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Sensitivity analysis of burns integrals

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In the paper the numerical analysis of thermal processes proceeding in the domain of biological tissue subjected to an external heat source is presented. Heat transfer in the skin tissue was assumed to be transient and two-dimensional. The bioheat transfer in the domain considered is described by the system of Pennes equations determining the temperature field in successive skin layers. Between the layers the ideal contact is assumed. On the selected part of skin surface the Neumann condition determining the value of external heat source is given, on the conventionally assumed internal surface of the tissue the no-flux condition is accepted. For time $t = 0$ the initial distribution of temperature is known. The degree of the skin burn can be predicted on the basis of the so-called Henriques integrals and the main subject of the paper is the sensitivity analysis of these integrals with respect to the skin parameters. On the stage of numerical computations the boundary element method has been used. In the final part of the paper the results obtained are shown.

Keywords: bioheat transfer, burn integrals, sensitivity analysis, boundary element method

1. Governing equations

The skin is treated as a multi-layer domain, in which one can distinguish the following sub-domains: epidermis with thermophysical parameters λ_1 [W/mK] (thermal conductivity), c_1 [J/m^{3}K] (volumetric specific heat), dermis with parameters λ_2, c_2 and sub-cutaneous region with parameters λ_3, c_3 – Fig. 1.

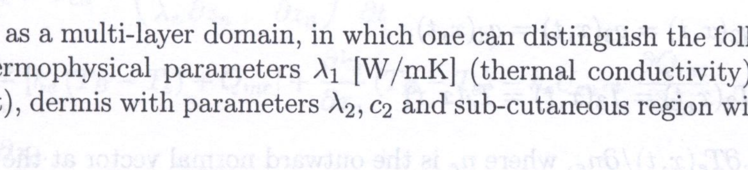

Fig. 1. Skin tissue

The transient bioheat transfer in domain of skin is described by the system of equations [1, 2] – Fig. 2

$$x \in \Omega_e: \quad c_e \frac{\partial T_e(x,t)}{\partial t} = \lambda_e \nabla^2 T_e(x,t) + k_e\left[T_B - T_e(x,t)\right] + Q_{me} \tag{1}$$

where $e = 1, 2, 3$ correspond to epidermis, dermis and sub-cutaneous regions, $k_e = G_e c_B$ is the product of blood perfusion rate and volumetric specific heat of blood, T_B is the blood temperature, Q_{me} is the metabolic heat source, $x = (x_1, x_2)$. It should be pointed out that $k_1 = 0$ and $Q_{m1} = 0$ [3].

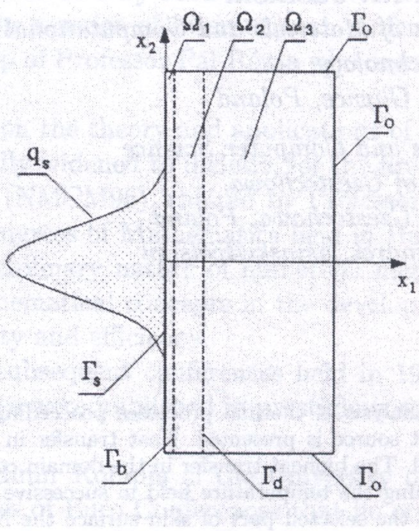

Fig. 2. Domain considered

On the contact surfaces between sub-domains considered the continuity conditions are assumed, namely

$$x \in \Gamma_b: \begin{cases} q_1(x,t) = q_2(x,t) = q_b(x,t), \\ T_1(x,t) = T_2(x,t) = T_b(x,t) \end{cases} \tag{2}$$

and

$$x \in \Gamma_d: \begin{cases} q_2(x,t) = q_3(x,t) = q_d(x,t), \\ T_2(x,t) = T_3(x,t) = T_d(x,t) \end{cases} \tag{3}$$

where $q_e(x,t) = -\lambda_e \partial T_e(x,t)/\partial n_e$, where n_e is the outward normal vector at the boundary point x.

On the skin surface the following boundary condition can be accepted

$$x \in \Gamma_s: \begin{cases} q_1(x,t) = q_s, & t \leq t_s, \\ q_1(x,t) = 0, & t > t_s \end{cases} \tag{4}$$

where q_s is the known boundary heat flux, t_s is the exposure time. Along the remaining parts of the boundary the non-flux condition

$$x \in \Gamma_0: \quad q_e(x,t) = 0 \tag{5}$$

is given. The initial condition takes a form

$$t = 0: \quad T_e(x,t) = T_0(x). \tag{6}$$

The knowledge of transient temperature field in the domain of skin tissue subjected to the external heat source allows to determine the Henriques integrals [3, 4]. The time-dependent values of these integrals can be applied for the prediction of burn degree. The temperatures appearing in the Henriques integrals correspond to the local temperatures of basal layer Γ_b (the surface between epidermis and dermis) and dermal base Γ_d (the surface between dermis and sub-cutaneous region) – Fig. 2. The basal layer and dermal base are parallel to the skin surface $\Gamma_s(x_1 = x_{1b}, x_1 = x_{1d})$. Thermal damage of skin begins when the temperature at the basal layer rises above 44[°C] (317 [K]). Henriques [4] found that the degree of skin damage could be predicted on the basis of the integrals

$$x \in \Gamma_b: \quad I_b = \int_0^\tau P_b \exp\left[-\frac{\Delta E}{RT_b(x,t)}\right] \mathrm{d}t \tag{7}$$

and

$$x \in \Gamma_d: \quad I_d = \int_0^\tau P_d \exp\left[-\frac{\Delta E}{RT_d(x,t)}\right] \mathrm{d}t \tag{8}$$

where $\Delta E/R$ [K] is the ratio of activation energy to universal gas constant, P_b, P_d [1/s] are the pre-exponential factors, $[0, \tau]$ is the time interval considered.

