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Optimization of torsional systems with self-regulated pneumatic clutches

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In this paper possibilities of optimization of two torsional mechanical systems with one and two differential pneumatic clutches with self-regulation are shown. The systems are excited by harmonic components of the periodic moment caused by an engine. The advantage of the differential pneumatic clutch lies in the fact that its torsional stiffness can be controlled by the pressure of a gas medium in it. Optimization of such systems enables not only minimization of vibrations and dynamic effects but also avoiding resonance regimes in relatively wide frequency intervals (speeds of rotation of the system). As objective function the mean total amplitude of relative vibration is used. The constraints on the amplitudes of dynamic moments and also anti-resonance constraints are considered.

Keywords: mechanical system, torsional vibration, optimization, pneumatic clutch

1. Introduction

The papers [1] and [2] showed possibilities of using pneumatic clutches with self-regulation in torsionally vibrating mechanical systems (with various engines, compressors, pumps, screw propellers etc.). These clutches enable to avoid dangerous resonance regimes. In comparison with classical elastic clutches they enable to change the clutch stiffness according to the speed of rotation of the system (or its loading moment).

In this paper we will concentrate on possibilities how to avoid resonance regimes in torsional mechanical systems with two degrees of freedom. The optimization method will be illustrated by a system with one and two differential pneumatic clutches with self-regulation. Such a clutch enable to keep the angular displacement between its both parts constant thanks to the change of the pressure of a gas medium in it which causes the change of the clutch stiffness. Only steady-state regimes described by linear mathematical models will be considered here. Experimental results (see e.g. [1]) showed negligible nonlinear effects in the pneumatic clutches considered here.

2. Dynamic models

In Figs. 1 and 2 dynamic models of torsional systems with three discs are shown, where is: I_1 – mass moment of inertia of an engine about its axis of rotation, I_2 – mass moment of inertia, I_3 – mass moment of inertia of the driven part of the system, k_S – torsional stiffness of the pneumatic clutch, k – torsional stiffness of the shaft or another clutch (see Fig. 2), M_i – the i-th harmonic component of the periodic exciting moment (caused by the engine), M – the constant loading moment, ω – angular frequency, $\varphi_1, \varphi_2, \varphi_3$ – angular displacements of the discs I_1, I_2 and I_3.

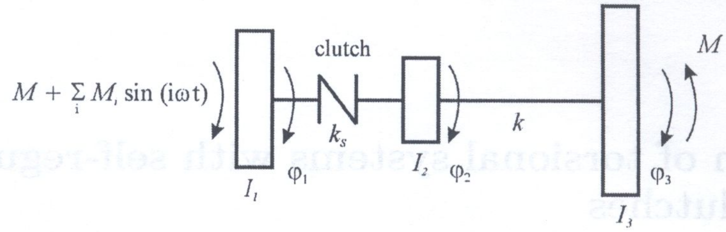

Fig. 1. Dynamic model (with one clutch)

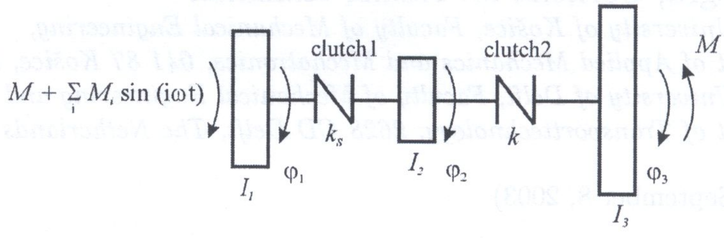

Fig. 2. Dynamic model (with two clutches)

Damping will not be considered here, because we assume that optimization will allow the system to avoid all resonances, where damping would not be negligible in damping vibration amplitudes.

3. Mathematical models

The equations of rotational motion of the systems shown in Fig. 1 are

$$\begin{aligned} &I_1\ddot{\varphi}_1 - k_S\,(\varphi_2 - \varphi_1) = M + \sum_i M_i \sin(i\omega\,t)\,, \\ &I_2\ddot{\varphi}_2 + k_S\,(\varphi_2 - \varphi_1) \;- k\;(\varphi_3 - \varphi_2) = 0, \\ &I_3\ddot{\varphi}_3 + k\;(\varphi_3 - \varphi_2) = -M. \end{aligned} \tag{1}$$

