q}$ *of the form:* $([x_0], [x_1], \dots, [x_k])$, *is said to be a* nondecreasing sequence, *when*

$$[x_i] \le [x_{i+1}], \qquad \textit{for all} \quad i = 0 \dots k-1.$$

We assume here the following definition of the "≤" relation between qualitative values:

$$[-] \le [-], \quad [-] \le [0], \quad [-] \le [+], \quad [0] \le [+], \quad [+] \le [+].$$

Definition 4 (Nonincreasing sequence) *The sequence of qualitative values* $[x_i] \in \mathbf{Q}$ *of the form:* $([x_0], [x_1], \dots, [x_k])$, *is said to be* nonincreasing sequence, *when*

$$[x_i] \ge [x_{i+1}], \qquad \textit{for all} \quad i = 0 \dots k-1$$

We assume here the following definition of the "≥" relation between qualitative values:

$$[+] \ge [+], \quad [+] \ge [0], \quad [+] \ge [-], \quad [0] \ge [-], \quad [-] \ge [-].$$

Note that in Definitions 3 and 4 the relations $[0] \ge [0]$ and $[0] \le [0]$ are not considered valid, so that sequences containing neighbouring zeros are note allowed in our application.

Definition 5 (Monothonic sequence) *We call the sequence* monothonic *if it is either nonincreasing or nondecreasing.*

To solve some truss problem we can use the set **Q** to represent all elements of the truss solution, i.e., both axial loads in bars and node displacements. Yet for some situations, the full set **Q** is not needed. For example, in the case of an unsupported node, the situation in which the node does not move in any direction, altough theoretically possible, has no practical significance, as it can take place only for some precisely chosen stiffnesses of the bars not known in general. In such circumstances, the smaller set $\mathbf{Q}_2$ of qualitative values can be useful:

$$\mathbf{Q}_2 = \{-, +\}. \tag{2}$$

5. QUALITATIVE MODEL OF A TRUSS

The concept of qualitative model-based truss analysis relies on the validity of three fundamental physical laws:

- material properties of the bars,
- equilibrium condition of axial forces and loads,
- geometrical admissibility of node displacements.

5.1. Material properties

In truss structures bars are joined using flexible, rotary joints and hence there is no bending of bars—under the load of external forces acting at nodes they can only be stretched or compressed.

Assuming linear elastic constitutive law, the axial force (P_i) in the bar i depends on the elongation of the bar (Δ_i), in accordance with the *Hooke's* law:

$$P_i = \frac{\Delta_i}{l_i} A_i E_i = \Delta_i s_i \,, \tag{3}$$

where l_i denotes the length of the bar, A_i is its cross-sectional area, E_i Young modulus, and $s_i = A_i E_i / l_i$ denotes the stiffness of the bar. We adopt here the convention that $\Delta_i > 0$ ($P_i > 0$) when the bar is stretched, and $\Delta_i < 0$ ($P_i < 0$) when it is compressed.

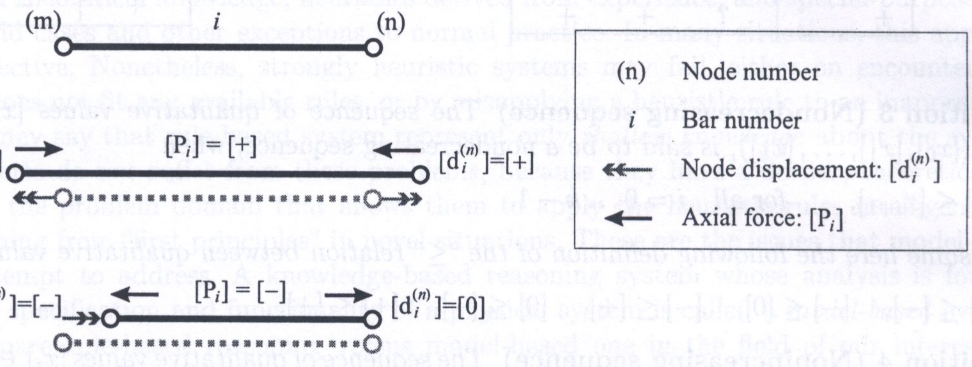

Fig. 1. Qualitative representation of a bar

The transition to the set of qualitative values (the set **Q**) is straightforward. The displacement of the node in the direction of the bar (with a possibility to compress it) will be indicated by the value of [−]. The displacement in the opposite direction by the value [+] (see Fig. 1). This simple model of the bar will allow us to formulate the first rule of the system, viz. *material law.*