First degree burns are said to occur when the burn integral (7) is from the interval $0.53 < I_b \leq 1$, while the second degree burns when $I_b > 1$ [3, 4]. The third degree appears when the integral $I_d > 1$.

2. Sensitivity analysis

In the paper the direct approach of sensitivity analysis [5–8] is used. So, the basic equations constituting the model of the process are differentiated with respect to the tissue thermal parameters z_n, where $z_1 = \lambda_1, z_2 = \lambda_2, z_3 = \lambda_3, z_4 = c_1, z_5 = c_2, z_6 = c_3, z_7 = k_2, z_8 = k_3, z_9 = Q_{m2}, z_{10} = Q_{m3}$. At first, the Eqs. (1) are differentiated, and then

$$\frac{\partial c_e}{\partial z_n}\frac{\partial T_e}{\partial t} + c_e \frac{\partial}{\partial z_n}\left(\frac{\partial T_e}{\partial t}\right) = \frac{\partial \lambda_e}{\partial z_n}\nabla^2 T_e + \lambda_e \frac{\partial}{\partial z_n}\left(\nabla^2 T_e\right) + \frac{\partial k_e}{\partial z_n}[T_B - T_e] - k_e\frac{\partial T_e}{\partial z_n} + \frac{\partial Q_{me}}{\partial z_n}. \tag{9}$$

Using again the Eqs. (1) we obtain the following form of (9)

$$\begin{aligned} c_e\frac{\partial U_{en}}{\partial t} &= \lambda_e \nabla^2 U_{en} + \left(\frac{c_e}{\lambda_e}\frac{\partial \lambda_e}{\partial z_n} - \frac{\partial c_e}{\partial z_n}\right)\frac{\partial T_e}{\partial t} \\ &- \frac{1}{\lambda_e}\frac{\partial \lambda_e}{\partial z_n}\left[k_e\left(T_B - T_e\right) + Q_{me}\right] + \frac{\partial k_e}{\partial z_n}\left(T_B - T_e\right) - k_e U_{en} + \frac{\partial Q_{me}}{\partial z_n} \end{aligned} \tag{10}$$

where $U_{en} = \partial T_e / \partial z_n$.

Differentiation of the boundary conditions gives

- for basal layer

$$x \in \Gamma_b: \begin{cases} V_{1n}(x,t) = V_{2n}(x,t) + \left(\dfrac{1}{\lambda_2}\dfrac{\partial \lambda_2}{\partial z_n} - \dfrac{1}{\lambda_1}\dfrac{\partial \lambda_1}{\partial z_n}\right) q_b(x,t), \\ U_{1n}(x,t) = U_{2n}(x,t) = U_{bn}(x,t); \end{cases} \tag{11}$$

- for dermal base

$$x \in \Gamma_d: \begin{cases} V_{2n}(x,t) = V_{3n}(x,t) + \left(\dfrac{1}{\lambda_3}\dfrac{\partial \lambda_3}{\partial z_n} - \dfrac{1}{\lambda_2}\dfrac{\partial \lambda_2}{\partial z_n}\right) q_d(x,t), \\ U_{2n}(x,t) = U_{3n}(x,t) = U_{dn}(x,t); \end{cases} \tag{12}$$

- for skin surface

$$x \in \Gamma_s: \begin{cases} V_{1n}(x,t) = -\dfrac{1}{\lambda_1}\dfrac{\partial \lambda_1}{\partial z_n} q_s, & t \le t_s, \\ V_{1n}(x,t) = 0, & t > t_s; \end{cases} \tag{13}$$

- for the remaining parts of the boundary

$$x \in \Gamma_0: \quad V_{en}(x,t) = 0, \tag{14}$$

where $V_{en}(x,t) = -\lambda_e \partial U_{en}(x,t)/\partial n_e$.

Finally, we differentiate the initial condition, this means

$$t = 0: \quad U_{en}(x,t) = 0 \tag{15}$$

Taking into account the form of I_b and I_d (c.f. Eqs. (7), (8)), the sensitivity of these integrals with respect to the parameter z_n should be calculated using the formulas

$$\frac{\partial I_b}{\partial z_n} = \int_0^{\tau} P_b \frac{\Delta E}{R T_b^2(x,t)} \exp\left[-\frac{\Delta E}{R T_b(x,t)}\right] U_{bn}(x,t)\,\mathrm{d}t \tag{16}$$

and

$$\frac{\partial I_d}{\partial z_n} = \int_0^{\tau} P_d \frac{\Delta E}{R T_d^2(x,t)} \exp\left[-\frac{\Delta E}{R T_d(x,t)}\right] U_{dn}(x,t)\,\mathrm{d}t. \tag{17}$$

