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Evaluation of the threshold values for the propagation of a fatigue crack starting at a V-notch[1]

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This paper presents a simple method for evaluating the threshold value for fatigue cracks that emanate from a V-notch. The proposed method is based on the similarities between the elastic-stress fields around the tip of a crack and the tip of a V-notch. Threshold values for fatigue cracks that emanate from a V-notch are expressed by means of the threshold value for the propagation of a high-cycle-fatigue crack and the opening angle of the V-notch. The corresponding calculations were performed by the finite-element method.

1. INTRODUCTION

Fatigue cracks usually start at some form of geometrical discontinuity such as a notch, for example. Sharp or V-notches (i.e., notches with their root radius equal to zero) can be described as singular stress concentrators. Near the tip of a sharp notch there are high stresses and strains that may cause failure of the structure. The V-notch (see Fig. 1) and sharp-crack models are really dealing with the same problem from the point of view of continuum mechanics, namely that both are represented by the singular-stress field in the vicinity of the tip. In both cases, the stress distribution at the tip is of the form

$$\sigma_{ij} = \frac{H}{\sqrt{2\pi}} r^{-p} f_{ij}(\vartheta, \alpha), \tag{1}$$

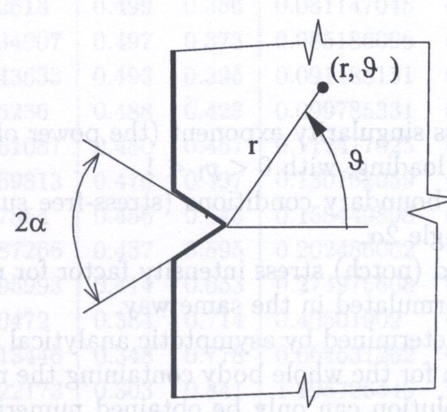

Fig. 1. V-notch with open angle 2α and corresponding coordinate system

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where $0 < p = p(\alpha) \leq 1/2$ is the value of the stress singularity exponent, H is a generalized (notch) stress intensity factor, r is the distance from the notch tip with an opening angle of 2α and $f_{ij}(\vartheta, \alpha)$ is a known function of the polar angle ϑ, see Fig. 1. Note that for cracks (where $2\alpha = 0$), $p = 1/2$ and $H = K$, the stress intensity factor. The fact that the stress-singularity exponent for a notch is, in general, no longer $1/2$ means that linear-elastic-fracture-mechanics arguments cannot be applied in this case. On the other hand, if the failure mechanism is the same in both cases, it is possible to formulate a unifying theoretical model that describes the behavior of both cracks and notches [4, 8, 11].

This paper is concerned with prediction of the behavior of a notched body under cyclic loading. The aim of the contribution is to describe a method for evaluating the threshold value ΔH_{th} for cracks that emanate from V-notches. Knowing the value of ΔH_{th} makes it possible to estimate the fatigue limit, or high-cycle endurance limit, $\Delta\sigma_c$, for the notched body. A necessary step in this process is estimation of the value of the generalized stress intensity factor H. Contrary to the known routines for estimating the stress intensity factor K, see, e.g. [6, 9], procedures for calculating the H value are not generally available. In this paper a direct method for estimating H is applied and its accuracy is discussed. The corresponding numerical calculations are performed by the finite-element system ANSYS [2]. The results of the paper contribute to a connection between the analysis of notch and the fracture mechanics of fatigue cracks.

2. Singular stress field due to a sharp notch

In the following Section, the solution for the stress distribution around a V-notch tip for normal mode I is presented i.e., $H = H_{\mathrm{I}}$, $p = p_{\mathrm{I}}$, etc. The problem of a V-notch in homogeneous material has been treated by a number of authors using various methods (see e.g. [3, 5, 12]). In the present paper, the results based on the solution of the Airy stress function [12] are used. Only singular stress terms are used for further considerations.

With reference to plane problem and V-notch with stress-free surfaces, the analytical expressions for the distributions of singular stress and strain are well known [12].

Let us consider a polar coordinate system (r, ϑ) with its origin at the notch tip. The notch is a V-notch with an opening angle of 2α (Fig. 1).

The Airy stress function $\Phi(r, \vartheta)$ is a bi-harmonic function, i.e., it must satisfy the equation

$$\Delta\Delta\Phi = 0. \tag{2}$$

The solution for normal (I) loading can be expressed in the form (only singular terms are taken into account)

$$\Phi_{\mathrm{I}}(r, \vartheta) = \frac{H_{\mathrm{I}}}{\sqrt{2\pi}} r^{2-p_{\mathrm{I}}} f_{\mathrm{I}}(\vartheta), \tag{3}$$

where $p_{\mathrm{I}}(\alpha)$ represents the stress singularity exponent (the power of the singularity) corresponding to the normal (mode I) type of loading, with $0 < p_{\mathrm{I}} < 1$.

