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A quasi-evolutionary identification attempt to modelling of steel frames†

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The work presents a process of analytical identification via a standard steel frame example. Some experimental tests are made to verify the identification process. Under controlled external loadings the values of displacements and strains are recorded and an approximate FEM-based model is formulated. The poly-optimization approach is employed to analyze that model. The compatibility criteria for comparison of theoretical and experimental models are assumed as square sums of differences between displacements and strains. The whole problem is proceeded in three cycles of evolution suggested by the authors.

1. Introduction

The first stage of designing process is working out an analytical model of structure. The computer implementation of such a model has a great influence on analysis and output data processing. Usually, some simplified schemes are adopted to solve the governing equation. In the case of more complex structure, however, even the smallest simplifications may be a source of wrongdoing. In such a situation the most appropriate computational model of structure is required. As an example, real support conditions frequently go far beyond those conventional schemes. The reason for those discrepancies may occur due to hinge friction or/and mismatching. Such parameters are difficult to compute without identification analysis.

The identification procedure may establish more precise conditions according to requirements for the structural system at hand. The whole problem can then be modelled and solved for as an optimization problem. In other words, it seems to be reasonable to create the best possible model in terms of optimization techniques [10]. They are particularly useful when complex objects are analyzed. Additionally, an identification-based process some characteristic features of the optimization problem should be taken into account. It is known that the identification procedure is usually considered as monocriteria problem, but the multicriteria approach may give better results in accordance with the real structure. A typical property of the multicriteria problem is co-operation of criteria. The converge criteria of the theoretical model and real object are expressed via an objective functions vector, defined usually as a square sum of differences of experimental and theoretical results. Identification parameters such as supports flexibility, range of local stiffening, stiffness of connections, etc. form a vector of decision variables. The block scheme of identification procedure is shown in Fig. 1 [4].

Another fact that makes the identification process more complicated is that each structure must be analyzed separately. At the beginning of each analysis the available knowledge concerning the design parameters is usually insufficient, since there is no information returning from the existing objects that stands for the basic reason for the lack of data. Some guesses and assumptions are thus

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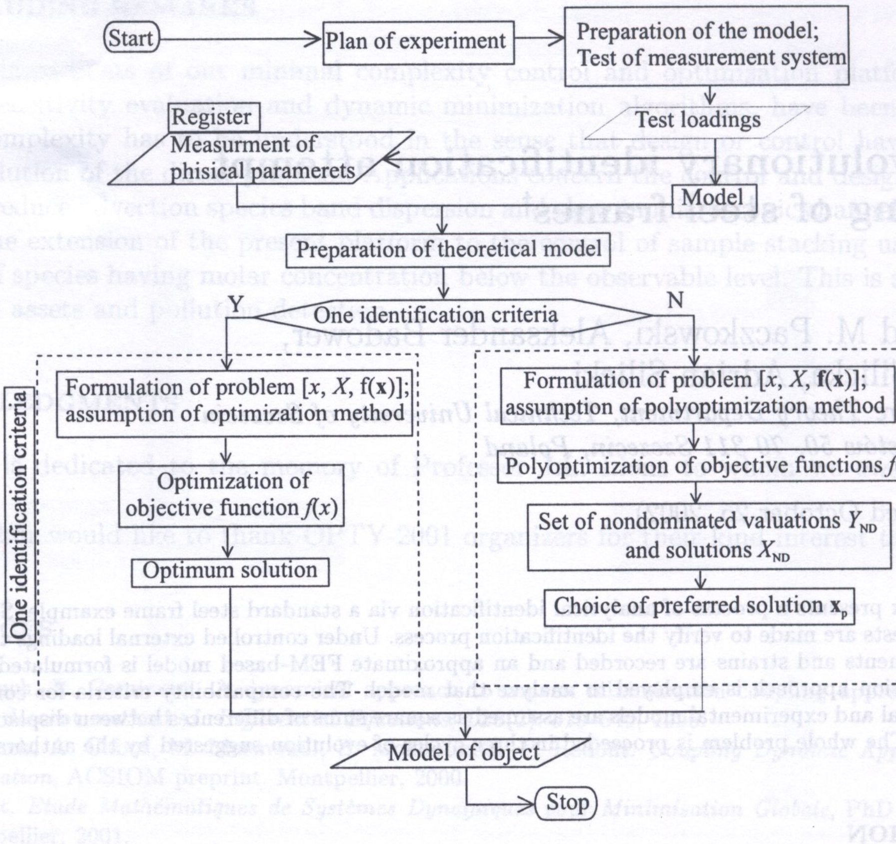


Fig. 1. Block scheme presenting quasi-evolutionary identification process

required at the start of the optimization process. More precise parameters of the considered object can then be find out only by further analysis. The process often requires some assumption to be successively modified during analysis. This fact implies the idea of a quasi-evolutionary approach to optimization process [10].