Rule 1 (Material law) *Let* $[d_i^{(m)}]$ *and* $[d_i^{(n)}]$ *denote qualitative values of displacements of the nodes m and n in the directions indicated by the bar i. The qualitative value of the axial force in the bar i is then given by the equation*

$$[P_i] = [d_i^{(m)}] + [d_i^{(n)}]. \tag{4}$$

The result of qualitative addition may be equal to zero in one of the three cases: [0]+[0], [+]+[−], and [−]+ [+] (see Definition 1). From formal (mathematical) point of view all these cases should be taken into consideration. However, when one end of the bar moves, it is very unlikely that the other one moves by exactly the same value. Although such a case is possible, it can happen only for some quantitatively precise values of stiffness parameters of bars. But we do not assume any concrete values of them, therefore we can exclude the cases [+] + [−] = [0] and [−] + [+] = [0] from the Rule 1. As a result, in real applications material rules can be usually implemented using simpler and faster form.

It is also not difficult to write different qualitative rules for other types of components, e.g. soft strings.

5.2. Equilibrium of forces

For truss structures, the equilibrium condition at a node is as follows,

$$\sum_i \mathbf{P}_i + \mathbf{F} = 0, \tag{5}$$

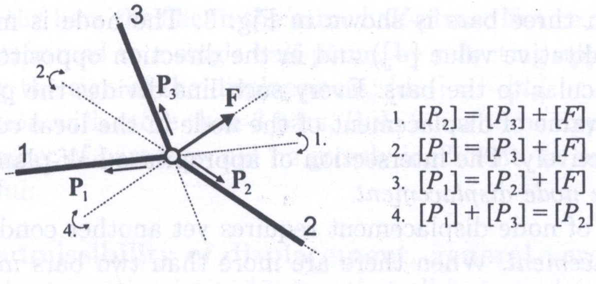

Fig. 2. Qualitative force equilibrium conditions

where $\mathbf{P}_i$ are reaction forces of the bars, $i = 1, 2, \ldots, n$, with n being the number of bars meeting at the node, and $\mathbf{F}$ is the external load.

In qualitative formulation, direct interpretation of Eq. (5) (although possible) would lead to great uncertainty of the result. A more adequate solution exists. Figure 2 shows an example configuration of the node with three bars and one external load vector. Let us select one direction indicated by the load vector or one of bars. Let it be the bar 1, for example. Consider projections of the forces onto a line perpendicular to the bar. The projection of $\mathbf{P}_1$ is now zero, and $\mathbf{P}_2$ projects in the opposite direction than $\mathbf{P}_3$ and $\mathbf{F}$. Therefore, from the equilibrium condition it follows that forces $\mathbf{P}_3$ and $\mathbf{F}$ have to be balanced by the force $\mathbf{P}_2$, i.e., the equation $[P_2] = [P_3] + [F]$ must be valid. The same equilibrium condition must hold for every other bar as well (see Fig. 2).

Rule 2 (Equilibrium condition) *Let $[R_i]$ denote qualitative value of the force acting at the node (it could be a reaction force of the bar, or the external load). Consider the straight line collinear with the direction of $\mathbf{R}_i$, dividing the plane into two half-planes $\mathcal{A}_i$ and $\mathcal{B}_i$. For every i (i.e., for every bar or load) we have*

$$\forall_i \quad \sum_{k \in \mathcal{A}_i} [R_k] = \sum_{l \in \mathcal{B}_i} [R_l], \tag{6}$$

where $k \in \mathcal{A}_i$ ($k \in \mathcal{B}_i$) indicates that the bar (or force vector) $\mathbf{R}_i$ lies in the half-plane $\mathcal{A}_i$ (respectively $\mathcal{B}_i$).

5.3. Displacement of nodes

The *material rule* (as described in the Sec. 5.1) relates the axial force in the bar to displacements of its ends (nodes). The displacement of a node is usually represented by a vector. In qualitative formulation, representing the displacements as two-dimentional vectors is not practical: it would only introduce additional uncertainty into the result.

Thus, the displacement of a node will be represented by the list of qualitative (sign-valued) displacements in the directions of the bars meeting at that node (see Fig. 3). Such a description is often even more precise then the representation by the qualitative mean of the vectors of qualitative values. The more bars meet at the node, the more exact it becomes.