Note that for a given set of boundary conditions (stress-free surfaces) the value of p_{I} depends only on the V-notch opening angle 2α.

The variable H_{I} is generalized (notch) stress intensity factor for mode I. The basic equations for shear loading mode II can be formulated in the same way.

The values of H_{I} cannot be determined by asymptotic analytical analysis and must be estimated using the corresponding solution for the whole body containing the notch under the given boundary conditions. Generally, such a solution can only be obtained numerically and the values of H_{I} can then be evaluated using the distributions of stress and strain.

In the present paper the value of the generalized stress intensity factor H_{I} for a notch is defined in such a way that for $\alpha = 0$ (i.e. for a crack) $H_{\mathrm{I}} = K_{\mathrm{I}}$ where K_{I} is the corresponding value of the stress intensity factor. Let us note that in the case of a crack $p_{\mathrm{I}} = 1/2$.

Inserting Eq. (3) into biharmonic Eq. (2) leads to an ordinary differential equation for functions $f_{\rm I}(\alpha)$,

$$f_{\rm I}'''' + 2(p_{\rm I}^2 - 2p_{\rm I} + 2) f_{\rm I}'' + p_{\rm I}^2 (2 - p_{\rm I})^2 f_{\rm I} = 0, \tag{4}$$

where prime denotes differentiation with respect to ϑ. Such a linear equation with constant coefficients can be solved by using trigonometric functions. The symmetry of loading mode I permits only cosines, while mode II is described by sines. Then we have

$$f_{\rm I}(\vartheta) = \cos(p_{\rm I}\vartheta) + q_{\rm I}\cos(2 - p_{\rm I})\vartheta, \tag{5}$$

The stress components in polar coordinates are given by

$$\sigma_{rr}^{\rm I} = \frac{1}{r}\frac{\partial \Phi_{\rm I}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Phi_{\rm I}}{\partial \vartheta^2}, \qquad \sigma_{\vartheta\vartheta}^{\rm I} = \frac{\partial^2 \Phi_{\rm I}}{\partial r^2}, \qquad \sigma_{r\vartheta}^{\rm I} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \Phi_{\rm I}}{\partial \vartheta}\right). \tag{6}$$

From the boundary conditions on the stress-free surfaces of the notch:

$$\sigma_{\vartheta\vartheta}^{\rm I}(\vartheta = \pi - \alpha) = \sigma_{\vartheta\vartheta}^{\rm I}(\vartheta = \pi + \alpha) = \sigma_{r\vartheta}^{\rm I}(\vartheta = \pi - \alpha) = \sigma_{r\vartheta}^{\rm I}(\vartheta = \pi + \alpha) = 0, \tag{7}$$

and it follows for parameter $p_{\rm I}$ that

$$\sin(2(1 - p_{\rm I})(\pi - \alpha)) + (1 - p_{\rm I})\sin(2(\pi - \alpha)) = 0 \tag{8}$$

and for constant $q_{\rm I}$

$$q_{\rm I} = -\cos(p_{\rm I}(\pi - \alpha))/\cos((2 - p_{\rm I})(\pi - \alpha)). \tag{9}$$

The singular stress distribution for a V-notch is then given by the value of the generalized stress intensity factor $H_{\rm I}$, obtained by numerical solution, and by the value of the stress singularity parameter $p_{\rm I}$, obtained from Eq. (8). The values of $p_{\rm I}$ and $q_{\rm I}$ are given in Table 1 as functions of the notch opening angle 2α.

Table 1. Stress singularity exponents $p_{\rm I}$ (Eq.(8)) and constants $q_{\rm I}$ (Eq. (9)); $f_v(\alpha)$ is the area of the plastic zone calculated for $H_{\rm I} = 1$ MPa m$^{p_{\rm I}}$ and $\sigma_0 = 1$ MPa; PSS holds for plane stress, PSN corresponds to plane strain and the normal mode of loading; the notch opening angle is 2α