The quasi-evolutionary optimization process consists of few cycles, each of them stands for a standard numerical problem. The term 'quasi' is used here to avoid misleading with the traditional evolutionary optimization procedure in which the genetic algorithms are required. In the quasi-evolutionary formulation each solution gives information and experience that allow one to obtain more precise assumptions and select more effective solution methods excluding any insignificant elements of solution. On the base of the former results the problem is analyzed again in the next cycle. In this context, the quasi-evolutionary approach is similar to the evolution of the outer world, where only those better and more adaptive forms give good start to next generation.

2. Problem statement

The main idea the identification formulation proposed in the text is best presented through a steel frame model. Its dimensions are shown in Fig. 2a. The cross-sections of the cold-formed profiles are shown in Fig. 2b. The goal is to simulate the load-carrying structure of a real one-bay hall by a loaded frame structure, that, in turn, is modelled as a plane system. This system is a part of a test-stand shown in Fig. 3.

The model consists of a symmetric two-sloped spandrel beam, jointed at the roof ridge using a screw M16 and two posts. The support area of the posts may be modelled alternatively as fixed (Fig. 2a-A) or hinged (Fig. 2a-B) to obtain three static schemes — the frame without or with one or two posts with the hinged supports. The connections between the spandrel beam and the posts are made fixed with an additional steel plate inside the posts. The test-stand includes two