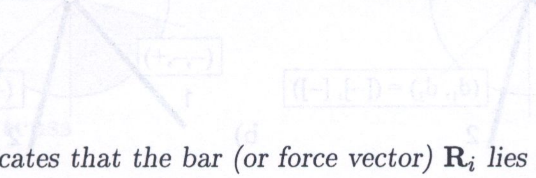

Fig. 3. Qualitative description of the node displacement—using local configuration instead of vector components in a global coordinate system

An example node with three bars is shown in Fig. 3. The node is moving in the direction of the bars 1 and 2 (the qualitative value [−]) and in the direction opposite to the bar 3 (value [+]). Dotted lines are perpendicular to the bars. Every such line divides the plane into two half-planes, for which the qualitative value of displacement of the node in the local configuration of that bar is equal to [+] or [−], respectively. The intersection of appropriate half-planes for all the bars defines thus the region of *possible node displacement*.

The above description of node displacement requires yet another condition, namely *geometrical admissibility of the displacement*. When there are more than two bars meeting at the node, some of the combinations of the qualitative values of bar displacements are not geometrically possible. For example, in the situation shown in Figure 3 the displacement $([d_1], [d_2], [d_3]) = ([+], [-], [+])$ is impossible.

Denoting by φ_{ij} the (unsigned) smaller angle between bars i and j, we have:

Rule 3 (Geometrical admissibility of displacement, simple version) *For every triple of bars i, j, k (meeting at the node) such that $\varphi_{ij} + \varphi_{jk} = \varphi_{ik} < \pi$ (i.e., for bars that lie inside the same half-plane), there holds*

$$[d_i] + [d_j] = [d_k], \tag{7}$$

where $[d_i]$ denotes qualitative value of the node displacement in the direction of the i-th bar.

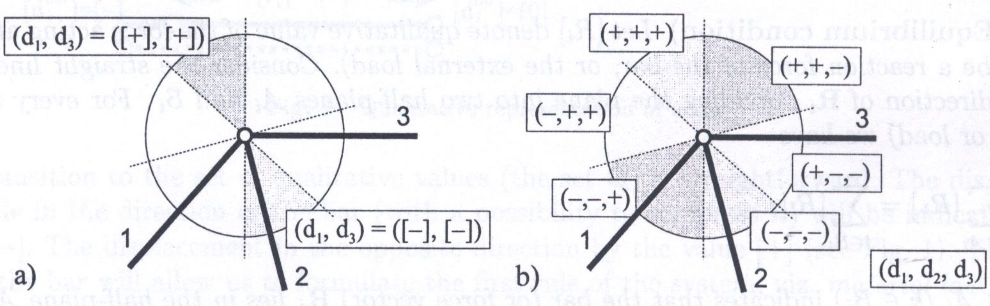

Fig. 4. Geometrical admissibility of displacement rule: $[d_1] + [d_3] = [d_2]$

Proof 1. An example configuration of a node with three bars is shown in Fig. 4a. When the displacement of the node into the directions $([d_1], [d_3])$ equals $([+], [+])$, then the only possible (geometrically feasible) qualitative value of the displacement $[d_2]$ will be [+]. Similar condition holds for $([d_1], [d_3]) = ([-], [-])$. In this case, the displacement $[d_2]$ must equal [−]. For other combinations of displacements $([d_1], [d_3])$, all qualitative values (from the set **Q**) of displacement $[d_2]$ are acceptable (see Table 1, the [?] value). Thus, all of them fulfill the condition: $[d_1] + [d_3] = [d_2]$. □

Proof 2. An example configuration of a node has been shown in Fig. 4b. The 6 gray areas indicate possible values of displacements $([d_1], [d_2], [d_3])$: $(+,+,+)$, $(+,+,-)$, $(+,-,-)$, $(-,-,-)$, $(-,-,+)$, $(-,+,+)$. Additional displacement codes cover the situations when the node does not move $(0,0,0)$ and when some displacements equal zero: $(0,+,+)$, $(+,+,0)$, $(+,0,-)$, $(0,-,-)$, $(-,-,0)$, $(-,0,+)$. In the table below all 13 possible displacement combinations are presented. One can easily notice that: $[d_1] + [d_3] = [d_2]$. □

$[d_1]$ \ $[d_3]$	−	0	+
−	−	−	− 0 +
0	−	0	+
+	− 0 +	+	+

Rule 3 applies only to the bars that lie in the same half-plane. Notice, however, that any configuration of bars can be transformed to a single half-plane, by reflecting appropriate bars with respect to the node and changing the sign of the displacement: $[d_{-i}] = -[d_i]$.