The change of burn integrals connected with the change of parameter z_n results from the Taylor formula limited to the first-order sensitivity, this means

$$I_b(z_n \pm \Delta z_n) = I_b(z_n) \pm \frac{\partial I_b}{\partial z_n} \Delta z_n \tag{18}$$

and

$$I_d(z_n \pm \Delta z_n) = I_d(z_n) \pm \frac{\partial I_d}{\partial z_n} \Delta z_n. \tag{19}$$

3. Boundary element method

The primary and also the additional problems resulting from the sensitivity analysis have been solved using the 1st scheme of the BEM for 2D transient heat diffusion. At first, the following Fourier equations are considered

$$x \in \Omega_e: \quad c_e \frac{\partial F_e(x,t)}{\partial t} = \lambda_e \nabla^2 F_e(x,t) + S_e(x,t), \qquad e = 1,2,3 \tag{20}$$

where $F_e(x,t)$ denotes the temperature or functions resulting from the sensitivity analysis, $S_e(x,t)$ are the source functions. The functions $S_e(x,t)$ take a form

- for the primary problem (c.f. Eq. (1))

$$S_e(x,t) = k_e\left[T_B - T_e(x,t)\right] + Q_{me}; \tag{21}$$

- for the problems concerning the sensitivity with respect to z_n (c.f. Eq. (10))

$$\begin{aligned} S_e(x,t) = {} & \left(\frac{c_e}{\lambda_e}\frac{\partial \lambda_e}{\partial z_n} - \frac{\partial c_e}{\partial z_n}\right)\frac{\partial T_e(x,t)}{\partial t} - \frac{1}{\lambda_e}\frac{\partial \lambda_e}{\partial z_n}\left[k_e T_B - k_e T_e(x,t) + Q_{me}\right] \\ & + \frac{\partial k_e}{\partial z_n}\left[T_B - T_e(x,t)\right] - k_e U_{en}(x,t) + \frac{\partial Q_{me}}{\partial z_n}. \end{aligned} \tag{22}$$

We introduce the time grid

$$0 = t^0 < t^1 < \ldots < t^{f-1} < t^f < \ldots < \infty, \qquad \Delta t = t^f - t^{f-1}. \tag{23}$$

If the 1st scheme of the BEM is taken into account [2, 7, 9, 10] then the boundary integral equations corresponding to transition $t^{f-1} \rightarrow t^f$ are of the form

$$\begin{aligned} B_e(\xi)F_e(\xi, t^f) &+ \frac{1}{c_e}\int_{t^{f-1}}^{t^f}\int_{\Gamma_e} F_e^*(\xi, x, t^f, t) J_e(x,t)\,\mathrm{d}\Gamma_e\,\mathrm{d}t \\ &= \frac{1}{c_e}\int_{t^{f-1}}^{t^f}\int_{\Gamma_e} J_e^*(\xi, x, t^f, t) F_e(x,t)\,\mathrm{d}\Gamma_e\,\mathrm{d}t \\ &+ \iint_{\Omega_e} F_e^*(\xi, x, t^f, t^{f-1}) F_e(x, t^{f-1})\,\mathrm{d}\Omega_e + \frac{1}{c_e}\int_{t^{f-1}}^{t^f}\iint_{\Omega_e} S_e(x,t) F_e^*(\xi, x, t^f, t)\,\mathrm{d}\Omega_e\,\mathrm{d}t \end{aligned} \tag{24}$$

where Γ_e denotes the boundary limiting sub-domain Ω_e, $e = 1, 2, 3$ and $B_e(\xi)$ is the coefficient from the interval $(0, 1)$.

In Eq. (24) F_e^* are the fundamental solutions [9, 10]:

$$F_e^*(\xi, x, t^f, t) = \frac{c_e}{4\pi\lambda_e(t^f - t)} \exp\left[-\frac{c_e r^2}{4\lambda_e(t^f - t)}\right] \tag{25}$$

where r is the distance from the point under consideration x to the observation point ξ, while

$$J_e^*(\xi, x, t^f, t) = -\lambda_e \frac{\partial F_e^*(\xi, x, t^f, t)}{\partial n_e} \tag{26}$$

and

$$J_e(x,t) = -\lambda_e \frac{\partial F_e(x,t)}{\partial n_e}. \tag{27}$$

For the constant elements with respect to time [9, 10] the boundary integral equations (24) take a form

$$\begin{aligned} B_e(\xi)F_e(\xi, t^f) &+ \int_{\Gamma_e} J_e(x, t^f) g_e(\xi, x)\,\mathrm{d}\Gamma_e = \int_{\Gamma_e} F_e(x, t^f) h_e(\xi, x)\,\mathrm{d}\Gamma_e \\ &+ \iint_{\Omega_e} J_e^*(\xi, x, t^f, t^{f-1}) F_e(x, t^{f-1})\,\mathrm{d}\Omega_e + \iint_{\Omega_e} S_e(x, t^{f-1}) g_e(\xi, x)\,\mathrm{d}\Omega_e \end{aligned} \tag{28}$$

where

$$h_e(\xi, x) = \frac{1}{c_e}\int_{t^{f-1}}^{t^f} J_e^*(\xi, x, t^f, t)\,\mathrm{d}t \tag{29}$$

and

$$g_e(\xi, x) = \frac{1}{c_e}\int_{t^{f-1}}^{t^f} F_e^*(\xi, x, t^f, t)\,\mathrm{d}t. \tag{30}$$