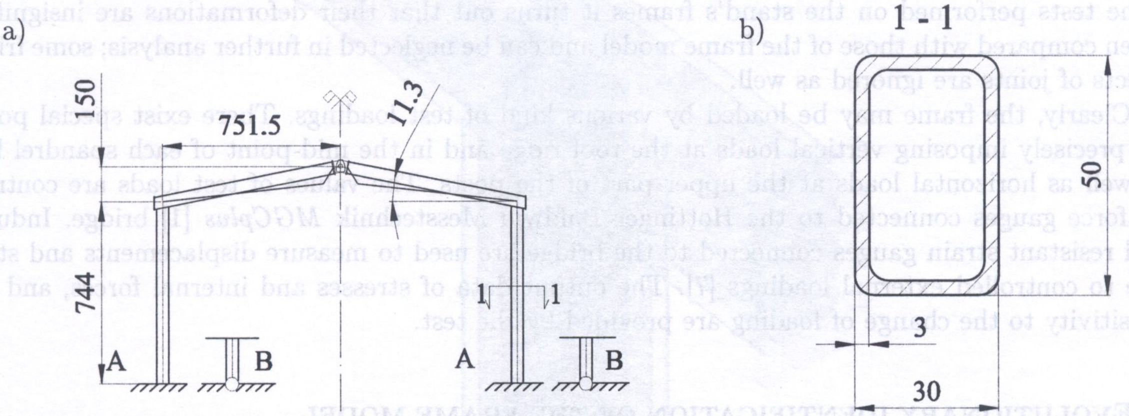


Fig. 2. Dimensions of the analyzed frame

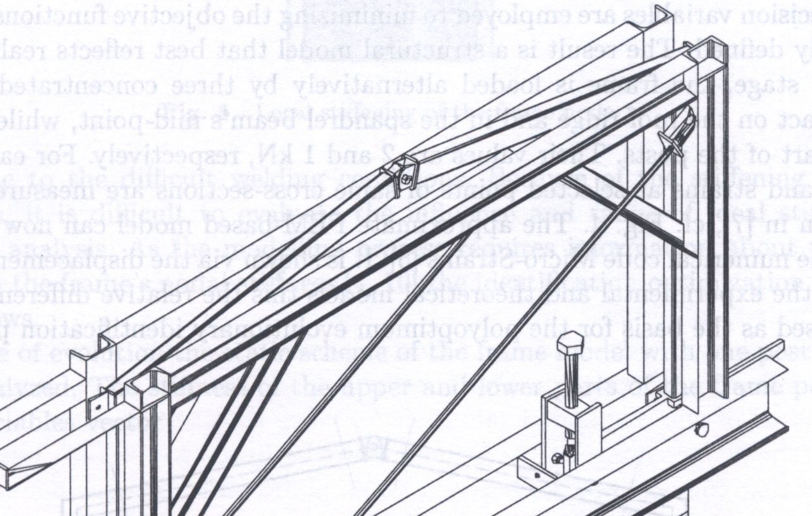


Fig. 3. Analyzed frame in test-stand

symmetric quasi-rigid frames, i.e. of stiffness much larger than that of the analyzed frame. After some tests performed on the stand's frames it turns out that their deformations are insignificant when compared with those of the frame model and can be neglected in further analysis; some friction effects of joints are ignored as well.

Clearly, the frame may be loaded by various kind of test loadings. There exist special pockets for precisely imposing vertical loads at the roof ridge and in the mid-point of each spandrel beam as well as horizontal loads at the upper part of the posts. The values of test loads are controlled by force gauges connected to the Hottinger Baldwin Messtechnik *MGCplus* [1] bridge. Inductive and resistant strain gauges connected to the bridge are used to measure displacements and strains due to controlled external loadings [7]. The output data of stresses and internal forces, and their sensitivity to the change of loading are provided by the test.

3. Evolutionary identification of the frame model

3.1. The scope of the problem

The identification process consists of many stages. First, the frame is loaded. Next, a numerical model is formed with the identification parameters defined as our decision variables. Further, the values of the decision variables are employed to minimizing the objective functions as an optimization problem already defined. The result is a structural model that best reflects reality [4].

At the first stage, the frame is loaded alternatively by three concentrated forces — the two vertical forces act on the roof ridge and in the spandrel beam's mid-point, while the horizontal one in the upper part of the posts. Their values are 2 and 1 kN, respectively. For each external loading displacements and strains at selected points of same cross-sections are measured according to the procedure given in [7], cf. Fig. 4. The approximate FEM-based model can now be formulated and analyzed via the numerical code Micro-Strains [5]. It is shown via the displacement and strain results obtained from the experimental and theoretical models this the relative differences are about 30%. They will be used as the basis for the polyoptimum evolutionary identification process as discussed bellow.

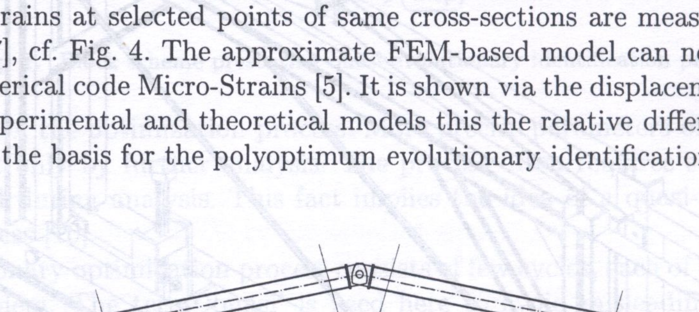


Fig. 4. Specified cross-sections of the frame

3.2. First evolution cycle

At this stage the local stiffenings of frame nodes (shown in Fig. 5) are considered as the main reason for the differences between theoretical and experimental test results. Since the dimensions of the cross-sections are rather small the connections between the plates and the posts may be