α [deg]	α [rad]	$p_{\rm I}$	$q_{\rm I}$	f_v (PSS)	f_v (PSN)
0	0	0.500	0.333	0.076468977	0.022324465
5	0.08727	0.500	0.336	0.076979034	0.022633185
10	0.17453	0.500	0.343	0.078344314	0.023462321
15	0.2618	0.499	0.356	0.081147045	0.025018965
20	0.34907	0.497	0.373	0.085186096	0.027173062
25	0.43633	0.493	0.395	0.091188151	0.030175621
30	0.5236	0.488	0.423	0.099785331	0.03454016
35	0.61087	0.480	0.457	0.112477625	0.040699476
40	0.69813	0.470	0.497	0.130752059	0.049564114
45	0.7854	0.456	0.543	0.158449898	0.062555865
50	0.87266	0.437	0.595	0.202486062	0.082462822
55	0.95993	0.414	0.653	0.274975668	0.11471773
60	1.0472	0.384	0.714	0.40501909	0.169239885
65	1.13446	0.348	0.778	0.664531262	0.272187672
70	1.22173	0.303	0.841	1.295726446	0.497024168
75	1.309	0.248	0.901	3.475121019	1.155219552
80	1.39626	0.181	0.951	19.37601587	4.617728837
85	1.48353	0.100	0.986	1793.845975	150.2210756
90	1.5708	0.000	1.000		

3. Distribution of stress at the tip of a V-notch

The singular stress components for mode I can be derived from Eqs. (3), (5) and (6). The stress components σ^{I}_{ij} for mode I are then given by the following expressions

$$\begin{aligned}
\sigma^{\mathrm{I}}_{rr} &= \frac{H_{\mathrm{I}}}{\sqrt{2\pi r^{p_{\mathrm{I}}}}}\left[(2-p_{\mathrm{I}}-p_{\mathrm{I}}^2)\cos(p_{\mathrm{I}}\vartheta)-q_{\mathrm{I}}(2-3p_{\mathrm{I}}+p_{\mathrm{I}}^2)\cos((2-p_{\mathrm{I}})\vartheta)\right],\\
\sigma^{\mathrm{I}}_{\vartheta\vartheta} &= \frac{H_{\mathrm{I}}}{\sqrt{2\pi r^{p_{\mathrm{I}}}}}\left[(2-3p_{\mathrm{I}}+p_{\mathrm{I}}^2)\cos(p_{\mathrm{I}}\vartheta)+q_{\mathrm{I}}(2-3p_{\mathrm{I}}+p_{\mathrm{I}}^2)\cos((2-p_{\mathrm{I}})\vartheta)\right],\\
\sigma^{\mathrm{I}}_{r\vartheta} &= \frac{H_{\mathrm{I}}}{\sqrt{2\pi r^{p_{\mathrm{I}}}}}\left[p_{\mathrm{I}}(1-p_{\mathrm{I}})\sin(p_{\mathrm{I}}\vartheta)+q_{\mathrm{I}}(2-3p_{\mathrm{I}}+p_{\mathrm{I}}^2)\sin((2-p_{\mathrm{I}})\vartheta)\right],\\
\sigma^{\mathrm{I}}_{zz} &= \frac{-2\sqrt{2}\nu H_{\mathrm{I}}}{\sqrt{\pi r^{p_{\mathrm{I}}}}}\cos(p_{\mathrm{I}}\vartheta)(p_{\mathrm{I}}-1) \qquad \text{for plane strain.}
\end{aligned} \tag{10}$$

4. Theoretical model for estimating the threshold value

In the following it is supposed that any cyclic stresses applied to a structure are so small that the plastic zone ahead of the fatigue crack or the notch tip is a minor perturbation in an otherwise elastic field, i.e., the assumptions of linear elastic fracture mechanics and high cycle fatigue are valid. For simplicity we limit our considerations to zero-tension loading (the minimum applied load value $\sigma_{\min}=0$ and $\Delta K_{\mathrm{I}}=K_{\mathrm{I}}$, $\Delta H_{\mathrm{I}}=H_{\mathrm{I}}$).

The expression for the fatigue crack growth rate $\mathrm{d}a/\mathrm{d}N$ is based on the stress intensity factor range $\Delta K = K_{\max}-K_{\min}$, where $K_{\max}$ and $K_{\min}$ are the extreme values of the K-factor in the stress cycle. Note that all variables are related to the normal mode of loading, i.e. $K=K_{\mathrm{I}}$ etc. In the case of zero-tension loading $K_{\min}=0$ and $\Delta K = K_{\max}$. In the region of very small crack propagation rates the existence of the threshold value of the stress intensity factor range $\Delta K_{\mathrm{th}} = K_{\mathrm{th}}$ has to be taken into account. The value of K_{th} implies that fatigue cracks will not grow if

$$K_{\max} < K_{\mathrm{th}}, \tag{11}$$

i.e. $\mathrm{d}a/\mathrm{d}N \to 0$ as $(K_{\max}-K_{\mathrm{th}}) \to 0$. Note that the value of K_{th} is a material constant.

The models for thresholds based on the assumptions of continuum mechanics relate to critical values of the energy, the crack-tip displacement, the plastic-zone size, the plastic strain, etc, see e.g. [7, 10]. In the case of a V-notch the stress singularity differs from 1/2 and the controlling variable for the initiation of a fatigue crack at the notch tip can be expressed by means of the generalized stress intensity factor H.