In numerical realization the constant boundary elements and constant internal cells have been used. We assume that the boundary of sub-domain Ω_1 is divided into N_1 constant boundary elements $\Gamma_j, j = 1, 1, \ldots, N_1$ and the interior Ω_1 is divided into L_1 constant internal cells. Similarly, the sub-domain Ω_2 is divided into $N_2 - N_1$ boundary elements $\Gamma_j, j = N_1+1, N_1+2, ..., N_2$ and $L_2 - L_1$ constant internal cells, while the sub-domain Ω_3 is divided into $N - N_1 - N_2$ boundary elements $\Gamma_j, j = N_2 + 1, N_2 + 2, ..., N$ and $L - L_1 - L_2$ internal cells. Then we obtain the following discrete forms of the boundary integral equations (28)

- for epidermis sub-domain

$$\sum_{j=1}^{N_1} G_{ij}^1 J_j^f = \sum_{j=1}^{N_1} H_{ij}^1 F_j^f + \sum_{l=1}^{L_1} P_{il}^1 F_l^{f-1} + \sum_{l=1}^{L_1} Z_{il}^1 S_l^{f-1}; \tag{31}$$

- for dermis sub-domain

$$\sum_{j=N_1+1}^{N_2} G_{ij}^2 J_j^f = \sum_{j=N_1+1}^{N_2} H_{ij}^2 F_j^f + \sum_{l=L_1+1}^{L_2} P_{il}^2 F_l^{f-1} + \sum_{l=L_1+1}^{L_2} Z_{il}^2 S_l^{f-1}; \tag{32}$$

- for sub-cutaneous region

$$\sum_{j=N_2+1}^{N} G_{ij}^3 J_j^f = \sum_{j=N_2+1}^{N} H_{ij}^3 F_j^f + \sum_{l=L_2+1}^{L} P_{il}^3 F_l^{f-1} + \sum_{l=L_2+1}^{L} Z_{il}^3 S_l^{f-1}, \tag{33}$$

where

$$G_{ij}^e = \int_{\Gamma_j} g_e(\xi^i, x)\, \mathrm{d}\,\Gamma_j, \tag{34}$$

$$H_{ij}^e = \begin{cases} \displaystyle\int_{\Gamma_j} h_e(\xi^i, x)\, \mathrm{d}\,\Gamma_j, & i \neq j \\ -0.5, & i = j \end{cases} \tag{35}$$

and

$$P_{il}^e = \iint_{\Omega_l} F_e^*(\xi^i, x, t^f, t^{f-1})\, \mathrm{d}\,\Omega_l, \tag{36}$$

$$Z_{il}^e = \iint_{\Omega_l} g_e(\xi^i, x)\, \mathrm{d}\,\Omega_l. \tag{37}$$

The system of equations (31)–(33) can be written in the matrix form, namely

$$\mathbf{G}^e \mathbf{J}_e^f = \mathbf{H}^e \mathbf{F}_e^f + \mathbf{P}^e \mathbf{F}_e^{f-1} + \mathbf{Z}^e \mathbf{S}_e^{f-1}, \qquad e = 1, 2, 3. \tag{38}$$

If we separate in these system the fragments concerning the skin surface Γ_s, the common boundary Γ_b between Ω_1 and Ω_2 and the common boundary Γ_d between Ω_2 and Ω_3 (c.f. Fig. 2) then we have

- for epidermis sub-domain

$$\begin{bmatrix} \mathbf{G}_s^1 & \mathbf{G}^1 & \mathbf{G}_b^1 \end{bmatrix} \begin{bmatrix} \mathbf{J}_s^f \\ \mathbf{J}_1^f \\ \mathbf{J}_{1b}^f \end{bmatrix} = \begin{bmatrix} \mathbf{H}_s^1 & \mathbf{H}^1 & \mathbf{H}_b^1 \end{bmatrix} \begin{bmatrix} \mathbf{F}_s^f \\ \mathbf{F}_1^f \\ \mathbf{F}_{1b}^f \end{bmatrix} + \mathbf{P}^1 \mathbf{F}_1^{f-1} + \mathbf{Z}^1 \mathbf{S}_1^{f-1}, \tag{39}$$

- for dermis sub-domain

$$\begin{bmatrix} \mathbf{G}^2 & \mathbf{G}_b^2 & \mathbf{G}_d^2 \end{bmatrix} \begin{bmatrix} \mathbf{J}_2^f \\ \mathbf{J}_{2b}^f \\ \mathbf{J}_{2d}^f \end{bmatrix} = \begin{bmatrix} \mathbf{H}^2 & \mathbf{H}_b^2 & \mathbf{H}_d^2 \end{bmatrix} \begin{bmatrix} \mathbf{F}_2^f \\ \mathbf{F}_{2b}^f \\ \mathbf{F}_{2d}^f \end{bmatrix} + \mathbf{P}^2 \mathbf{F}_2^{f-1} + \mathbf{Z}^2 \mathbf{S}_2^{f-1}, \tag{40}$$

