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Neural network for constitutive modelling in finite element analysis

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Finite element method has, in recent years, been widely used as a powerful tool in analysis of engineering problems. In this numerical analysis, the behavior of the actual material is approximated with that of an idealized material that deforms in accordance with some constitutive relationships. Therefore, the choice of an appropriate constitutive model, which adequately describes the behavior of the material, plays a significant role in the accuracy and reliability of the numerical predictions. Several constitutive models have been developed for various materials. Most of these models involve determination of material parameters, many of which have no physical meaning [1, 2].

In this paper a neural network-based finite element analysis will be presented for modeling engineering problems. The methodology involves incorporation of neural network in a finite element program as a substitute to conventional constitutive material model. Capabilities of the presented methodology will be illustrated by application to practical engineering problems. The results of the analyses will be compared to those obtained from conventional constitutive models.

Keywords: finite element, neural network, constitutive modelling, soil.

1. Intelligent finite element method

An intelligent finite element method has been developed, based on the integration of neural network in a finite element framework. In the proposed methodology, a neural network is incorporated in the finite element analysis as a substitutive to constitutive material model. A neural network is trained using the raw experimental (or in-situ) data representing the mechanical response of the material to applied load. The trained network is then used in the finite element analysis to predict the relationship between the stress and strain in the material. This is illustrated in Fig. 1.

To illustrate the computational methodology, three numerical examples of application of the developed intelligent finite element code (NeuroFE program) to engineering problems are presented.

2. Numerical examples

To illustrate the computational methodology, three numerical examples of application of the developed intelligent self-learning finite element code (NeuroFE program) to engineering problems are presented. In the first example, application of the methodology to a simple case of linear elastic material behaviour is illustrated.