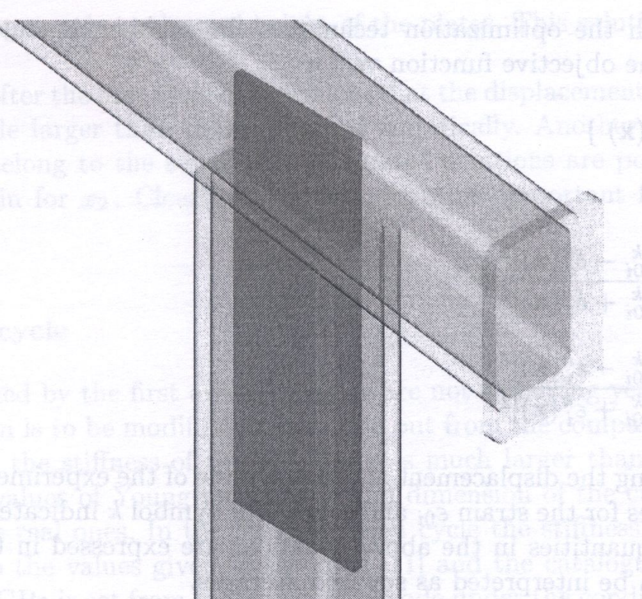


Fig. 5. Local stiffening of the frame node

non-monolithic due to the difficult welding conditions. Because of the stiffening plates and the welding conditions, it is difficult to evaluate the influence and range of local stiffening with no additional specific analysis. As the modelling process requires information about the influence of stiffening plates on the frame's nodal stiffness [4, 10] the identification optimization problem can be formulated as follows.

In the first cycle of evolution the static scheme of the frame model with one post hinged and the second fixed is analyzed. The stiffness of the upper and lower parts of the frame posts are defined by the decision variables vector

$$\mathbf{x} = \{\, x_1 \;\, x_2 \,\} \tag{1}$$

where x_1 denotes the effective thickness and x_2 the effective length of the nodal plates (the length of welds). In this way the real connection conditions between nodal plate and the wall of posts are considered in the model. The upper and lower bounds of their allowable values are given in Table 1 with n_1 and n_2 being positive integers.

Table 1. Bounds of feasible domain

Cube restrain [mm]	Discrete restrain [mm]
$0 \le x_1 \le 3.0$	$x_1 = n_1 \cdot 0.3$
$0 \le x_2 \le 150.0$	$x_2 = n_2 \cdot 5.0$

For the variable x_1 the upper restrain, being the real thickness of the nodal plates, describes the full monolithic connection between the plates and the posts, while the lower one with the zero thickness describes the zero connection and zero local stiffening. For the decision variable x_2 the upper restrain is the real height of the nodal plates (150 mm), while the lower one — zero height. The code Micro-Strains requires discretizing of the decision variables. The thickness x_1 and length x_2 are changed with the increments 0.3 mm and 5.0 mm, respectively. By those restrains the solution and valuation spaces stand for the discrete ones and the problem stands for a discrete optimization one.

In accordance with the optimization techniques the problem is stated with two optimization criteria, defined by the objective function vector

$$\mathbf{f}(\mathbf{x}) = \{\ f_1(\mathbf{x})\ \ f_2(\mathbf{x})\ \} \tag{2}$$

where

$$f_1(\mathbf{x}) = \sum_{k=1}^{3}\sum_{i=1}^{n}\frac{[\delta_{0i}^k - \delta_i^k(\mathbf{x})]^2}{[\delta_{0i}^k + \delta_i^k(\mathbf{x})]^2}\,, \tag{3}$$

$$f_2(\mathbf{x}) = \sum_{k=1}^{3}\sum_{i=1}^{n}\frac{[\varepsilon_{0i}^k - \varepsilon_i^k(\mathbf{x})]^2}{[\varepsilon_{0i}^k + \varepsilon_i^k(\mathbf{x})]^2}\,, \tag{4}$$

with δ_{0i} and $\delta_i(\mathbf{x})$ being the displacement at the i-th node of the experimental and numerical model, respectively, so as does for the strain ε_{0i} and $\varepsilon_i(\mathbf{x})$. The symbol k indicates the load scheme number. It is noted that the quantities in the above equations are expressed in the non-dimensional form; the denominators can be interpreted as squared averages.

The numerical algorithm to this problem consists of two steps. The first is to find out the solution $\check{\mathbf{x}}_1$ that minimizes function $f_1(\mathbf{x})$ by using an iterative procedure typical of the Gauss–Seidel method [2]. An optimum solution that minimizes the objective function is to be searched in the orthogonal contiguity of the start point. When the contiguity contains a solution that decreases the value of the objective function the direction of the search is determined. If such a solution does not exist, the direction of the search is changed in the orthogonal way. At the second step the sets of non-dominated solutions and valuations are calculated by ortho-diagonal method [8, 10]. Both the orthogonal and the diagonal contiguity are now analyzed to get a solution that decreases the value of the other function(s). From the set of obtained solutions the ones that cannot be uniquely improved are selected. They create a set of non-dominated solutions and its inverse images, i.e. the set of non-dominated valuations.