- for sub-cutaneous region

$$\begin{bmatrix} \mathbf{G}^3 & \mathbf{G}_d^3 \end{bmatrix} \begin{bmatrix} \mathbf{J}_3^f \\ \mathbf{J}_{3d}^f \end{bmatrix} = \begin{bmatrix} \mathbf{H}^3 & \mathbf{H}_d^3 \end{bmatrix} \begin{bmatrix} \mathbf{F}_3^f \\ \mathbf{F}_{3d}^f \end{bmatrix} + \mathbf{P}^3\mathbf{F}_3^{f-1} + \mathbf{Z}^3\mathbf{S}_3^{f-1}. \tag{41}$$

The boundary conditions given on the surfaces Γ_s, Γ_b and Γ_d can be written in the form

- for basal layer (c.f. Eqs. (2), (11))

$$x \in \Gamma_b : \quad \begin{cases} \mathbf{J}_{1b}^f = \mathbf{J}_{2b}^f + \mathbf{R}_b^f, \\ \mathbf{F}_{1b}^f = \mathbf{F}_{2b}^f = \mathbf{F}_b^f; \end{cases} \tag{42}$$

- for dermal base (c.f. Eqs. (3), (12))

$$x \in \Gamma_d : \begin{cases} \mathbf{J}_{2d}^f = \mathbf{J}_{3d}^f + \mathbf{R}_d^f, \\ \mathbf{F}_{2d}^f = \mathbf{F}_{3d}^f = \mathbf{F}_d^f; \end{cases} \tag{43}$$

- for the skin surface (c.f. Eqs. (4), (13))

$$x \in \Gamma_s : \quad \mathbf{J}_s^f = A\mathbf{q}_s^f. \tag{44}$$

We introduce the conditions (42)–(44) to the Eqs. (39)–(41). Taking into account the remaining boundary conditions (Eqs. (5), (14)) we obtain

- for epidermis sub domain

$$\begin{bmatrix} -\mathbf{H}^1 & -\mathbf{H}_s^1 & -\mathbf{H}_b^1 & \mathbf{G}_b^1 \end{bmatrix} \begin{bmatrix} \mathbf{F}_1^f \\ \mathbf{F}_s^f \\ \mathbf{F}_b^f \\ \mathbf{J}_{1b}^f \end{bmatrix} = -\mathbf{G}_s^1 A\mathbf{q}_s^f + \mathbf{P}^1\mathbf{F}_1^{f-1} + \mathbf{Z}^1\mathbf{S}_1^{f-1}; \tag{45}$$

- for dermis sub-domain

$$\begin{bmatrix} -\mathbf{H}_b^2 & \mathbf{G}_b^2 & \mathbf{H}^2 & -\mathbf{H}_d^2 & \mathbf{G}_d^2 \end{bmatrix} \begin{bmatrix} \mathbf{F}_b^f \\ \mathbf{J}_{1b}^f \\ \mathbf{F}_2^f \\ \mathbf{F}_d^f \\ \mathbf{J}_{2d}^f \end{bmatrix} = \mathbf{G}_b^2\mathbf{R}_b^f + \mathbf{P}^2\mathbf{F}_2^{f-1} + \mathbf{Z}^2\mathbf{S}_2^{f-1}; \tag{46}$$

- for sub-cutaneous region

$$\begin{bmatrix} -\mathbf{H}_d^3 & \mathbf{G}_d^3 & -\mathbf{H}^3 \end{bmatrix} \begin{bmatrix} \mathbf{F}_d^f \\ \mathbf{J}_{2d}^f \\ \mathbf{F}_3^f \end{bmatrix} = \mathbf{H}^3\mathbf{R}_d^f + \mathbf{P}^3\mathbf{F}_3^{f-1} + \mathbf{Z}^3\mathbf{S}_3^{f-1}. \tag{47}$$

Joining the above systems of equations finally we have

$$\begin{bmatrix} -\mathbf{H}^1 & -\mathbf{H}^1_s & -\mathbf{H}^1_b & \mathbf{G}^1_s & 0 & 0 & 0 & 0 \\ 0 & 0 & -\mathbf{H}^2_b & \mathbf{G}^2_b & -\mathbf{H}^2 & -\mathbf{H}^2_d & \mathbf{G}^2_d & 0 \\ 0 & 0 & 0 & 0 & 0 & -\mathbf{H}^3_d & \mathbf{G}^3_d & -\mathbf{H}^3 \end{bmatrix} \begin{bmatrix} \mathbf{F}^f_1 \\ \mathbf{F}^f_s \\ \mathbf{F}^f_b \\ \mathbf{J}^f_{1b} \\ \mathbf{F}^f_2 \\ \mathbf{F}^f_d \\ \mathbf{J}^f_{2d} \\ \mathbf{F}^f_3 \end{bmatrix} = \begin{bmatrix} -\mathbf{G}^1_s A \mathbf{q}^1_s + \mathbf{P}^1\mathbf{F}^{f-1}_1 + \mathbf{Z}^1\mathbf{S}^{f-1}_1 \\ \mathbf{G}^2_b\mathbf{R}^f_i + \mathbf{P}^2\mathbf{F}^{f-1}_2 + \mathbf{Z}^2\mathbf{S}^{f-1}_2 \\ \mathbf{H}^3\mathbf{R}^f_d + \mathbf{P}^3\mathbf{F}^{f-1}_3 + \mathbf{Z}^3\mathbf{S}^{f-1}_3 \end{bmatrix}. \tag{48}$$