In the present paper it is suggested that the controlling variable for the initiation of a fatigue crack relates to the size of the plastic zone. The threshold value of the V-notch is then calculated using the equality of the areas of the plastic zones in the cases of cracks and V-notches.

In the case of a crack, the purely elastic estimates of the elastic-plastic boundaries for small-scale yielding for plane stress are given by the Mises yield condition, and the area of the plastic zone corresponding to the threshold loading is (e.g. [1])

$$R_p = \left(\frac{K_{\mathrm{th}}}{\sigma_0}\right)^4 \frac{123}{512\pi} \tag{12}$$

where K_{th} the is corresponding value of the threshold value, and σ_0 is the yield stress of the material under consideration.

Analogously, the formula for the area of the plastic zone near the tip of the V-notch can be written in the following way,

$$R_p = \left(\frac{H_{\mathrm{th}}}{\sigma_0}\right)^{\frac{2}{p}} f_v(\alpha), \tag{13}$$

where f_v can be calculated by means of the Mises yield condition, see Table 1. Due to the same fracture mechanism the size of the plastic zones for the crack and the notch are the same. Comparing of Eqs. (12) and (13), we get the expression for the threshold value for crack initiation at the notch tip, H_{th}, in the form

$$H_{th} = K_{th}^{2p} \sigma_0^{(1-2p)} \left(\frac{f_v(\alpha = 0)}{f_v(\alpha)} \right)^{\frac{p}{2}} . \tag{14}$$

In a similar way the relation between H_{th} and K_{th} can be derived by means of the equality of the dimensions $r_p(\vartheta = 0)$ of the elastic–plastic boundary ahead of the crack and the notch tip.

The condition of notch stability can then be expressed in the form

$$H_I < H_{th}(K_{th}, \alpha). \tag{15}$$

If condition (15) holds, a fatigue crack at the notch tip is not initiated. The H_{th} value corresponds to the threshold value of H_I for a crack that emanates from a V-notch. In order to make it consistent with the fracture mechanics of cracks it can be called the generalized (notch) threshold value.

Calculation of the threshold value for the initiation of a crack at the notch tip based on condition (15) requires a procedure for estimating the generalized stress intensity factor H_I. This problem is solved in the next Section.

5. Numerical estimation of generalized stress intensity factor

Procedures for calculating the H_I value are not generally available. In the following, a procedure based on a direct method of estimating H_I is presented and discussed.

Direct methods, see e.g. [6, 9], are based on comparison of the numerical results with an analytical representation of the corresponding variables. This method can be used for 2D and 3D structures. The advantage of the method is that it utilizes standard numerical procedures (i.e., the standard finite element procedure) and thus can be used in conjunction with conventional numerical systems. The disadvantage of the method is that the accuracy of the results is relatively low and depends on the numerical model used (e.g., on the size of the mesh in the case of finite element methods).

In this paper the stress components σ_{ij} (Eqs. (10)) are used and compared.

In order to apply a direct method the stress distribution near the notch tip must be known. In the following, the FEM system ANSYS [2] was used for the numerical calculation of stresses. The V-notched body was modelled by using a half-notch model with symmetric boundary conditions (Fig. 2). The details of the finite-element mesh are shown in Fig. 3. Note the relatively highly refined mesh near the notch tip.

The values of H_I can be calculated by using the stresses in the near vicinity of the notch tip. For example if the stress component $\sigma_{\vartheta\vartheta}$ is used, it follows from Eq. (10) that

$$H_I = \sigma_{\vartheta\vartheta} \sqrt{2\pi} r^{p_I} / [(2 - 3p_I + p_I^2) \cos(p_I \vartheta) + q_I (2 - 3p_I + p_I^2) \cos((2 - p_I)\vartheta)] . \tag{16}$$

Similar relations can be found for other stress components or for displacements.

Substituting the numerically calculated value of the stress $\sigma_{\vartheta\vartheta}(r, \vartheta)$ and the corresponding values of p_I and q_I known from the analytical solution (see Table 1) into Eq. (16), we obtain the corresponding value of H_I.