In the node configuration with more then 3 bars, Rule 3 has to be applied to all combinations of three bars. For larger number of bars it would be grossly inefficient. In such cases, the more general version of the rule is useful:

Rule 4 (Geometrical admissibility of displacement, general version) *Consider the configuration of a node with n bars meeting in it. Assume that all bars are lying in the same half-plane—otherwise transform the bars to place them in the single half-plane by reflecting some of them with respect to the node and changing the sign of the displacement. Number the bars consecutively so that* $\varphi_{1i} < \varphi_{1j}$ *for all* $i, j = 1 \ldots n$, $i < j$. *The list of qualitative values* $([d_1], [d_2], \ldots, [d_n])$, *denoting the geometrical displacements of the node can take only the form of a monothonic sequence (see Definition 5) or a sequence* $([0], [0], \ldots, [0])$ *containing only the* $[0]$ *values.*

Proof The proof of this rule can be done with the help of a figure similar to Fig 4b (but with more bars). It is omited here for brevity. □

From Rule 4 it follows that for the node with n bars meeting at it (assuming that none of them are collinear), exactly $4n + 1$ possible combinations of qualitative node displacements are possible. This can be reduced even more. In the considered qualitative description, we do not assume any quantitative knowledge about stiffness coefficients of bars. Therefore, for most of the unsupported nodes, the displacement equal to zero is not possible, or it is not interesting from the practical point of view. Hence, the set $\mathbf{Q}_2$ might be used for the qualitative variables. With this simplification, the number of possible combinations is reduced to $2n$.

6. Examples

6.1. Simple three-node truss

Let us illustrate the working of our model-based system using first the simplest example of a three-node truss, shown in Fig. 5. Using the rules described in Section 5, we can write down all qualitative equations constraining the truss (see Table 3).

These equations can be easily solved with one of automated resolution systems. From the expression (E5) one can conclude that: $[P_1] = [-]$ and $[P_3] = [-]$. Then, from expression (E2), we conclude that $[d_2^{(1)}] = [-]$, and from (E3): $[d_3^{(2)}] = [-]$. At last we conclude that $[d_1^{(1)}] = [-]$ (from E6) and $[d_1^{(2)}] = [+]$ (from E7). Thus, the whole system has been solved (see Fig. 5b).

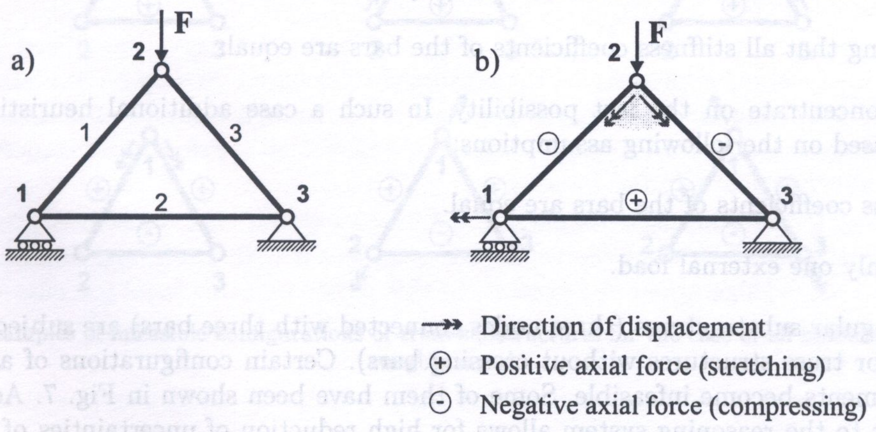

Fig. 5. A simple example of qualitative truss analysis

Table 3. Equations used to solve the truss structure shown in Fig. 5

No.	Equation
Material properties (Rule 1):	
E1	$[P_1] = [d_1^{(1)}] + [d_1^{(2)}]$
E2	$[P_2] = [d_2^{(1)}] + [d_2^{(3)}]$
E3	$[P_3] = [d_3^{(2)}] + [d_3^{(3)}]$
Equilibrium of forces (Rule 2):	
E4	$[P_1] = -[P_2]$
E5	$[P_1] + [+] = [0] \ \wedge \ [P_3] + [+] = [0] \ \wedge \ [P_1] = [P_3]$
Geometrical constraints (Rule 3):	
E6	$[d_1^{(1)}] = [d_2^{(1)}]$
E7	$[d_1^{(2)}] + [d_3^{(2)}] = [+]$
E8	$[d_2^{(3)}] = [d_3^{(3)}] = [0]$

□ For that simple example, the qualitative solution is determinate. All axial forces as well as node displacements have been determined, without the knowledge of stiffness coefficients of the bars.