After the determining the 'missing' boundary values of F and J, the values of function F at the internal points ξ^i for time t^f can be calculated using the formulas

- for epidermis sub-domain ($i = N+1, N+2, ..., N+L_1$)

$$T^f_i = \sum_{j=1}^{N_1} H^1_{ij}F^f_j - \sum_{j=1}^{N_1} G^1_{ij}J^f_j + \sum_{l=1}^{L_1} P^1_{il}F^{f-1}_l + \sum_{l=1}^{L_1} Z^1_{il}S^{f-1}_l; \tag{49}$$

- for dermis sub-domain ($i = N+L_1+1, N+L_1+2, ..., N+L_2$)

$$T^f_i = \sum_{j=N_1+1}^{N_2} H^2_{ij}F^f_j - \sum_{j=N_1+1}^{N_2} G^2_{ij}J^f_j + \sum_{l=L_1+1}^{L_2} P^2_{il}F^{f-1}_l + \sum_{l=L_1+1}^{L_2} Z^2_{il}S^{f-1}_l; \tag{50}$$

- for sub-cutaneous region ($i = N+L_2+1, N+L_2+2, ..., N+L$)

$$T^f_i = \sum_{j=N_2+1}^{N} H^3_{ij}F^f_j - \sum_{j=N_2+1}^{N} G^3_{ij}J^f_j + \sum_{l=L_2+1}^{L} P^3_{il}F^{f-1}_l + \sum_{l=L_2+1}^{L} Z^3_{il}S^{f-1}_l. \tag{51}$$

4. Results of computations

The symmetrical fragment of 2D domain of skin shown in Fig. 2 is considered. The dimensions of Ω equal 0.0121×0.02 [m]. The positions of basal layer and dermal base correspond to $x_1 = 0.0001$ [m] and $x_1 = 0.0021$ [m]. The thermophysical parameters of skin tissue are collected in Table 1 [3].

The ratio of activation energy to universal gas constant $\Delta E/R = 55000$ [K], the pre-exponential factors: $P_b = 0$ for $T(x_{1b}, x_2, t) < 317$ [K], $P_b = 1.43 \cdot 10^{72}$ [1/s] for $T(x_{1b}, x_2, t) \geq 317$ [K] and $P_d = 0$ for $T(x_{1d}, x_2, t) < 317$ [K], $P_d = 2.86 \cdot 10^{69}$ [1/s] for $T(x_{1d}, x_2, t) \geq 317$ [K] [3]. The initial temperature of skin: $T_0 = 37$ [°C].

The external boundary of tissue Γ_s is subjected to the heat flux given by formula (for $t \leq t_s$)

$$q_s(0, x_2, t) = \begin{cases} 0, & x_2 \in [-0.01, -0.005], \\ \sum_{k=0}^{4} a_k x_2^k, & x_2 \in [-0.005, 0.005], \\ 0, & x_2 \in [0.005, 0.01], \end{cases} \tag{52}$$

where a_k are the coefficients.

Table 1. Thermophysical parameters of skin and blood

Parameter	Unit	Sub-domain	Value
thermal conductivity	W/mK	epidermis	0.235
		dermis	0.445
		sub-cutaneous region	0.185
volumetric specific heat	J/kgm^3	epidermis	$4.31 \cdot 10^6$
		dermis	$3.96 \cdot 10^6$
		sub-cutaneous region	$2.67 \cdot 10^6$
		blood	$3.9962 \cdot 10^6$
metabolic heat source	W/m^3	epidermis	0
		dermis	245
		sub-cutaneous region	245
blood temperature	°C		37
blood perfusion coefficient	m^{3}blood/s/ m^{3}tissue	epidermis	0
		dermis	0.00125
		sub-cutaneous	0.00125

On the stage of numerical computations the interior of domain has been divided into 6640 internal cells, while the external and internal boundaries into 646 boundary elements.

In the first example of computations the following values have been assumed: $a_0 = -6500$, $a_1 = 0$, $a_2 = 5.2 \cdot 10^8$, $a_3 = 0$, $a_4 = -1.04 \cdot 10^{13}$ and exposure time $t_s = 18$ [s].

In Fig. 3 the temperature distribution in the domain considered for 15 [s] and 25 [s] is shown. The sensitivity analysis of temperature field and burn integral I_b has been done with respect to the all thermophysical parameters. It turned out that especially essential in the case considered are the changes of thermal conductivity and volumetric specific heat of the dermis sub-domain. In Fig. 4 the distribution of burn integral $I_b(x_{1b}, x_2, 16)$ and its sensitivity with respect to c_2 for $\Delta c_2 = 120000$ is shown.