Note that estimating the value of H_I by using the stress at nodal points too close to the notch tip can give poor results because the conventional elements do not adequately represent the singularity conditions at the notch tip. On the other hand, calculations based on the stresses at nodal points distant from the notch tip give poor estimates because the stresses equation (10) for the notch tip are accurate only for the limit as $r \to 0$. Analogous to the corresponding problem of estimating K_I for a crack, a good estimate of the value of H_I can be obtained from the $H_I = H_I(r)$ by extrapolating

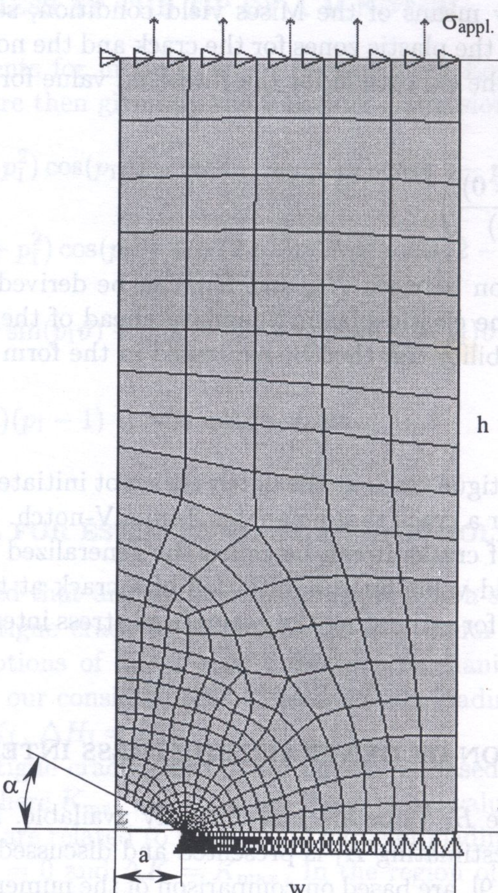

Fig. 2. The finite-element mesh for the V-notch (half model)