It is obvious that this system of equations has only two nonzero natural frequencies

$$\Omega_1 = \sqrt{\frac{b_2 - \sqrt{b_2^2 - 4\,b_1\,b_3}}{2\,b_1}}, \qquad \Omega_2 = \sqrt{\frac{b_2 + \sqrt{b_2^2 - 4\,b_1\,b_3}}{2\,b_1}}, \tag{2}$$

where

$$b_1 = \frac{I_1\,I_2\,I_3}{k_S\,k}, \qquad b_2 = \frac{(I_2 + I_3)\,I_1}{k_S} + \frac{(I_1 + I_2)\,I_3}{k}, \qquad b_3 = (I_1 + I_2 + I_3)\,. \tag{3}$$

If we introduce two relative coordinates $\varphi = \varphi_2 - \varphi_1$ and $\psi = \varphi_3 - \varphi_2$ and do not take into consideration the constant loading moment M (whose particular solution corresponds to the constant angular displacement of the system), then the equations of motion (1) can be written in the form

$$\begin{aligned} &\ddot{\varphi} + \left(\frac{k_S}{I_2} + \frac{k_S}{I_1}\right)\varphi \;- \frac{k}{I_2}\psi = -\sum_{i=0.5}^{6} \frac{M_i}{I_1}\,\sin(i\omega\,t)\,, \\ &\ddot{\psi} - \frac{k_S}{I_2}\varphi + \left(\frac{k}{I_3} + \frac{k}{I_2}\right)\psi = 0, \end{aligned} \tag{4}$$

where the first twelve harmonic components of the periodic exciting moment are taken into account.

Particular solution to these equations (corresponding to the forced vibration) can be determined by superposition

$$\varphi = \sum_{i=0.5}^{6} \Phi_i \sin(i\omega t), \tag{5a}$$

$$\psi = \sum_{i=0.5}^{6} \Psi_i \sin(i\omega t), \tag{5b}$$

where the amplitudes of vibration Φ_i and Ψ_i are

$$\Phi_i = \frac{-M_i}{D I_1}\left(\frac{k}{I_3} + \frac{k}{I_2} - i^2\omega^2\right), \tag{6a}$$

$$\Psi_i = \frac{-M_i k_S}{D I_1 I_2}, \tag{6b}$$

and

$$D = \left(\frac{k_S}{I_2} + \frac{k_S}{I_1} - i^2\omega^2\right)\cdot\left(\frac{k}{I_3} + \frac{k}{I_2} - i^2\omega^2\right) - \frac{k k_S}{I_2^2}. \tag{7}$$

The total amplitude of the dynamic torsional moment in the clutch is

$$M_S = k_S \sum_{i=0.5}^{6} |\Phi_i| \tag{8}$$

and the total amplitude of the torsional dynamic moment in the shaft (between the discs I_2 and I_3, Fig. 1), or in the second clutch (see Fig. 2) is

$$M = k \sum_{i=0.5}^{6} |\Psi_i|. \tag{9}$$

4. Formulations of the optimization problems

Next two optimization problems (A and B) will be solved. In the problem A (Fig. 1) we will consider only one design variable φ^* – relative angular displacement of both parts of the pneumatic clutch (depending on the pressure in it) which will be constant in the whole frequency interval. The torsional stiffness of the shaft between the discs I_2 and I_3 will be constant (specified). In the problem B two pneumatic clutches will be used (Fig. 2). So, we will have two design variables in this problem: φ_1^* and φ_2^*.

The frequency interval from ω_A to ω_B, corresponding to the lowest and highest revolutions of the system, will be devided into $N-1$ subintervals of the constant length. In this way we get N discrete frequencies ω_j ($\omega_1 \equiv \omega_A$, ω_2, ω_3, Λ, $\omega_N \equiv \omega_B$).

The objective function is determined in such a way, that it expresses the mean total amplitude of the relative vibration between the discs I_1 and I_2

$$f_{op} = \frac{1}{N}\sum_{j=1}^{N}\left(\sum_{i=0.5}^{6} |\Phi_i|\right), \tag{10}$$

where is (see equation (6a))

$$\Phi_i = \frac{-M_i}{D_j I_1}\left(\frac{k}{I_3} + \frac{k}{I_2} - i^2\omega_j^2\right). \tag{11}$$

$$D_j = \left(\frac{k_{Sj}}{I_2} + \frac{k_{Sj}}{I_1} - i^2\omega_j^2\right) \cdot \left(\frac{k}{I_3} + \frac{k}{I_2} - i^2\omega_j^2\right) - \frac{k\,k_{Sj}}{I_2^2}, \tag{12}$$

and

$$k_{Sj} = \frac{a\omega_j^2}{\varphi^*}. \tag{13}$$

For the problem B the following equations hold

$$D_j = \left(\frac{k_{Sj}}{I_2} + \frac{k_{Sj}}{I_1} - i^2\omega_j^2\right) \cdot \left(\frac{k_j}{I_3} + \frac{k_j}{I_2} - i^2\omega_j^2\right) - \frac{k_j\,k_{Sj}}{I_2^2}, \tag{14a}$$