The Gauss–Seidel and ortho-diagonal solutions include three-entry set of the non-dominated solutions and valuations, as shown in Fig. 6, points 16, 17 and 19. The preferred valuation $\mathbf{y}_{17}$ is selected from the set of non-dominated valuations by using the distance function method with norm $\|p\| = 2$ [9, 12]. The inverse image of $\mathbf{y}_{17}$ is the preferred solution $\mathbf{x}_{17} = \{2.7, 150.0\}$, whose physical interpretation is that the connections between nodal plates and posts are non-monolithic, and the

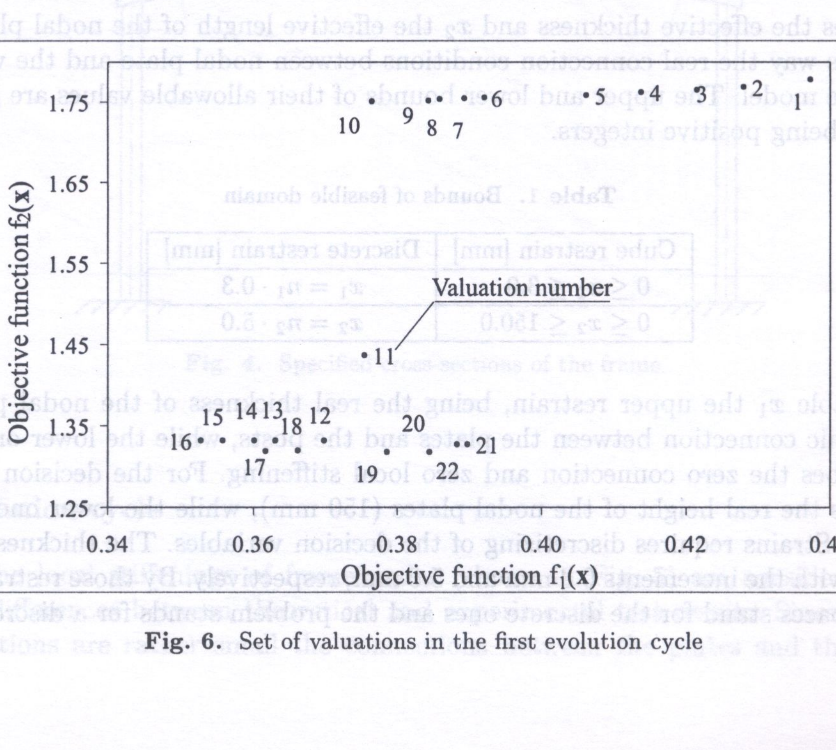


Fig. 6. Set of valuations in the first evolution cycle

range of the local stiffening equals to the real height of the plates. This solution conforms both the criteria in best way.

It is pointed out here, after the first cycle of evolution, that the displacements and strains obtained numerically are as a whole larger than those obtained empirically. Another important conclusion is that all the elements belong to the set of non-dominated solutions are positioned at the upper bound of the cube restrain for x_2. Clearly, there may be other important factors not taken into account yet.

3.3. Second evolution cycle

Since the solutions obtained by the first evolution cycle are not satisfying yet the above-mentioned formulation of the problem is to be modified. It is turned out from the comparison of numerical and experimental results that the stiffness of the real frame is much larger than that of its numerical model. It is because the values of Young modulus E and dimension of the cross-section have been assumed smaller than the real ones. In the first evolution cycle the stiffness parameters are taken into account according to the values given in the code [11] and the catalogue [3]. As an example, Young modulus $E = 205$ GPa is set from the Polish steel code under the condition of the probability of its exceeding being large enough. Moreover, the experiment carried out for testing bars sampled from the same elements as the analyzed model shows that the value of E for the samples are about 215 GPa. The real cross-sectional dimensions measured in several sections are also differed slightly from those in the catalogue. That is why the stiffness of cross sections may be greater than that from the first cycle.