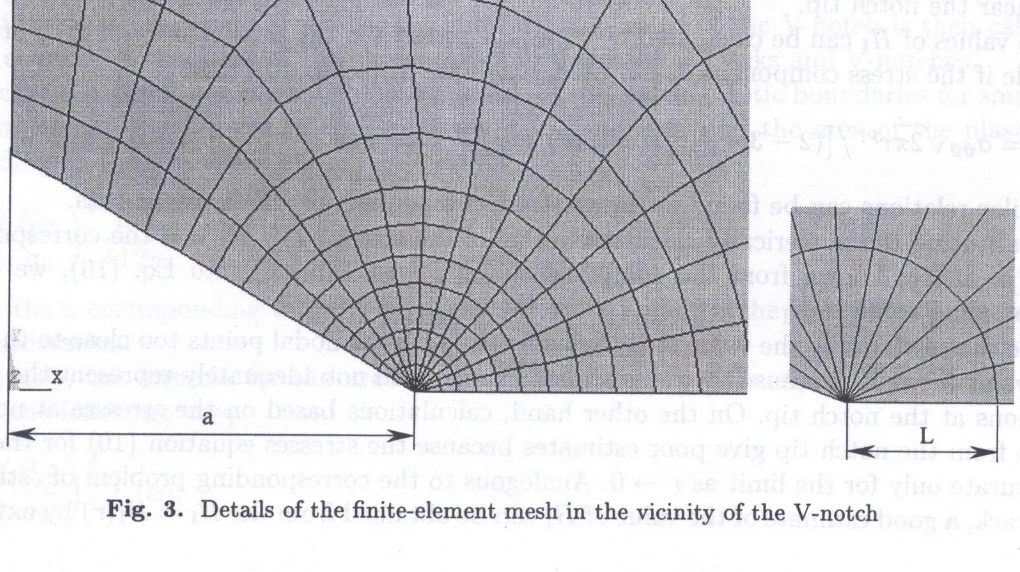

Fig. 3. Details of the finite-element mesh in the vicinity of the V-notch

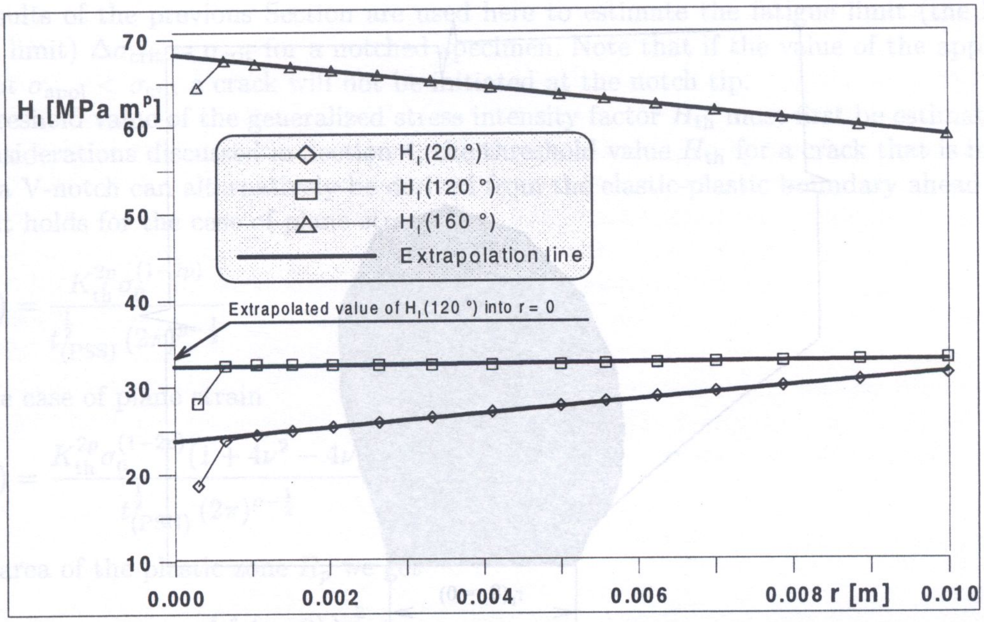

Fig. 4. Estimates of H_{I} by extrapolation for V-notch opening angles $2\alpha = 20°$, $120°$ and $160°$

the straight portion of the curve to the notch tip, i.e. to the point $r = 0$, as shown in Fig. 4. The $H_{\mathrm{I}} = H_{\mathrm{I}}(r)$ have been calculated according to Eq. (16) for different angles of the notch opening and different mesh refinements. A great number of numerical simulations with different mesh refinements have been performed in the framework of this paper. The degree of the mesh refinement has been expressed by the ratio of the size L of the smallest triangular elements at the V-notch tip and the notch depth a (Fig. 3). The ratio L/a has been varied in the interval $0.03 < L/a < 0.2$, the corresponding meshes have been generated, and the numerical calculations have been performed.

It can be concluded from the results of the numerical simulations performed that a refinement of the finite-element mesh which gives correct values of the stress intensity factor K_{I} is also suitable for estimating the values of H_{I}. Thus for a body with a given geometry and a notch, a finite element mesh is first generated where in the notch is replaced by a crack with a length corresponding to the depth of the notch. Then the K_{I} value of the cracked body is estimated by the direct method. The error in the value of H_{I} calculated for the same geometry with the same mesh refinement is then the same as the error for the K_{I} value.

Another method of computing the generalized stress intensity factor H_{I} is based on determination of the size of plastic zone. Plastic zone size in the assumed direction of propagation of the crack from the V-notch tip $r_p(\vartheta{=}0)$ is used to describe the plastic zone as shown in Fig. 5. H_{I} can then be written in the form

$$H_{\mathrm{I}} = \sigma_0 r_p^{p_{\mathrm{I}}} \sqrt{\frac{2\pi}{t}} \tag{17}$$

where σ_0 is the yield stress, and t is given by the following expressions,

$$t(\mathrm{PSS}) = (p_{\mathrm{I}} - 1)^2 (3p_{\mathrm{I}}^2 + 3p_{\mathrm{I}}^2 q_{\mathrm{I}}^2 + 6p_{\mathrm{I}}^2 q_{\mathrm{I}} - 12q_{\mathrm{I}}^2 p_{\mathrm{I}} - 12q_{\mathrm{I}} p_{\mathrm{I}} + 12q_{\mathrm{I}}^2 + 4) \quad \text{(for plane stress)}$$

$$t(\mathrm{PSN}) = (p_{\mathrm{I}} - 1)^2 (3p_{\mathrm{I}}^2 + 3p_{\mathrm{I}}^2 q_{\mathrm{I}}^2 + 6p_{\mathrm{I}}^2 q_{\mathrm{I}} - 12q_{\mathrm{I}}^2 p_{\mathrm{I}} - 12q_{\mathrm{I}} p_{\mathrm{I}} + 12q_{\mathrm{I}}^2 + 4 + 16\nu^2 - 16\nu) \quad \text{(for plane strain).}$$

Similarly, using the area of the plastic zone we get

$$H_{\mathrm{I}} = \left(\frac{Rp}{f_v(\alpha)} \right)^{\frac{p_{\mathrm{I}}}{2}} \sigma_0 \tag{18}$$