where the stiffnesses of the first and second clutch are

$$k_{Sj} = \frac{a\omega_j^2}{\varphi_1^*}, \tag{14b}$$

$$k_j = \frac{a\omega_j^2}{\varphi_2^*}. \tag{14c}$$

In Eqs. (13), (14a,b,c) equality of the loading moment quadratically dependent on ω

$$M = a\omega^2 \tag{15}$$

(a is a constant) and the moments in the clutches

$$\text{problem } A: \qquad M_S = k_S\varphi^*, \tag{16a}$$

$$\text{problem } B: \qquad M_{S1} = k_S\varphi_1^*, \qquad M_{S2} = k\,\varphi_2^*. \tag{16b}$$

was used.

4.1. Constraints on the dynamic moment amplitude

Meaning of this constraint is apparent. If the dynamic moment amplitudes in the shaft between the discs I_2 and I_3 (problem A), or in the clutch between the same discs (problem B), are too high, then serious damage of the system can occur. This constraint will be considered in the form

$$M_{\mathrm{dyn},j} \le M_D, \qquad j = 1, 2, ..., M, \tag{17}$$

where M_D is the maximum dynamic moment amplitude. The dynamic moment amplitude $M_{\mathrm{dyn},j}$ is

$$M_{\mathrm{dyn},j} = k_j \sum_{i=0.5}^{6} |\Psi_j|, \qquad j = 1, 2, ..., M, \tag{18}$$

and M ($M \le N$) is the number of discrete frequencies (from ω_A to ω_B) for which conditions (17) are applied. It can be expected that M can be much less than N.

The stiffness k_j in (18) is determined by equation (14c) and Ψ_i is (see Eq. (6b))

$$\Psi_i = \frac{-M_i\,k_{Sj}}{D_j\,I_1\,I_2}, \tag{19}$$

where k_{Sj} is determined by Eq. (14b) and D_j by Eq. (12) for problem A, or by Eq. (14a) for problem B.

4.2. Anti-resonance constraints

We want the first natural frequency of the system $\Omega_1(\omega)$ in the whole frequency interval (Fig. 3) to lie below the straight line corresponding to $i = 0.5$ and $\Omega_2(\omega)$ between the straight lines corresponding to $i = 1$ and $i = 1.5$. To ensure these demands the following inequalities must be fullfiled

$$\Omega_{1,j} \leq 0.4\,\omega_j, \qquad j = 1, 2, ..., M, \tag{20a}$$

$$\Omega_{2,j} \leq 1.4\,\omega_j, \qquad j = 1, 2, ..., M, \tag{20b}$$

and

$$\Omega_{2,j} \geq 1.1\,\omega_j, \qquad j = 1, 2, ..., M. \tag{20c}$$

If the dependence of the clutch stiffness k_S on the frequency ω is suitable then the line of the dependence $\Omega = \Omega\,(\omega)$will not intersect any straight line (passing through the origin of the coordinate system $O(\omega, \Omega)$) corresponding to the harmonic components M_i (i= 0.5, 1, 1.5, ..., 6) in the frequency interval from ω_A to ω_B.