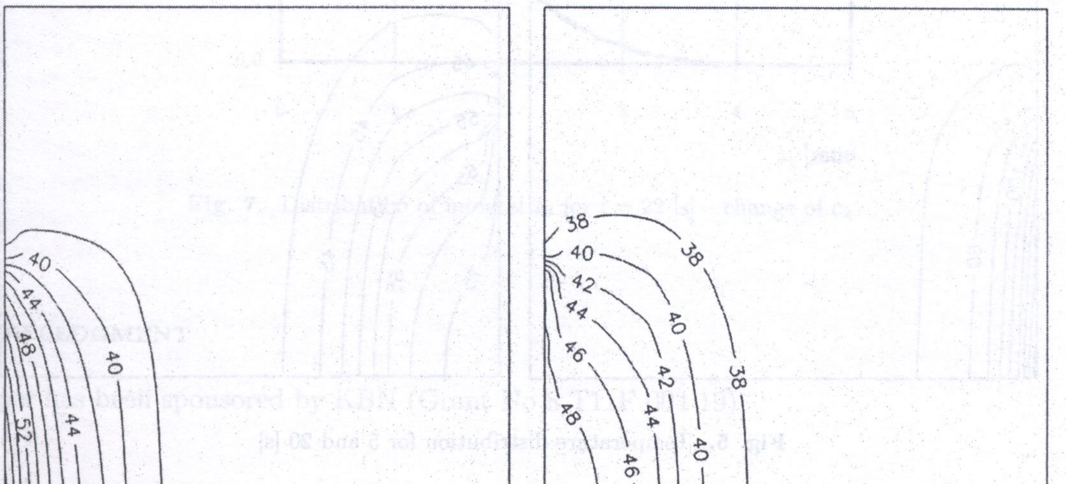

Fig. 3. Temperature distribution for 15 and 25 [s]

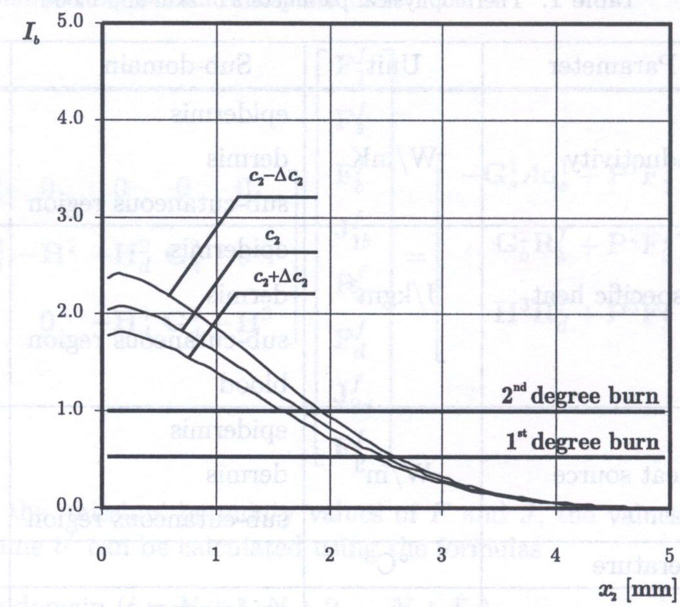

Fig. 4. Distribution of integral I_b for $t = 16$ [s] – change of c_2

In the second example the following values of coefficients (c.f. Eq. (52)) have been assumed $a_0 = -80000, a_1 = 0, a_2 = 6.4 \cdot 10^9, a_3 = 0, a_4 = 1.28 \cdot 10^{14}$ and exposure time $t_s = 5$ [s].

In Fig. 5 the temperature distribution in the domain considered for 5 and 20 [s] is shown. The sensitivity analysis of temperature field and burn integral I_d has been also done with respect to the all thermophysical parameters. It turned out that especially essential in the case considered are the changes of thermal conductivity and volumetric specific heat of the dermis and sub-cutaneous region. In Fig. 6 and 7 the distribution of burn integral $I_d(x_{1d}, x_2, 22)$ and its sensitivity with respect to the λ_2 and c_2 for $\Delta\lambda_2 = 0.025$ and $\Delta c_2 = 12000$ are shown.

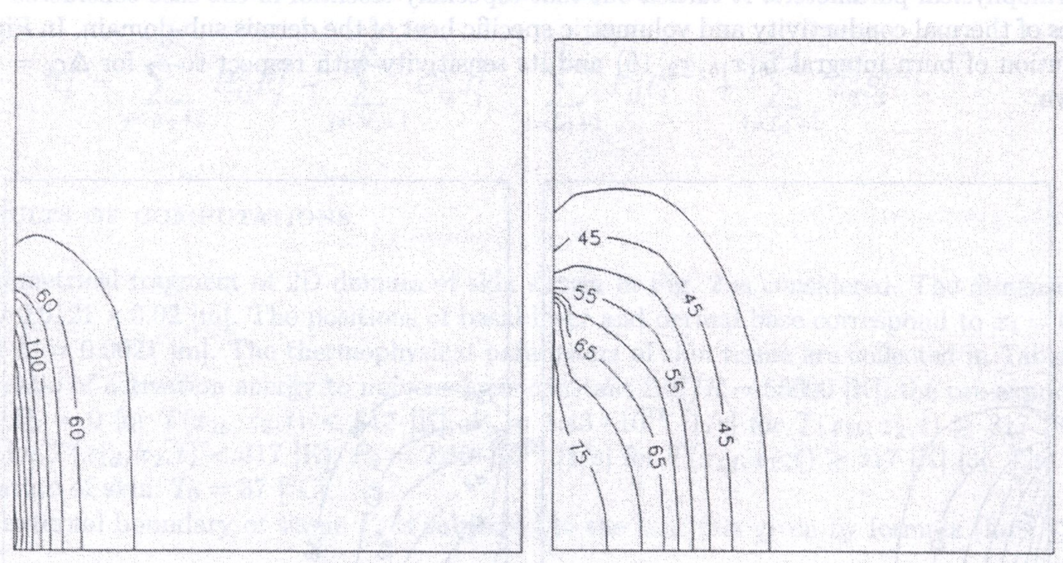

Fig. 5. Temperature distribution for 5 and 20 [s]