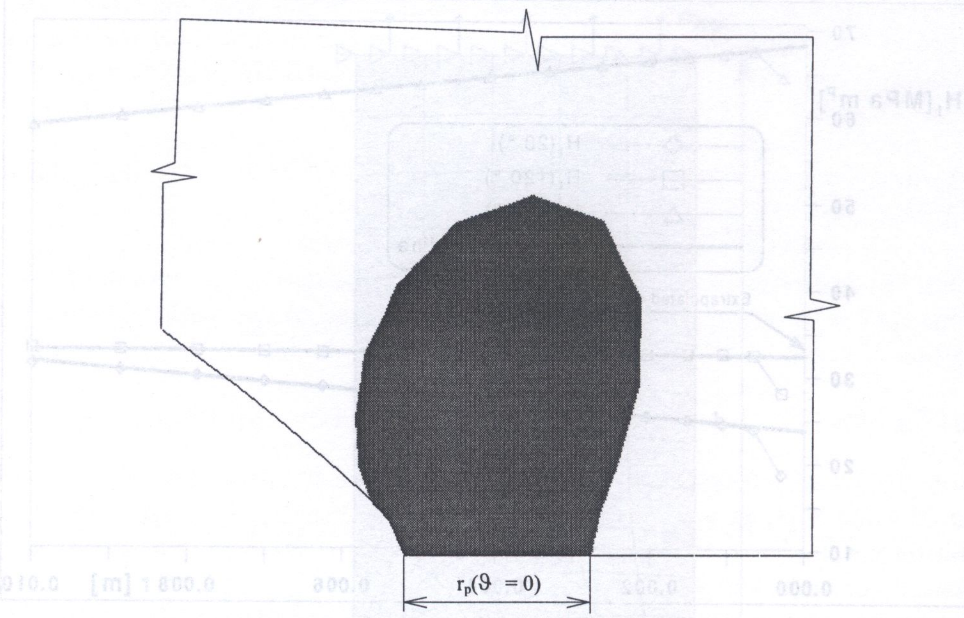

Fig. 5. The plastic zone at a deformed V-notch tip (half model, ANSYS)

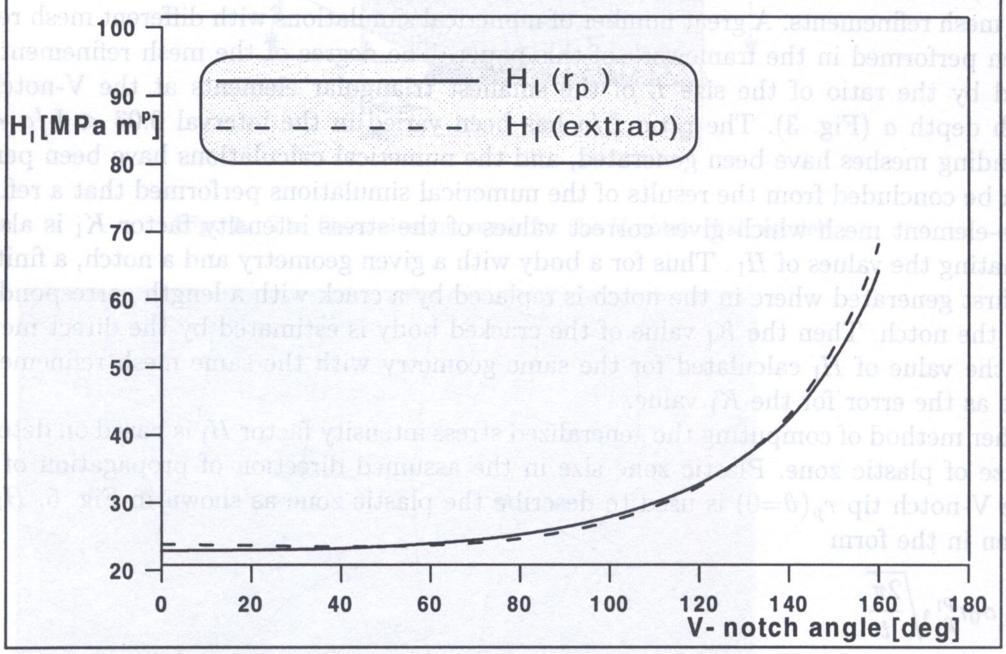

Fig. 6. The dependence of the generalized stress intensity factor on the opening angle of the V-notch

where R_p is the area of the plastic zone and f_v is normalized area of the plastic zone. Values of f_v for 5° increments of angle are shown in Table 1. This procedure for estimating H_I is again based on a direct method. Instead of Eq. (16) the expression for the Mises equivalent stress is used and the calculations are performed for the elastic–plastic boundary

The dependence of the generalized stress intensity factor $H_\mathrm{I}(2\alpha)$ as estimated by extrapolation (Eq. (18)) and as calculated using the area of the plastic zone (Eq. (17)) on the opening angle of the V-notch is shown in Fig. 6. The value of the applied stress $\sigma_{\mathrm{appl}} = 100$ MPa, while $a = 10$ mm, $w = 50$ mm, and $h = 125$ mm.

The results of the previous Section are used here to estimate the fatigue limit (the high-cycle endurance limit) $\Delta\sigma_{\text{crit}} = \sigma_{\text{crit}}$ for a notched specimen. Note that if the value of the applied stress is such that $\sigma_{\text{appl}} < \sigma_{\text{crit}}$ a crack will not be initiated at the notch tip.