6.2. More complex example

The second example truss structure consists of 9 nodes and 15 bars (see Fig. 6). The derivation of equations describing the truss is left as an exercise for the reader. Here, we concentrate only on the results of the analysis.

The model-base analysis of the truss has been shown in Fig. 6a. Almost all axial forces in bars have been determined—only for a single bar (namely 7–9) the axial force was not found.

Unfortunately, the node displacements remained mostly undetermined. The reason for that is that no constraints on stiffness coefficients of the bars were assumed. It means that some of them might be extremely soft while the other extremely stiff. For such indeterminate data, a more precise result cannot be expected. To solve that problem we must increase accuracy of the model. There are at least three general methods of doing that:

- By creating a *hybrid* reasoning system [4, 9], combining qualitative and quantitative knowledge (e.g., when at least some of the truss parameters are known with greater accuracy); among others, *interval methods* can be used in this case [8, 9].
- By determining some qualitative *relations* between parameters of the system; an *order-of-magnitude calculus* can be used in this case [11].
- By assuming that all stiffness coefficients of the bars are equal.

Here we concentrate on the last possibility. In such a case additional heuristic rules can be formulated based on the following assumptions:

1. All stiffness coefficients of the bars are equal.
2. There is only one external load.

Now the triangular substructures (three nodes connected with three bars) are subject to additional constraints (for truss structures without crossing bars). Certain configurations of axial forces and node displacements become infeasible. Some of them have been shown in Fig. 7. Addition of such heuristic rules to the reasoning system allows for high reduction of uncertainties of the result (see Fig. 6b).

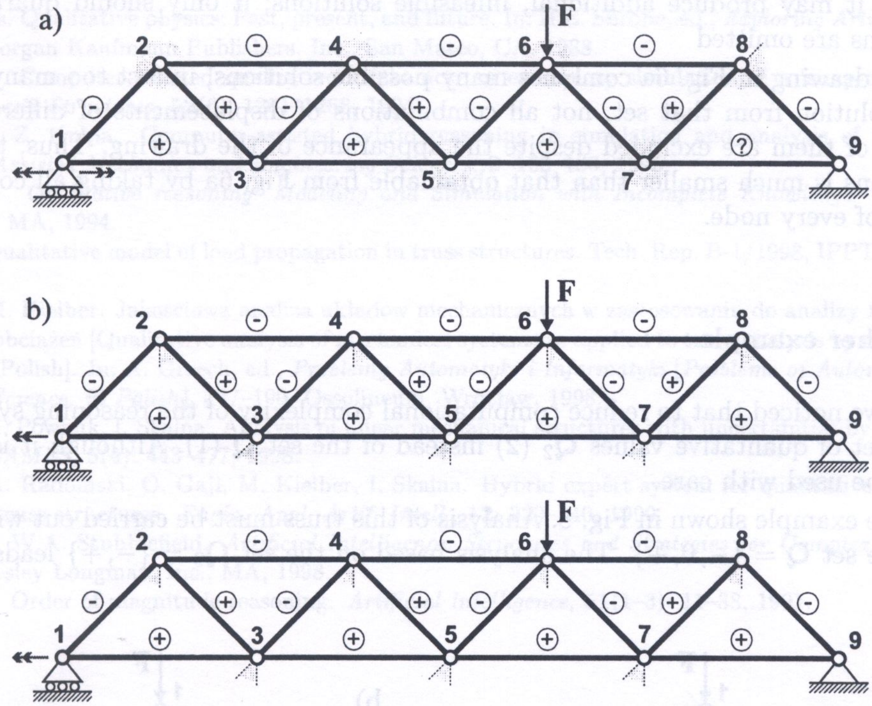

Fig. 6. Qualitative analysis of a truss structure: using only model-based rules (a), and with additional heuristics assuming that all the bars have the same stiffness coefficients (b)

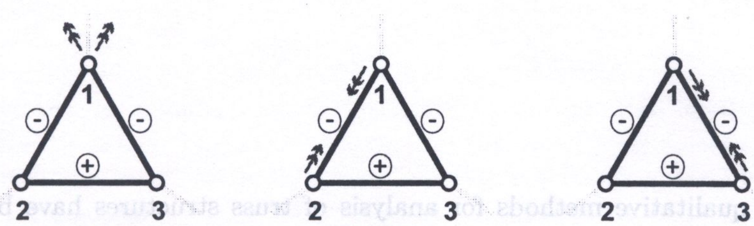

Fig. 7. Examples of infeasible configurations of truss substructures for the case of all stiffness coefficients roughly equal