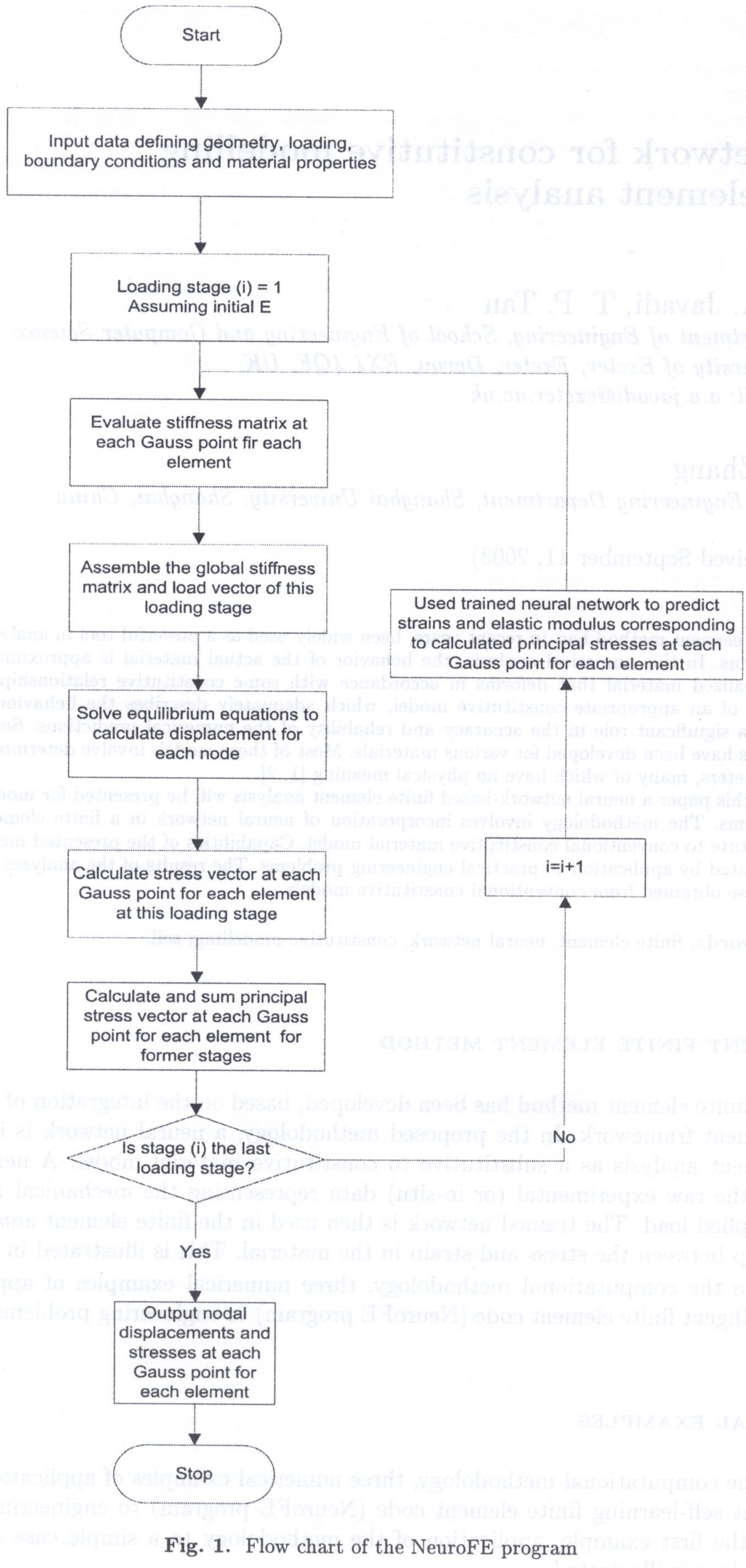

Fig. 1. Flow chart of the NeuroFE program

2.1. Example 1

The first example involves a beam of 1 [m] thickness and length of 15 [m]. This is shown in Fig. 3a. In the example, the beam is subjected to a point load of 1 [kN], as shown in the figure. This example was selected to confirm the computational methodology by evaluating the results to the analytical solution.

The problem is solved using 9 eight-node isoparametric elements. Figure 2a shows a linear elastic stress-strain relationship with a slope of 1000, which corresponds to the assumption of an elastic modulus of 1000 [kPa]. The data from this figure was used to train the neural network. A back-propagation neural network was trained to capture the linear stress-strain relationship. The architecture of the neural network used composed of an input layer, a hidden layer and an output layer. The learning rate for the neural network was set to 0.0000001 and the tolerance for the error measure was 0.001%.

Figure 2b shows the stress-strain relationship as predicted by the trained neural network from the data, together with the measured one. It is seen that after training, the neural network has successfully captured the stress-strain relationship.

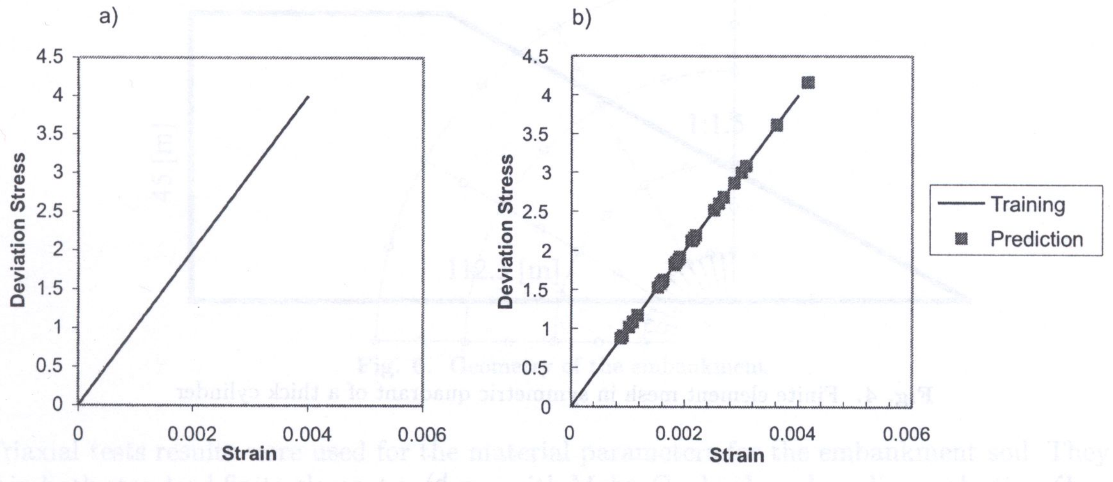

Fig. 2. a) linear stress-strain relationship used for training, b) Comparison between stress-strain relationships for training and prediction