To modify the cross-sectional stiffness at the second stage of evolution, a scale coefficient ξ is introduced as the additional third entry of the vector of decision variables

$$\mathbf{x} = \{\, x_1 \ x_2 \ x_3 \,\} \tag{5}$$

The x_3 is defined to take uncertainties of E and I into account. It is available then to create better model. Both the bending and longitudinal stiffnesses are to be scaled by multiplying by x_3. As before, x_3 is also to be restrained with appropriate lower and upper bounds. The lower limit is 1

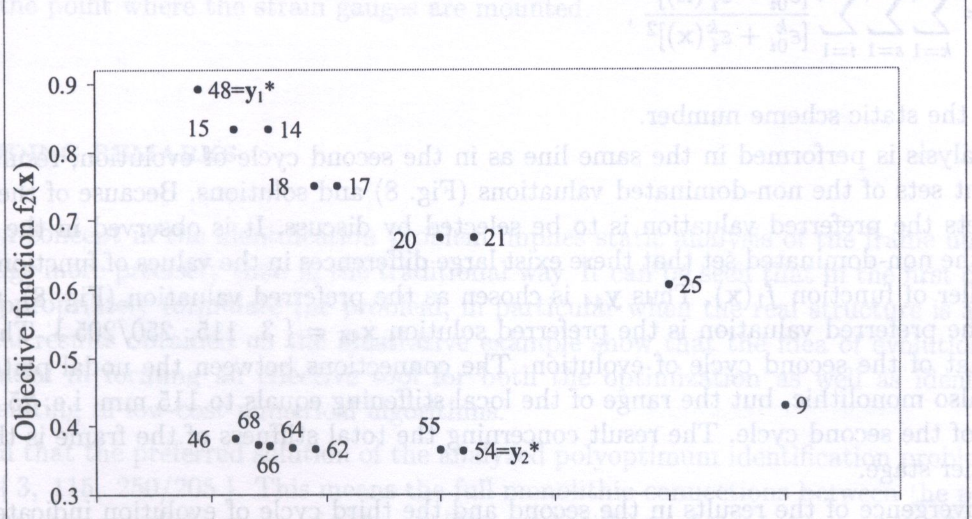


Fig. 7. Set of valuations in the second evolution cycle

and that means zero probability for the event that both values of E and I are smaller than those from the code and the catalogue [3, 11]. The upper limit appears to be $E_{\text{exp}}/E_{\text{norm}} = 1.22$, E_{exp} and E_{norm} being the experimental and 'normed' values, respectively. Since the value 5 GPa is a precision for E in the Polish code x_3 has to be changed at step $5/205 = 0.024$. Apart from the vector (5) other quantities involved in formulation remain unchanged.

In order to improve the analysis procedure and graphical interpretation of results the vector $\mathbf{x}$ can now be decomposed [10] into two local vectors $\mathbf{x} = \{\, \mathbf{x}^{(1)}\ \mathbf{x}^{(2)} \,\}$, with $\mathbf{x}^{(1)} = \{\, x_1\ x_2 \,\}$ and $\mathbf{x}^{(2)} = \{\, x_3 \,\}$. The solution proceeded with a few local stages, with the x_3 assumed as a parameter each successive problem is solved for $\mathbf{x}^{(1)}$ employing both the Gauss–Seidel and ortho-diagonal methods. The results obtained at each stage stand for the local sets of non-dominated solutions and valuations. The global 7-elements sets of non-dominated solutions and valuations will be the sum of the local ones (Fig. 7). The preferred valuation is selected via the distance method. Three elements with the highest values of objective functions have to be withdrew in that process. The preferred solution is the vector $\mathbf{x}_{68} = \{\, 3,\ 90,\ 250/205 \}$ shown in Fig. 7. This means that the connections between the nodal plates and the posts are monolithic with the range 90 mm. The total stiffness of the structure is of 22% larger than that from the norm and catalogue. The values of the objective functions are lower than those of the first cycle of evolution.