Some additional clarifications concerning the solutions shown in Fig. 6 are done. First, of the two solutions produced under the assumption of approximate equality of stiffnesses (Fig. 6b) only one (the second) is physically valid. It goes in accord with the general principle of qualitative analysis [2, 5]: it may produce additional, infeasible solutions; it only should quarantee that no feasible solutions are omitted

Second, the drawing in Fig. 6a combines many possible solutions, in fact too many to be drawn here. In any solution from that set, not all combinations of displacements of different nodes are allowed. Many of them are excluded despite the appearance of the drawing. Thus, the number of possible solutions is much smaller than that obtainable from Fig. 6a by taking all combinations of displacements of every node.

6.3. Yet another example

In Section 5.3 we noticed that to reduce computational complexity of the reasoning system, one can use a smaller set of qualitative values $\mathbf{Q}_2$ (2) instead of the set $\mathbf{Q}$ (1). Although it is safe in most cases, it must be used with care.

Consider the example shown in Fig. 8. Analysis of this truss must be carried out with qualitative values from the set $\mathbf{Q} = \{-, 0, +\}$. The analysis based on the set $\mathbf{Q}_2 = \{-, +\}$ leads to an empty solution.

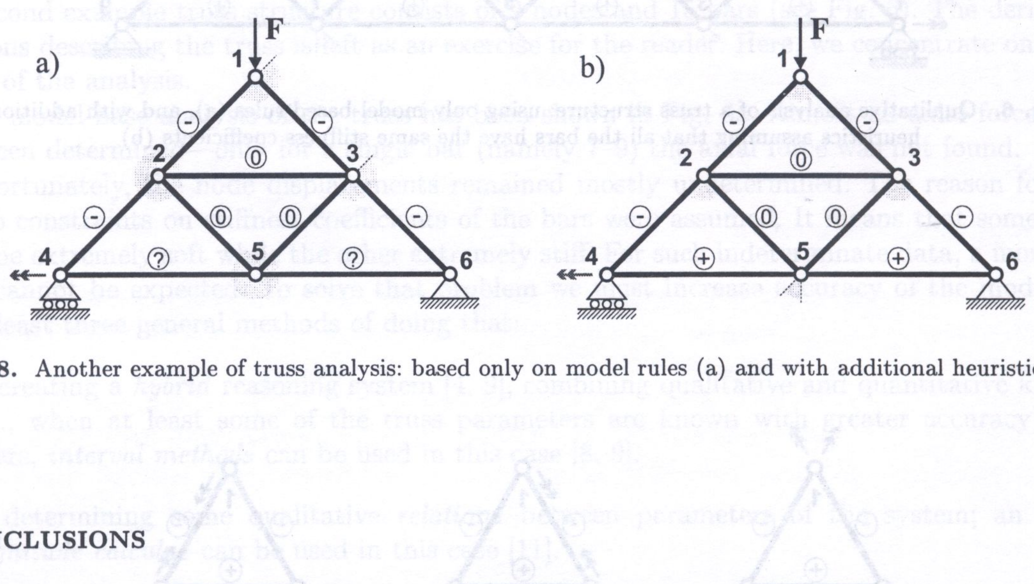

Fig. 8. Another example of truss analysis: based only on model rules (a) and with additional heuristics (b)

7. Conclusions

In this paper some qualitative methods for analysis of truss structures have been proposed. We consider here a *model-based* approach, i.e. the reasoning rules based directly on fundamental *physical laws* (sometimes called the "first principles" approach). The main advantage of methods of this kind over strictly heuristic ones is that it works with equal ease for all types of truss structures, not only for some types of structures selected in advance.

The qualitative formulation of truss parameters based only on *sign calculus* is usually too inaccurate for practical purposes. Therefore we have to augment it with additional heuristic rules based on more accurate knowledge about the system. In this paper we considered the case when stiffness coefficients of bars are close to each other. Other approaches are also possible (see Sec. 6.2), and will be subjects of further research.

Another advantage of *model-based* approach is the possibility to easily combine qualitative methods with heuristic rules.

All examples presented in the paper have been solved using a program implemented in PROLOG.

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