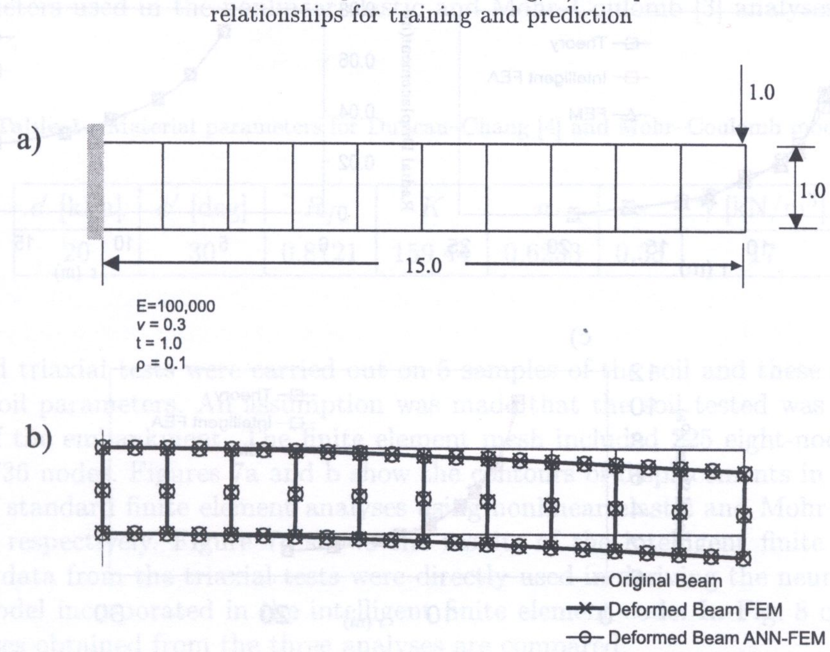

Fig. 3. a) Plane Stress Beam, b) comparison of results obtained by conventional FEM and by NeuroFEM

Figure 3b shows the results obtained both by conventional Finite Element Method and by the intelligent finite element method.

2.2. Example 2

Details of the second example are given in Fig. 4. This example involves a thick cylinder, 5 [m] in diameter and 15 [m] thickness, subject to uniformly distributed loading of 10 [kPa]. This example was also deliberately selected to verify the computational methodology by comparing the results to the analytical solution.

Like the beam example, the cylinder is made of linear elastic material. It has a Young's modulus of E=1000 [kPa] and Poisson's ratio of 0.3.

Results obtained by using the NeuroFEM have been compared with results obtained from a standard linear elastic finite element analysis [3] as well as with analytical solution. Figure 5 shows the tangential stresses, radial displacements and radial stresses along the radius of the cylinder,