3.4. Third evolution cycle

In the third cycle of evolution two additional static schemes are taken into account (Fig. 2a). For that purpose the experimental tests are made on the real model with appropriate support conditions. Also the numerical models are prepared for the real frame with three hinges and two post fixed supports. These models are built in the way so that the results can be best verified. In accordance with these modifications the objective functions can be defined as:

$$f_1(\mathbf{x}) = \sum_{k=1}^{3}\sum_{s=1}^{3}\sum_{i=1}^{n} \frac{[\delta_{0i}^k - \delta_i^k(\mathbf{x})]^2}{[\delta_{0i}^k + \delta_i^k(\mathbf{x})]^2}, \tag{6}$$

$$f_2(\mathbf{x}) = \sum_{k=1}^{3}\sum_{s=1}^{3}\sum_{i=1}^{n} \frac{[\varepsilon_{0i}^k - \varepsilon_i^k(\mathbf{x})]^2}{[\varepsilon_{0i}^k + \varepsilon_i^k(\mathbf{x})]^2}, \tag{7}$$

where s is the static scheme number.

The analysis is performed in the same line as in the second cycle of evolution, resulting in the two-element sets of the non-dominated valuations (Fig. 8) and solutions. Because of the small size of those sets the preferred valuation is to be selected by discuss. It is observed in the valuations belong to the non-dominated set that there exist large differences in the values of function $f_2(\mathbf{x})$ and much smaller of function $f_1(\mathbf{x})$. Thus $\mathbf{y}_{44}$ is chosen as the preferred valuation (Fig. 8), the inverse image of the preferred valuation is the preferred solution $\mathbf{x}_{44} = \{\, 3,\ 115,\ 250/205 \,\}$. This result is close to that of the second cycle of evolution. The connections between the nodal plates and the posts are also monolithic, but the range of the local stiffening equals to 115 mm, i.e. 25 mm larger than that of the second cycle. The result concerning the total stiffness of the frame is the same as in the former stage.

The convergence of the results in the second and the third cycle of evolution indicates that the formulated mathematical model reflects well the real structure. After the third cycle of evolution another experimental tests are made to control the numerical model. The frame is subjected to different test loadings and the difference of the structure the mathematical models are about 5%. This means that by employing the results of the identification process much better numerical model than the starting one can be obtained.

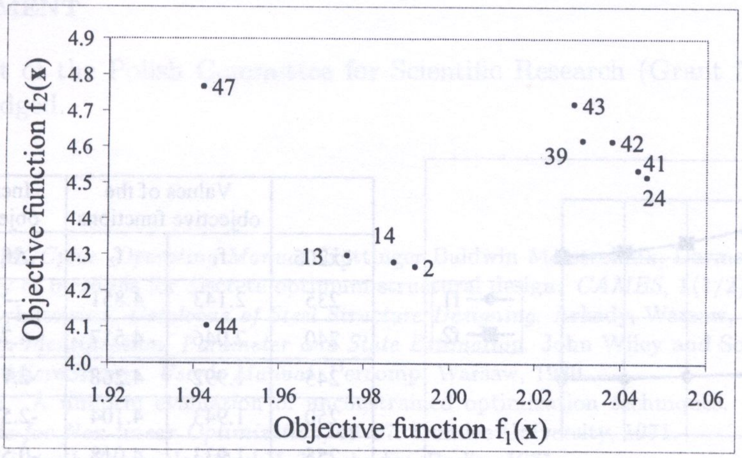


Fig. 8. Set of valuations in the third evolution cycle

4. Sensitivity analysis

The sensitivity of the objective functions to the changes of the decision variables is analyzed by using finite difference method. All the decision variables are assumed mutually independent to evaluate influence of the decision variables on the objective functions. The results are illustrated in Fig. 9.

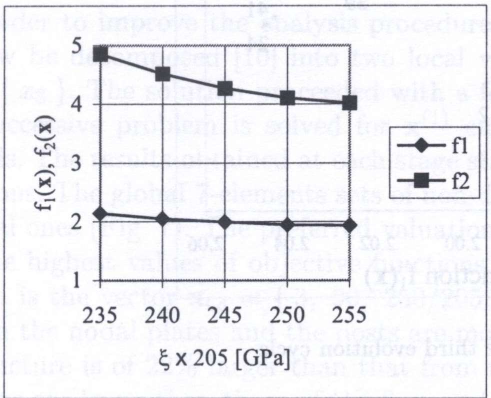
It is seen from the results that the variable x_3 has a great influence on both the functions $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$. Since the variables x_1 and x_2 determine only measures of local nodal stiffness the function $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$ are more sensitive to the variation of the x_3 than the x_1 and x_2 . The x_3 determines total stiffness of structure and that is the basic reason of its significance.