The threshold value of the generalized stress intensity factor H_{th} must first be estimated. Based on the considerations discussed in Section 4, the threshold value H_{th} for a crack that is initiated at the tip of a V-notch can alternatively be derived from the elastic-plastic boundary ahead of the tip $r_p(\vartheta{=}0)$. It holds for the case of plane stress that

$$H_{\text{th}}(r_p) = \frac{K_{\text{th}}^{2p}\sigma_0^{(1-2p)}}{t_{(\text{PSS})}^{\frac{1}{2}}(2\pi)^{p-\frac{1}{2}}} \tag{19}$$

and for the case of plane strain

$$H_{\text{th}}(r_p) = \frac{K_{\text{th}}^{2p}\sigma_0^{(1-2p)}(1+4\nu^2-4\nu)}{t_{(\text{PSN})}^{\frac{1}{2}}(2\pi)^{p-\frac{1}{2}}}\,.$$

From the area of the plastic zone R_p we get

$$H_{\text{th}}(R_p) = K_{\text{th}}^{2p}\sigma_0^{(1-2p)}\left(\frac{f_v(\alpha{=}0)}{f_v(\alpha)}\right)^{\frac{p}{2}}. \tag{20}$$

In Eqs. (19), (20), K_{th} is the threshold value of the stress intensity factor.

Then the critical stresses σ_{crit} are computed from the condition

$$\sigma_{\text{crit}} H_{\text{I}}(\sigma_{\text{appl}}) = \sigma_{\text{appl}} H_{\text{th}}\,. \tag{21}$$

Note that the case $2\alpha = 0$ corresponds to a crack. The computed values of the critical stresses σ_{crit} for the notched specimen (see Fig. 2 where $a = 10$ mm, $w = 50$ mm, and $h = 125$ mm) based on different approaches are presented in Fig. 7. The calculation was performed for $K_{\text{th}} = 50\ \text{MPa}{\cdot}\text{m}^{1/2}$ and an approximation of the plane stress. The corresponding values of H_{I} are taken from Fig. 6, where $\sigma_{\text{appl}} = 100$ MPa.

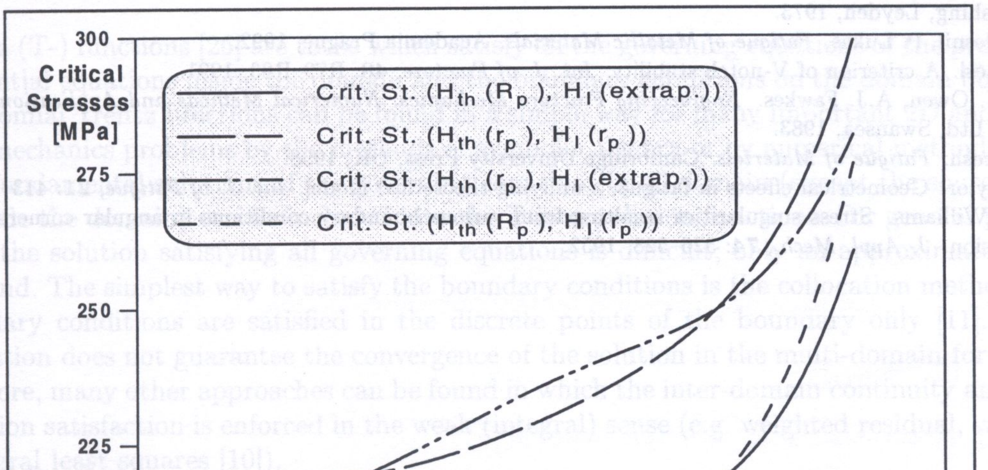

Fig. 7. The dependence of the critical stresses on the opening angle of the V-notch

6. Conclusions

In this paper a simple procedure for evaluating the threshold value H_{th} for the propagation of a fatigue crack that starts at a V-notch is suggested. The fatigue limit σ_{crit} of the notched body can then be estimated by means of the threshold value H_{th} of the generalized stress intensity factor H_{I}. In order to apply the procedure it is necessary to estimate of the value of H_{I} numerically for the given structure and boundary conditions. One of the most general and effective methods of estimating the H_{I} value, suggested and tested in this paper, is based on a direct comparison between the analytical solution and numerically obtained values of the stresses. This direct method can be used in connection with standard finite elements, but it needs a relatively highly refined finite element mesh. A recommended procedure for refining the mesh near the tip of a notch was obtained, see Section 5. Based on the considerations discussed in the paper, the value of the fatigue limit for a notched specimen was obtained. The suggested procedure is generally valid and can easily be adapted to mixed modes of loading.

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