Summing up, the methods of sensitivity analysis and application of BEM on the stage of numerical computations constitute the quite useful tool for estimation of thermal processes proceeding in the biological tissue subjected to an external heat source.

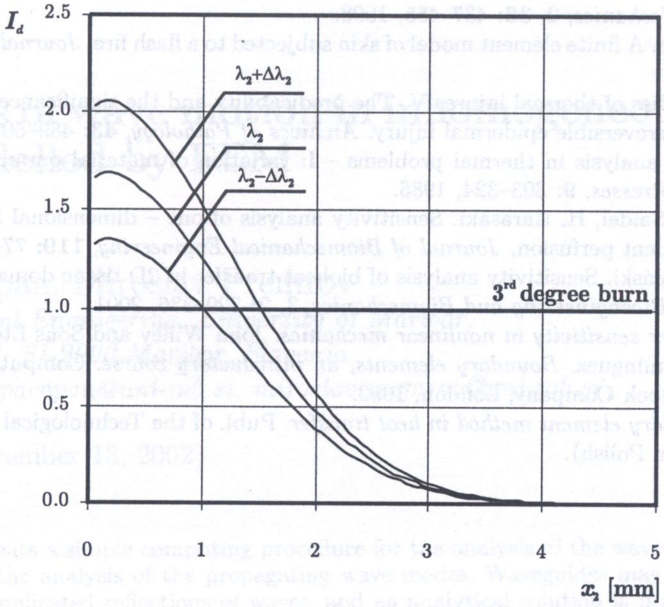

Fig. 6. Distribution of integral I_d for $t = 22$ [s] – change of λ_2

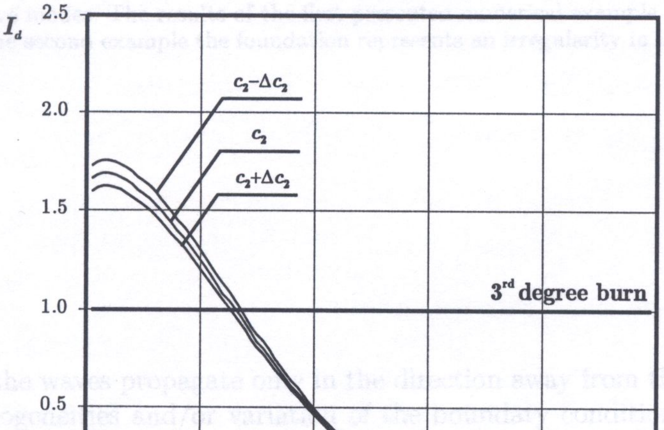

Fig. 7. Distribution of integral I_d for $t = 22$ [s] – change of c_2

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References

[1] H. W. Huang, C. L. Chan, R. B. Roemer. Analytical solutions of Pennes bioheat transfer equation with a blood vessel. *Journal of Biomechanical Engineering*, **116**: 208–212, 1994.

[2] E. Majchrzak. Numerical modelling of bio-heat transfer using the boundary element method. *Journal of Theoretical and Applied Mechanics*, 2, **36:** 437–455, 1998.
[3] D. A. Torvi, J. D. Dale. A finite element model of skin subjected to a flash fire. *Journal of Mechanical Engineering*, **116:** 250–255, 1994.
[4] F. C. Henriques. Studies of thermal injures V. The predicability and the significance of thermally inducted rate processes leading to irreversible epidermal injury. *Archives of Pathology*, **43:** 489–502, 1947.
[5] K. Dems. Sensitivity analysis in thermal problems – I: variation of material parameters within fixed domain. *Journal of Thermal Stresses*, **9:** 303–324, 1986.
[6] C. R. Davies, G. M. Saidel, H. Harasaki. Sensitivity analysis of one – dimensional heat transfer in tissue with temperature – dependent perfusion. *Journal of Biomechanical Engineering*, **119:** 77–80, 1997.
[7] E. Majchrzak, M. Jasiński. Sensitivity analysis of bioheat transfer in 2D tissue domain subjected to an external heat source. *Acta of Bioengineering and Biomechanics*, 3, **2:** 329–336, 2001.
[8] M. Kleiber. *Parameter sensitivity in nonlinear mechanics.* John Willey and Sons Ltd., Chichester, 1997.
[9] C. A. Brebbia, J. Dominguez. *Boundary elements, an introductory course.* Computational Mechanics Publications, McGraw-Hill Book Company, London, 1992.
[10] E. Majchrzak. *Boundary element method in heat transfer.* Publ. of the Technological University of Czestochowa, Czestochowa, 2001 (in Polish).