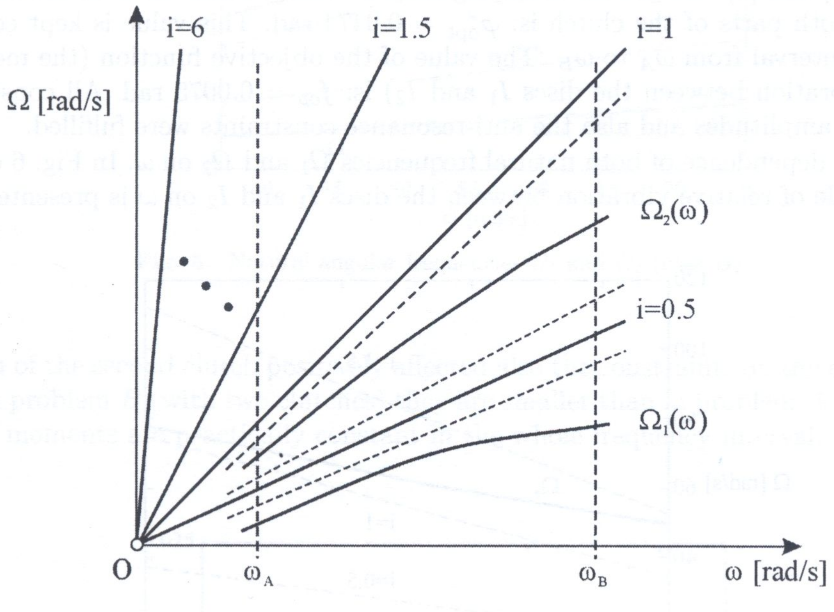

Fig. 3. Campbell diagram

These anti-resonance constraints will be applied for the same frequencies ω_j as the constraints on the dynamic moment amplitudes, equation (17). Inequalities (20a,b,c) involves safety intervals which prevent both natural frequencies $\Omega_1(\omega)$ and $\Omega_2(\omega)$ to lie too close to the straight lines corresponding to $i = 0.5$, 1 and 1.5, because for these cases resonances caused by corresponding harmonic components M_i could occur.

Both natural frequencies $\Omega_1(\omega)$ and $\Omega_2(\omega)$ are determined by Eq. (2), however the first two coefficients in Eq. (3) must be repeatidly computed for each ω_j. For problem A we have

$$b_{1j} = \frac{I_1\, I_2\, I_3}{k_{Sj}\; k}, \qquad b_{2j} = \frac{(I_2 + I_3)\; I_1}{k_{Sj}} + \frac{(I_1 + I_2)\; I_3}{k}, \qquad j = 1, 2, ..., M, \tag{21}$$

and for problem B

$$b_{1j} = \frac{I_1\, I_2\, I_3}{k_{Sj}\; k_j}, \qquad b_{2j} = \frac{(I_2 + I_3)\; I_1}{k_{Sj}} + \frac{(I_1 + I_2)\; I_3}{k_j}, \qquad j = 1, 2, ..., M. \tag{22}$$

5. Numerical solution

The following variables will be specified for both optimization problems: $\omega_A = 37$ rad/s (lower bound of the frequency interval), $\omega_B = 75$ rad/s (upper bound of the frequency interval), $I_1 = 26.6$ kgm^2, $I_2 = 8.1$ kgm^2, $I_3 = 92.5$ kgm^2, $a = 0.26$ Nms^2 (see Eq. (13) and (14b,c)), $M_D = 640$ Nm, $k = 12000$ Nm/rad (torsional stiffness of the shaft in problem A).

The harmonic components of the periodic exciting moment are: $i = 0.5$: $M_{05} = 48.5$ Nm, $i = 1$: $M_1 = 15.5$ Nm, $i = 1.5$: $M_{15} = 33.6$ Nm, $M_2 = 12.0$ Nm, $M_{25} = 24.1$ Nm, $M_3 = 2066.9$ Nm, $M_{35} = 16.5$ Nm, $M_4 = 5.9$ Nm, $M_{45} = 10.2$ Nm, $M_5 = 6.2$ Nm, $M_{55} = 11.5$ Nm, $M_6 = 931.9$ Nm. These moments correspond to an unbalanced vibration of individual engine cylinders.

5.1. Problem A

The Optimization Toolbox of Matlab [3] and also program GOOD developed at the TU Delft (The Netherlands) [4, 5, 6] were used to solve this optimization problem. Applications of the program GOOD (in the past based on the Monte Carlo method and at present on genetic algorithms) can be found e.g. in [7] and [8]. We got the following results: the optimum value of the relative angular displacement of both parts of the clutch is: $\varphi^*_{\mathrm{opt}} = 0.0474$ rad. This value is kept constant in the whole frequency interval from ω_A to ω_B. The value of the objective function (the mean amplitude of the relative vibration between the discs I_1 and I_2) is: $f_{op} = 0.0075$ rad. All constraints on the dynamic moment amplitudes and also the anti-resonance constraints were fulfilled.

Figure 4 shows dependence of both natural frequencies Ω_1 and Ω_2 on ω. In Fig. 6 dependence of the total amplitude of relative vibration between the discs I_1 and I_2 on ω is presented (curve A).

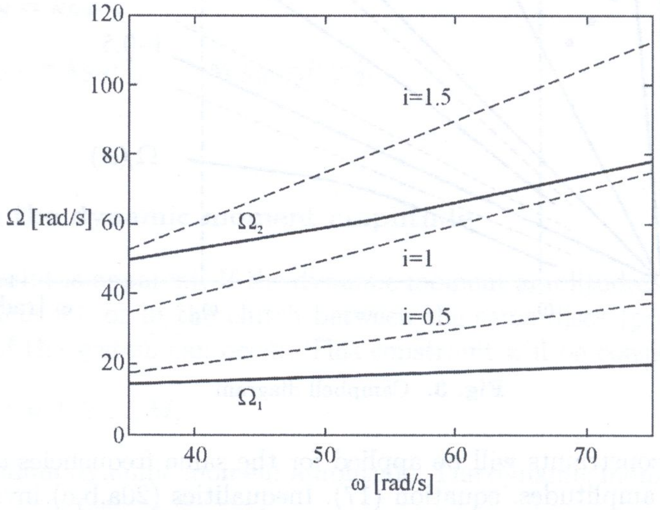

Fig. 4. Natural angular frequencies Ω_1 and Ω_2 (case A)