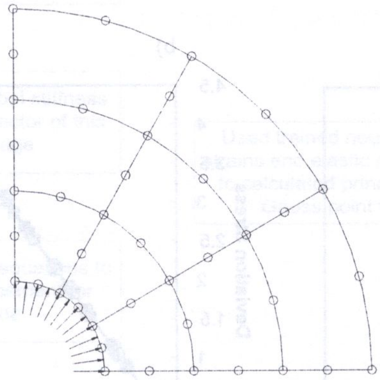

Fig. 4. Finite element mesh in symmetric quadrant of a thick cylinder

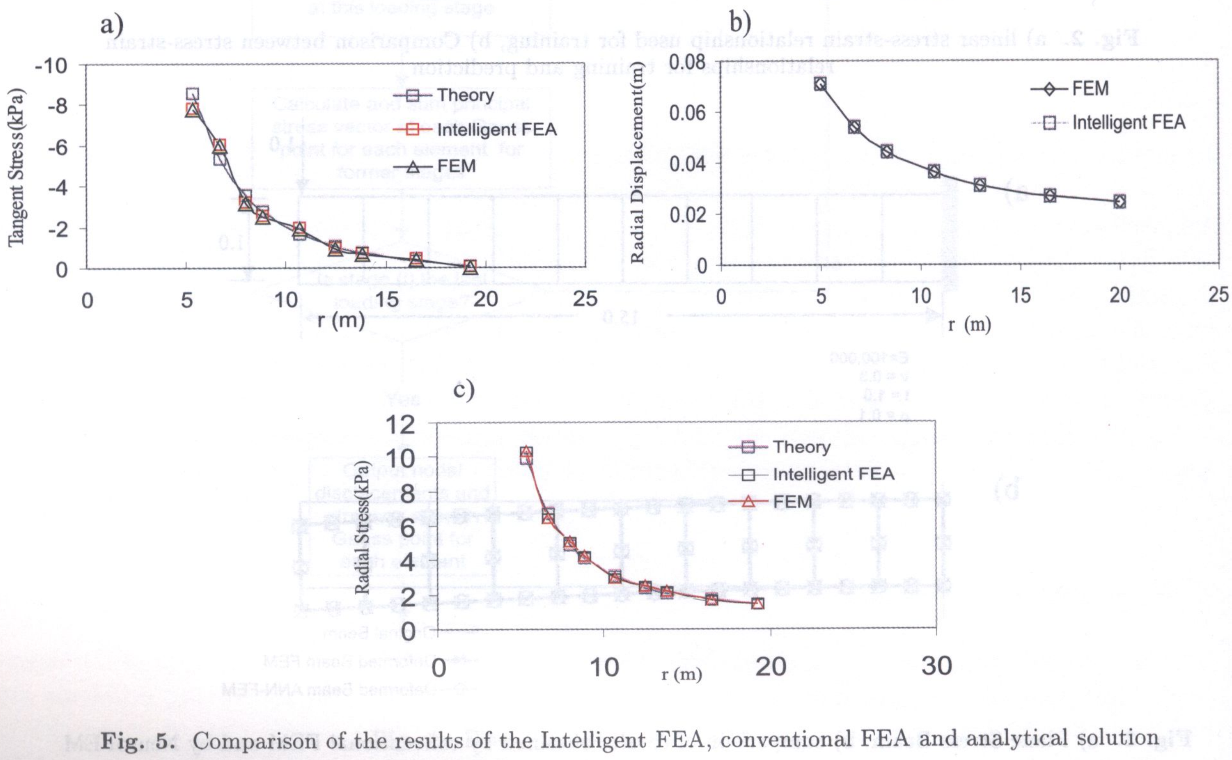

Fig. 5. Comparison of the results of the Intelligent FEA, conventional FEA and analytical solution

acquired from the three methods. Comparison between the results of the three methods shows that Intelligent FEA, with its obtained results, is in excellent agreement with the theoretical solution as well as with the standard finite element analysis.

From this example, it is again shown that in terms of deriving the implicit constitutive relationship from the raw data and then solving engineering problems, there is potential in the developed intelligent finite element method.

In the next and last example, application of the methodology to a more complex engineering problem will be demonstrated.

2.3. Example 3

The last example to be analysed is an embankment of Mohr-Coulomb material subjected to gravity loading. Details of the embankment are shown in Fig. 6.

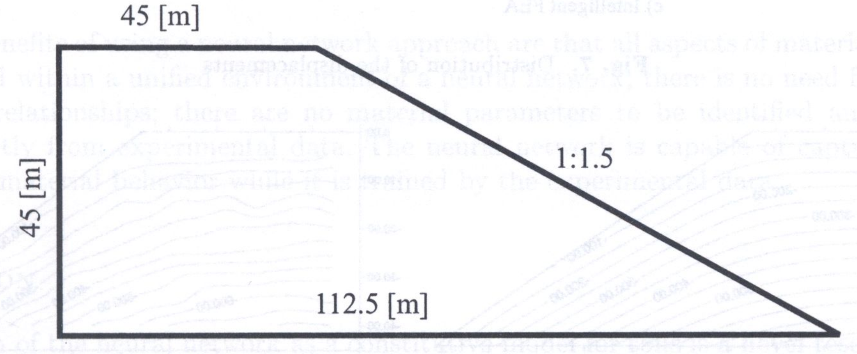

Fig. 6. Geometry of the embankment

Triaxial tests results were used for the material parameters for the embankment soil. They were used in both standard finite element analyses with Mohr–Coulomb and nonlinear elastic soil models. The soil parameters used in the nonlinear-elastic and Mohr–Coulomb [3] analyses are summarised in Table 1.