In addition, the function $f_2(\mathbf{x})$ is more sensitive to the decision variables in comparison with the function $f_1(\mathbf{x})$, because the stiffening plates affects significantly strains in the measuring points around to the nodes. These affects are more clearly seen when the measuring points are in the range or out of range of the nodal plates. The great changes of objective functions occur when x_2 exceeded 120 mm, at the point where the strain gauges are mounted.

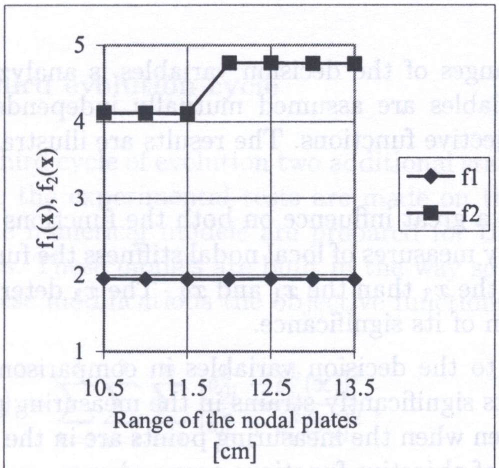
5. Concluding remarks

The evolution concept in the identification problem implies static analysis of the frame under controlled loadings more precisely than in the traditional way. It can be seen that in the first cycle it is difficult to appropriately formulate the problem, in particular when the real structure is analyzed. The numerical results obtained on the illustrative example show that the idea of evolution should become essential in forming an effective tool for both the optimization as well as identification problems, resulting in low-cost numerical algorithms.

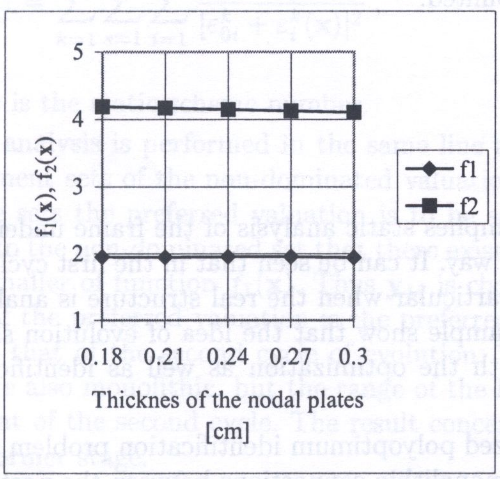
It is shown that the preferred solution of the analyzed polyoptimum identification problem is the vector $\mathbf{x}_p = \{\, 3,\ 115,\ 250/205 \,\}$. This means the full monolithic connections between the posts and the stiffening plates are of the range of 115 mm. The total stiffness of the frame is about 20% larger than given in the catalogue and code. The discrepancies are of the 5% when compared with the analytical solutions for displacements and strains. These results are much better than those from the starting model. The remaining differences between numerical and experimental results should be treated as effects of the random character of other values that are not taken into account in the analysis yet.



	Values of the objective functions		Increments of the objective functions	
ξx205	f_1	f_2	Δf_1	Δf_2
235	2.143	4.851	--	--
240	2.046	4.517	-4.74	-7.39
245	1.992	4.268	-2.71	-5.83
250	1.943	4.104	-2.52	-4.00
255	1.933	4.018	-0.52	-2.14



	Values of the objective functions		Increments of the objective functions	
Range of the plates	f_1	f_2	Δf_1	Δf_2
10.5	1.949	4.115		
11	1.943	4.110	-0.31	-0.12
11.5	1.943	4.104	0.00	-0.15
12	1.942	4.767	-0.05	13.91
12.5	1.945	4.765	0.15	-0.04
13	1.944	4.758	-0.05	-0.15
13.5	1.94	4.755	-0.21	-0.06



	Values of the objective functions		Increments of the objective functions	
Thickness of the plates	f_1	f_2	Δf_1	Δf_2
0.18	1.95	4.189	--	--
0.21	1.945	4.166	-0.26	-0.55
0.24	1.954	4.144	0.46	-0.53
0.27	1.955	4.124	0.05	-0.48
0.3	1.943	4.104	-0.62	-0.49

Fig. 9. Sensitivity analysis of the objective function on variations of decision variables

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