5.2. Problem B

This problem is characterized by using two pneumatic clutches. Now we have two design variables: φ^*_1 and φ^*_2. They both will be optimized. Again Optimization Toolbox of Matlab and also program GOOD were used and the final results of optimization process are: $\varphi^*_{1,\mathrm{opt}} = 0.0347$ rad and $\varphi^*_{2,\mathrm{opt}} = 0.1396$ rad. The value of the objective function is: $f_{op} = 0.0054$ rad. As expected, by application of the second clutch reduction of the mean total relative vibration amplitude between the discs I_1 and I_2 was reached (by 28 %).

In Fig. 5 dependence of both natural frequencies Ω_1 and Ω_2 on ω is graphically shown. It is remarkable that in this case both natural frequencies $\Omega_1(\omega)$ and $\Omega_2(\omega)$ are approximately linear and passing through the origin of the Campbell diagram together with the straight lines corresponding to $i = 0.5$, 1 and 1.5 (compare with the results of problem A, where this dependence is not linear and $\Omega_1(\omega)$, $\Omega_2(\omega)$ are not passing through the origin of the Campbell diagram). It means that application of the second clutch enables easier optimization of the system (and better results). However, we should note that these results are valid only for the loading moment quadratically dependent on ω.

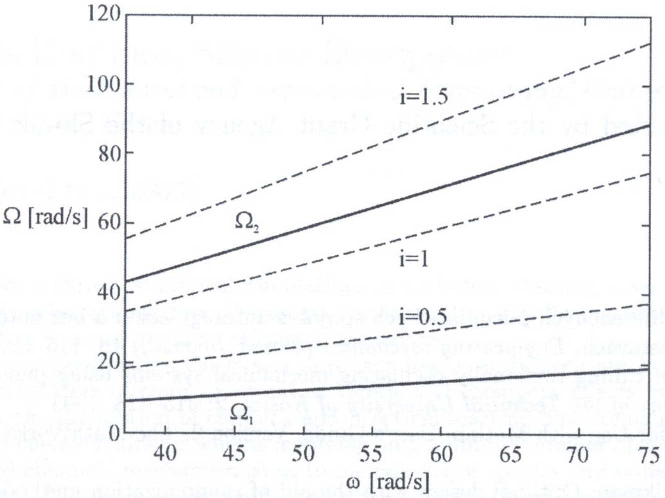

Fig. 5. Natural angular frequencies Ω_1 and Ω_2 (case B)

Application of the second clutch positively affected also the constraints on the dynamic moments amplitudes. In problem B (with two clutches) they are smaller than in problem A (with one clutch). Besides, these moments are practically constant in the whole frequency interval.

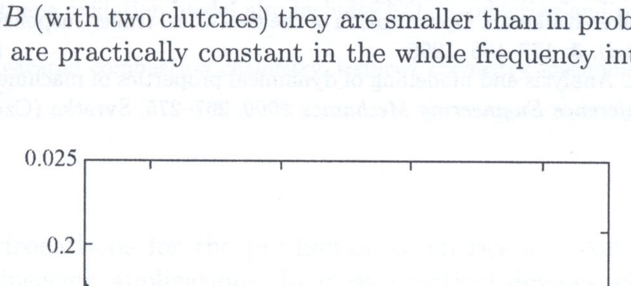

Fig. 6. Amplitudes of relative vibration (cases A and B)

In Fig. 6 dependence of the total amplitude of relative vibration between the discs I_1 and I_2 on ω is again shown (curve B). This figure shows that application of the second clutch positively affects the amplitudes of relative vibration between the discs I_1 and I_2 for lower frequencies.

6. Conclusion

In this paper optimization and comparison of two torsionally vibrating mechanical systems (with one or two clutches) was presented. It was shown that differential pneumatic clutches with self-regulation enable not only minimization of vibration in such systems, but they also enable to avoid resonance regimes in relatively wide frequency intervals. All this is possible thanks to the fact that stiffness of the pneumatic clutch can be controlled by the pressure of a gas medium in it. Comparison of the results showed advantages of using two pneumatic clutches in three-mass torsionally vibrating mechanical systems.

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