Table 1. Material parameters for Duncan–Chang [4] and Mohr–Coulomb models

c' [kPa]	ϕ' [deg]	R_f	K	n	ν	γ [kN/m^3]
20	30	0.8121	159.44	0.6233	0.33	17

Five drained triaxial tests were carried out on 5 samples of the soil and these results were used to derive the soil parameters. An assumption was made that the soil tested was representative of the material of the embankment. The finite element mesh included 225 eight-node iso-parametric elements and 736 nodes. Figures 7a and b show the contours of displacements in the embankment obtained using standard finite element analyses using nonlinear elastic and Mohr–Coulomb elasto-plastic models respectively. Figure 7c shows the results of the intelligent finite element analysis where the raw data from the triaxial tests were directly used in deriving the neural network-based constitutive model incorporated in the intelligent finite element code. In Fig. 8 contours of major principal stresses obtained from the three analyses are compared.

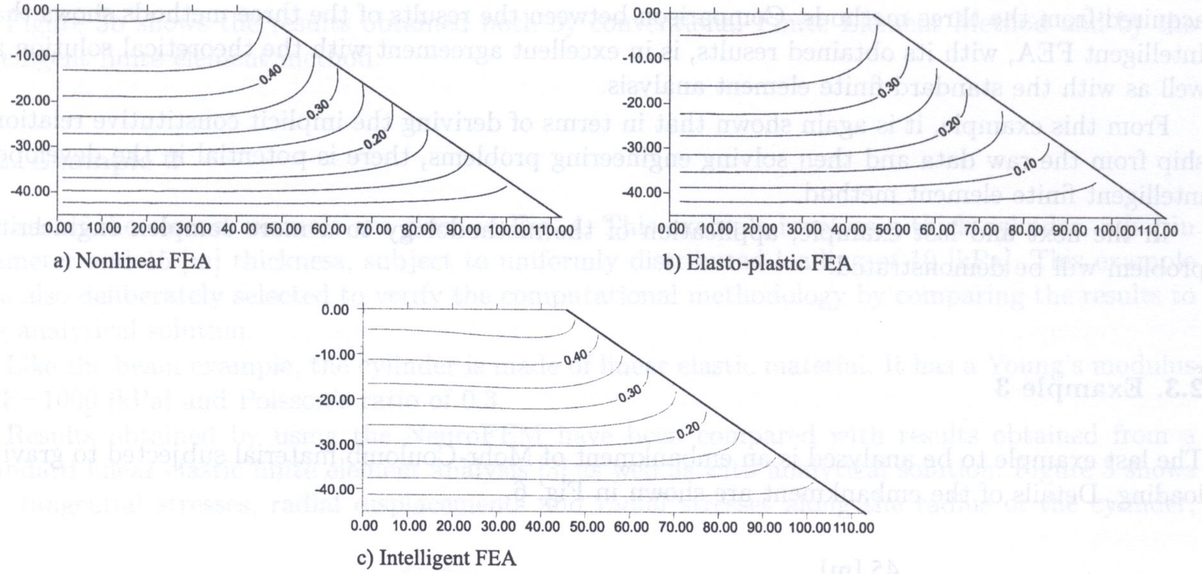

Fig. 7. Distribution of the displacements

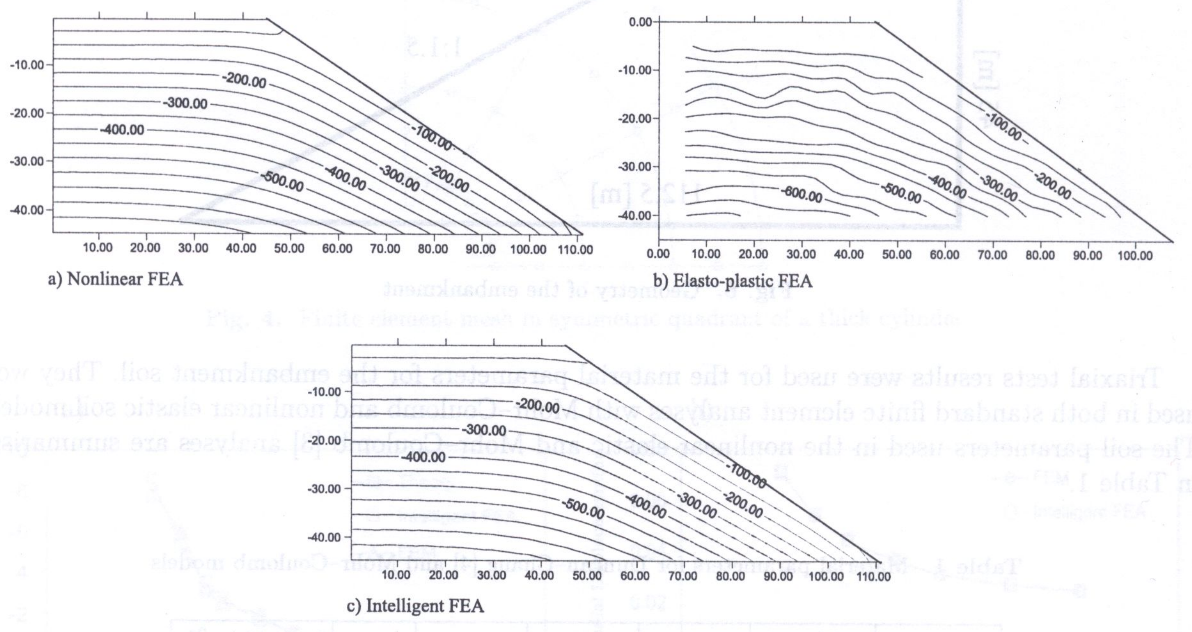

Fig. 8. Distribution of the major principal stresses

From the results obtained, it was shown that intelligent finite element method is also able to capture the more complex constitutive relationship of the soil and offer very good prediction of the behaviour of the structure.

3. Discussion

The neural network offers an effective alternative for constitutive models for soils in finite element analysis. An artificial neural network (ANN) has been integrated into a finite element program to substitute the mathematical constitutive model for the material. The efficiency and adaptability of the ANN model is demonstrated by successful analysis of three boundary value problems. The results of the analysis have been compared to those obtained from nonlinear as well as Mohr Coulomb elasto-plastic models. The results show that ANN can be successfully implemented in a finite element software as an effective substitute for complicated mathematical constitutive relationships.

As compared with the traditional constitutive models, the ANN model has the following salient features:

1. The model is only based on experimental data. The model is more objective than subjective, since no assumptions are made. In other words, the ANN model is not influenced by the shape of stress-strain curve.
2. Another advantage of ANN is that as more experimental data become available, the ANN will be able to store and train from more comprehensive information associated with the soil behaviour. And therefore, the ANN model will become more effective and robust.
3. The ANN model does not require calculation of the material parameters, as is the case for conventional mathematical constitutive models. These calculations result in errors in the application of most constitutive models. The ANN model is simple and effective if appropriate experimental data are available.

The main benefits of using a neural network approach are that all aspects of material behavior can be implemented within a unified environment of a neural network; there is no need for complicated mathematical relationships; there are no material parameters to be identified and the network is trained directly from experimental data. The neural network is capable of capturing the main features of the material behavior while it is trained by the experimental data.

4. Conclusion

Implementation of the neural network as a constitutive model for soils is a novel research area. The successful implementation of an ANN in a finite element program as a substitute for conventional constitutive models has demonstrated the capabilities of the ANN model. It has been shown that artificial neural network can be an efficient alternative to the complex mathematical constitutive models for soils in finite element analysis. Various aspects of the behaviour of soils can be captured by an ANN model.

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